Resonant Tunneling, Quantum Brownian Motion, and Multichannel Kondo Problem

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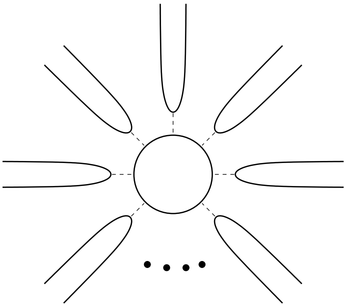
[H. Yi, PRB **65**, 195101 (2002)]

Resonant Tenneling

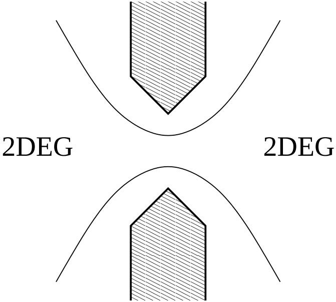
- Multilead Quantum Dot



- Coulomb blockade $\to$ conducting near resonance
- Strongly interacting $\to$ difficult to solve!!

  $\to$ bosonization, CFT, RG, Bethe ansatz, MC, $\cdots$

- Quantum Point Contact

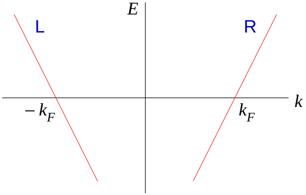


Single transverse mode (transmission probability $T \leq 1$) $\rightarrow$ 1DEG!

Bosonization of 1DEG

[J. von Delft and H. Schoeller, Am. J. Phys. **64**, 1968 (1996), cond-mat/9805275]

- Linear dispersion near Fermi points



Single electron energy:

$$v_F = \frac{1}{\hbar}\frac{d\mathcal{E}}{dk} \implies \mathcal{E} = \hbar v_F(k - k_F)$$

- Hamiltonian of a *spinless* 1DEG ($\hbar = 1$)

$$H = -iv_F \int dx : \left(\psi_+^\dagger \partial_x \psi_+ - \psi_-^\dagger \partial_x \psi_-\right) :$$

$$+ \int dx : \left[g_2 \psi_+^\dagger \psi_-^\dagger \psi_- \psi_+ + \frac{g_4}{2}\left(\psi_+^\dagger \psi_+^\dagger \psi_+ \psi_+ + \psi_-^\dagger \psi_-^\dagger \psi_- \psi_-\right)\right] :$$

- Hamiltonian (continued)

Fourier transform: $\psi_\pm(x) = \frac{1}{\sqrt{L}} \sum_k e^{ikx} c_\pm(k)$

$$
\begin{aligned}
H = v_F \sum_k k : \Big[c_+^\dagger(k) c_+(k) - c_-^\dagger(k) c_-(k) \Big] : \\
+ \frac{1}{L} \sum_{kpq} : \Big\{ g_2 c_+^\dagger(k+q) c_-^\dagger(p-q) c_-(p) c_+(k) \\
+ \frac{g_4}{2} \Big[c_+^\dagger(k+q) c_+^\dagger(p-q) c_+(p) c_+(k) \\
+ c_-^\dagger(k+q) c_-^\dagger(p-q) c_-(p) c_-(k) \Big] \Big\} :
\end{aligned}
$$

- Hamiltonian (continued)

$$\rho_{\pm}(q) \equiv \int dx\, e^{-iqx} : \psi_{\pm}^{\dagger}(x)\psi_{\pm}(x) : \; = \sum_{k} : \left[c_{\pm}^{\dagger}(k-q) c_{\pm}(k) \right] :$$

$$\left[\rho_{\pm}(q), \rho_{\pm}^{\dagger}(q') \right] = \pm \delta_{qq'} \frac{qL}{2\pi}, \quad \left[\rho_{+}(q), \rho_{-}^{\dagger}(q') \right] = 0$$

