SEOUL NATIONAL UNIVERSITY – SCHOOL OF PHYSICS

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SPRING SEMESTER 2004

Solid State Physics II

Chapter 6 Introduction to Superconductivity

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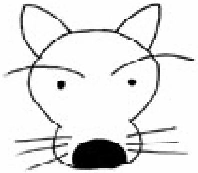