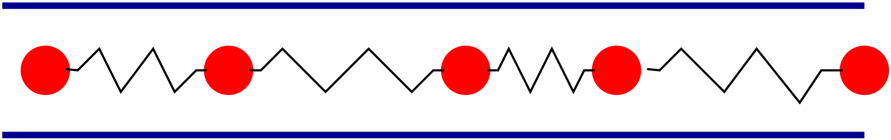
$$H = \frac{2\pi v_F + 2g_4}{L} \sum_{q} : \left[\rho_{+}(q)\rho_{+}(-q) + \rho_{-}(-q)\rho_{-}(q) \right] : + \frac{2g_2}{L} \sum_{q} : \rho_{+}(q)\rho_{-}(-q) :$$

Bosonization of 1DEG

- Charge density wave: $\rho_\pm = \dfrac{\partial_x \phi_\pm}{2\pi}$

$$\phi_\pm(x) = 2\pi \times \text{ spatial } \text{displacement} \text{ of electron at position } x$$

$$= \text{CDW phase}$$



$$\left[\phi_\pm(x), \partial_x \phi_\pm(x')\right] = \pm 2\pi \delta(x - x')$$

$$\psi_\pm(x) \propto F_\pm e^{\pm i\phi_\pm(x)} \ \cdots \text{ kink annihilation operator}$$

(vertex operator in CFT)

- Hamiltonian in terms of CDW

$$\theta = \phi_+ + \phi_- = \text{CDW phase}, \quad \phi = \phi_+ - \phi_- = \text{Josephson phase}$$

$$\Longrightarrow \frac{\partial_x \theta}{2\pi} = \text{charge density}, \quad \frac{\partial_x \phi}{2\pi} = \text{current density}$$

$$H = \frac{v}{8\pi} \int dx \left[g \left(\partial_x \phi\right)^2 + \frac{1}{g} \left(\partial_x \theta\right)^2 \right] = 2\pi v \int dx \left[g \Pi_\theta^2 + \frac{1}{g} \left(\frac{\partial_x \theta}{4\pi} \right)^2 \right]$$

$$v = \frac{1}{\pi} \sqrt{\left(\pi v_F + g_4\right)^2 - g_2^2}, \quad g = \sqrt{\frac{\pi v_F + g_4 - g_2}{\pi v_F + g_4 + g_2}}$$

repulsive $\rightarrow \; g_2 > 0 \; \rightarrow \; g < 1$, attractive $\rightarrow \; g_2 < 0 \; \rightarrow \; g > 1$

$$[\theta(x'), \partial_x \phi(x)] = i 4\pi \delta(x - x') \Longrightarrow \Pi_\theta = \frac{\partial_x \phi}{4\pi}$$

$\longrightarrow$ conjugation relation between number and phase!

- Bosonized action

$$\partial_t\theta = i[\theta, H] = 4\pi v g \Pi$$

Euclidean action: $S = \dfrac{1}{8\pi v g}\displaystyle\int d\tau dx \left[(\partial_\tau\theta)^2 + (v\partial_x\theta)^2\right]$

- Dual theory: $[\phi(x'), \partial_x\theta(x)] = i4\pi\delta(x - x') \Longrightarrow \Pi_\phi = \dfrac{\partial_x\theta}{4\pi}$

$$H = 2\pi v \int dx \left[\frac{1}{g}\Pi_\phi^2 + g\left(\frac{\partial_x\phi}{4\pi}\right)^2\right]$$

$$S = \frac{g}{8\pi v}\int d\tau dx \left[(\partial_\tau\phi)^2 + (v\partial_x\phi)^2\right]$$

- Multilead quantum dot action

$$S = S_0 + S_v + S_C$$

$$S_0 = \sum_{a=1}^{N} \frac{1}{8\pi v_F} \int d\tau dx \left[(\partial_\tau \theta_a)^2 + (v_F \partial_x \theta_a)^2 \right]$$

$$S_v = \sum_{a=1}^{N} \int \frac{d\tau}{\tau_c} v \cos\left[2\pi Q_a(\tau)\right], \quad \left(Q_a = \int_0^\infty dx \, \frac{\partial \theta(x)}{2\pi} = -\frac{\theta(x=0)}{2\pi} \right)$$

$$S_C = \frac{e^2}{2C} \int d\tau \left[-\sum_{a=1}^{N} Q_a(\tau) - n_0 \right]^2$$

Mapping I: Quantum Brownian Motion

- Effective action: Caldeira-Legget model of QBM in a periodic potention

$$S'_{\text{eff}}[\{Q_a\}] = \sum_{a=1}^{N} \left\{ \frac{1}{2} \int d\omega \, |\omega| |Q_a(\omega)|^2 - v \int \frac{d\tau}{\tau_c} \cos 2\pi Q_a(\tau) \right\}$$

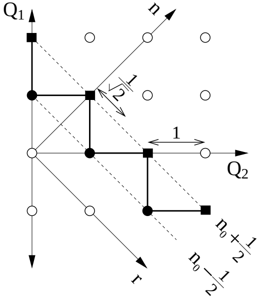
$$+ \frac{e^2}{2C} \int d\tau \left\{ - \left[\sum_{a=1}^{N} Q_a(\tau) \right] - n_0 \right\}^2$$

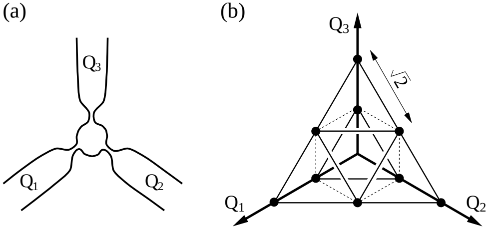
$T \ll e^2/C \implies$ Total charge in the dot ($\sum_a Q_a$) freezes out.

$\longrightarrow$ Coulomb blockade unless $n_0 =$ half integer

Mapping I: Quantum Brownian Motion

- Lattice models





Mapping I: Quantum Brownian Motion

- Lattice models (continued)

$$S = \frac{1}{2}\int d\omega\, |\omega| e^{|\omega|\tau_c} |\mathbf{r}(\omega)|^2 - \int \frac{d\tau}{\tau_c} \sum_{\mathbf{G}} v_{\mathbf{G}} e^{i2\pi \mathbf{G}\cdot\mathbf{r}(\tau)}$$

($\mathbf{G}$ = reciprocal lattice vector)

$$S = \frac{1}{2}\int d\omega\, |\omega| e^{|\omega|\tau_c} |\mathbf{k}(\omega)|^2 - \int \frac{d\tau}{\tau_c} \sum_{\mathbf{R}} t_{\mathbf{R}} e^{i2\pi \mathbf{R}\cdot\mathbf{k}(\tau)}$$

($\mathbf{R}$ = direct lattice vector, $\mathbf{G}\cdot\mathbf{R}$ = integer)

- Perturbative RG flow equations (small $v_{\mathbf{G}}$ or $t_{\mathbf{R}}$)

$$\frac{dv_{\mathbf{G}}}{d\ell} = \left(1 - |\mathbf{G}|^2\right) v_{\mathbf{G}} + \mathcal{O}\left(v^2\right), \qquad \frac{dt_{\mathbf{R}}}{d\ell} = \left(1 - |\mathbf{R}|^2\right) t_{\mathbf{R}} + \mathcal{O}\left(t^2\right)$$

- Linear response and mobility

uniform external force $\mathbf{F} \longrightarrow S_{\mathbf{F}} = -\int d\tau \mathbf{F} \cdot \mathbf{r}(\tau)$

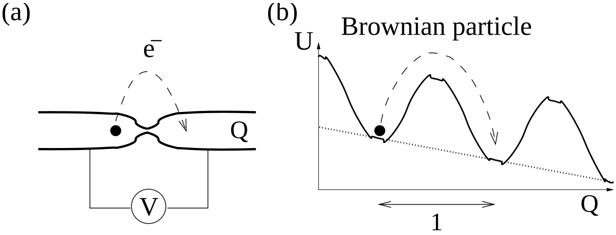
$$\mu_{ij} \equiv \lim_{\mathbf{F}\to 0} \frac{\partial}{\partial F_j} \langle \partial_t r_i \rangle = \lim_{\omega \to 0} \frac{1}{|\omega|} \int d\omega' \, \omega\omega' \langle r_i(\omega) r_j(-\omega') \rangle = \mu \delta_{ij}$$

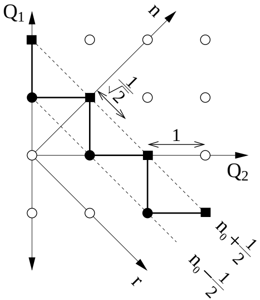
$$= \delta_{ij} - \lim_{\omega \to 0} \frac{1}{|\omega|} \int d\omega' \, \omega\omega' \langle k_i(\omega) k_j(-\omega') \rangle$$

$$\begin{cases} \mu = 0, & \text{if } t_{\mathbf{R}} = 0 \\ \mu = 1, & \text{if } v_{\mathrm{G}} = 0 \\ 0 < \mu < 1, & \text{otherwise} \end{cases}$$

Mapping I: Quantum Brownian Motion

- 1D lattice (2 leads with or without a quantum dot)

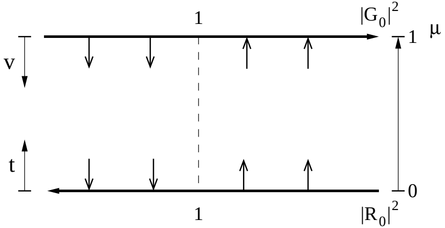




- 1D lattice (continued)

$$R = mR_0 = m/G_0, \quad G = nG_0 = n/R_0$$

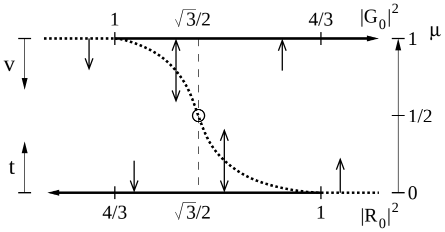
$$\frac{dv_G}{d\ell} = (1 - n^2 G_0^2) v_G, \qquad \frac{dt_R}{d\ell} = (1 - m^2/G_0^2) t_R, \qquad (m, n \neq 0)$$



Fixed point mobility: $\mu^* = \begin{cases} 0 & \text{if } G_0 < 1, \\ 1 & \text{if } G_0 > 1. \end{cases}$

- 2D triangular lattice (3 leads, off resonance)

  Simplification: symmetric contacts $t_{\mathbf{R}} = t$, for all $\mathbf{R}$

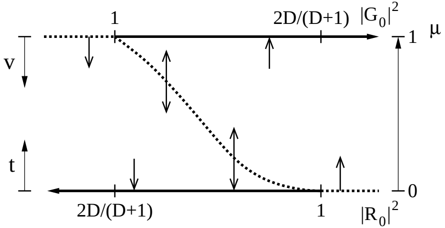


Unstable intermediate fixed point for $1 < |\mathbf{G}_0|^2 < 4/3$!

Fixed point mobility: $\mu^* = \begin{cases} 0 & \text{if } |\mathbf{G}_0|^2 < 1, \\ 0 \text{ or } 1 & \text{if } 1 < |\mathbf{G}_0|^2 < 4/3, \\ 1 & \text{if } |\mathbf{G}_0|^2 > 4/3. \end{cases}$

- D-D "hypertriangular" lattice ($D+1$ leads, off resonance)

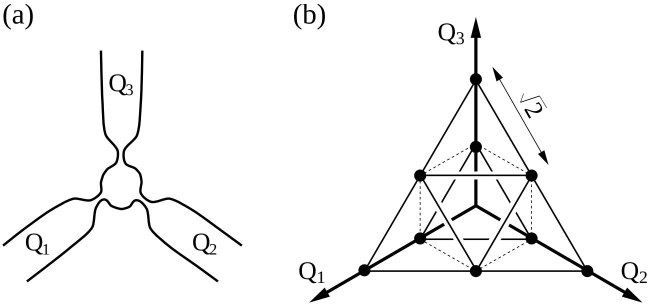
  (eg: 4 leads $\rightarrow$ $D=3$ $\rightarrow$ tetrahedron)



Fixed point mobility: $$\mu^* = \begin{cases} 0 & \text{if } |\mathbf{G}_0|^2 < 1, \\ 0 \text{ or } 1 & \text{if } 1 < |\mathbf{G}_0|^2 < 2D/(D+1), \\ 1 & \text{if } |\mathbf{G}_0|^2 > 2D/(D+1). \end{cases}$$

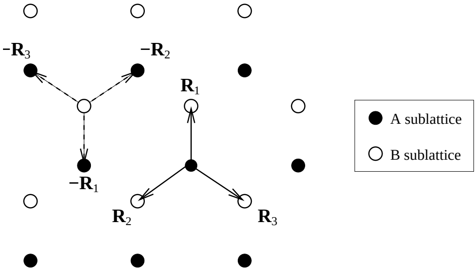
Mapping I: Quantum Brownian Motion

- 2D honeycomb lattice (3 leads, on resonance)



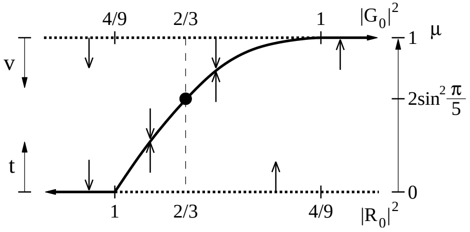
nonsymmorphic lattice!

- 2D honeycomb lattice (continued)



$$S_t = -t \int \frac{d\tau}{\tau_c} \sum_a \left(\frac{\sigma^+}{2} e^{i2\pi \mathbf{R}_a \cdot \mathbf{k}} + \frac{\sigma^-}{2} e^{-i2\pi \mathbf{R}_a \cdot \mathbf{k}} \right)$$

- 2D honeycomb lattice (continued)



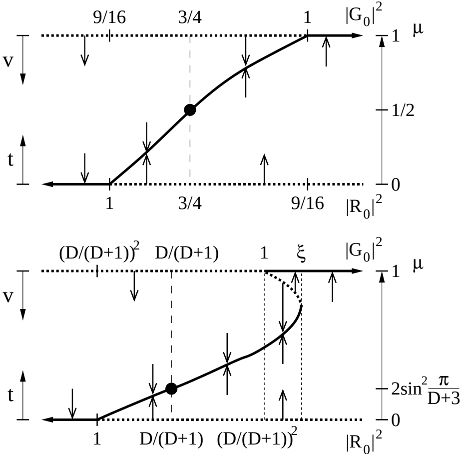
Stable intermediate fixed point!

Fixed point mobility:
$$\begin{cases} \mu^* = 0 & \text{if } |\mathbf{G}_0|^2 < 4/9, \\ 0 < \mu^* < 1 & \text{if } 4/9 < |\mathbf{G}_0|^2 < 1, \\ \mu^* = 1 & \text{if } |\mathbf{G}_0|^2 > 1. \end{cases}$$

Mapping I: Quantum Brownian Motion

- D-D "hyperhoneycomb" lattice ($D+1$ leads, on resonance)

$$S_t = -t \int \frac{d\tau}{\tau_c} \sum_{a=1}^{D+1} \left(\frac{\sigma^+}{2} e^{i2\pi \mathbf{R}_a \cdot \mathbf{k}} + H.c. \right)$$



- D-D "hyperhoneycomb" lattice (continued)

  Fixed point mobility:

$$
\begin{cases}
\mu^* = 0 & \text{if } |\mathbf{G}_0|^2 < D^2/(D+1)^2, \\
0 < \mu^* < 1 & \text{if } D^2/(D+1)^2 < |\mathbf{G}_0|^2 < 1, \\
0 < \mu^* < 1 \text{ or } \mu^* = 1 & \text{if } 1 < |\mathbf{G}_0|^2 < \xi, \\
\mu^* = 1 & \text{if } |\mathbf{G}_0|^2 > \xi.
\end{cases}
$$

Mapping II: Multichannel Kondo Problem

- Hamiltonian

$$H = H_0 + H_J$$

$$H_0 = i v_F \sum_{a,s} \int dx\, \psi^\dagger_{as} \partial_x \psi_{as}$$

$$H_J = 2\pi v_F \sum_a \left\{ J_z S^z_{\text{imp}} s^z_a(0) + \frac{1}{2} J_\perp \left[S^+_{\text{imp}} s^-_a(0) + \text{H.c.} \right] \right\}$$

Mapping II: Multichannel Kondo Problem

- Boson Hamiltonian

$$H' = \frac{v_F}{8\pi}\int dx\left[(\partial_x\phi^{\mathrm{s}})^2 + \sum_{i=1}^{N-1}(\partial_x\phi_i^{\mathrm{sf}})^2\right]$$

$$+\ \frac{J_\perp}{2\tau_c}\sum_{a=1}^{N}\left(S_{\mathrm{imp}}^{+}\exp\left\{-i\left[\frac{1}{\sqrt{N}}\left(1-\frac{N}{2}J_z\right)\phi^{\mathrm{s}}(0)+\ \sum_{i=1}^{N-1}\mathrm{O}_{ai}^{-1}\phi_i^{\mathrm{sf}}(0)\right]\right\}\right.$$

$$\left. +\ \mathrm{H.c.}\right)$$

- Generalized Toulouse limit: $1-\dfrac{N}{2}J_z = 0 \ \longrightarrow\ \phi^s$ is decoupled.

Mapping II: Multichannel Kondo Problem

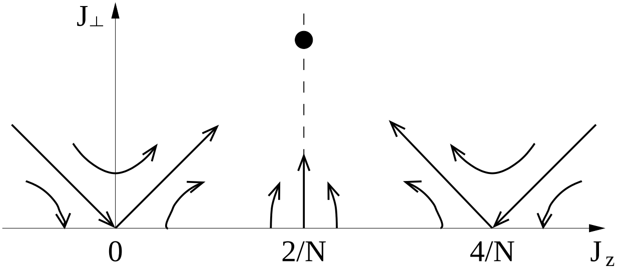
- Euclidean action

$$S_{\text{Kondo}} = \frac{1}{8\pi^2} \int d\omega\, |\omega| \sum_{i=1}^{N} |\phi_i^{\text{sf}}(\omega)|^2$$

$$+ \frac{J_\perp}{2} \int \frac{d\tau}{\tau_c} \sum_{a=1}^{N} \left\{ S_{\text{imp}}^{+} \exp\left[-i \sum_{i=1}^{N-1} \mathrm{O}_{ai}^{-1} \phi_i^{\text{sf}}(\tau) \right] + \text{c.c.} \right\}$$

$\rightarrow$ mapped to quantum Brownian motion and on-resonance tunneling!

- RG flow



Mapping II: Multichannel Kondo Problem

- Fixed point mobility of on-resonance tunneling

  From spin current correlation functions obtained from CFT,

  [A.W.W. Ludwig and I. Affleck, NPB **428**, 545 (1994); PRB **48**, 7297 (1993)]

$$\mu^*_{\text{on resonance}} = 2\sin^2\frac{\pi}{N+2}$$

- On-resonance conductance: $G = \frac{N\mu}{2}\frac{e^2}{h}$

- Universal resonance lineshape: $G(\delta n_0, T) = \tilde{G}\left(\frac{\delta n_0}{T^{1-\Delta_{\mathcal{H}}}}\right)$

Summary

- Mappings between resonant tunneling, QBM, and multichannel Kondo problem are explicitly shown. (Same universality class is shared by seemingly different models.)
- Nonperturbative results of one model may be obtained from known properties of another model.
- It helps to get familiar with many different physical models and systems!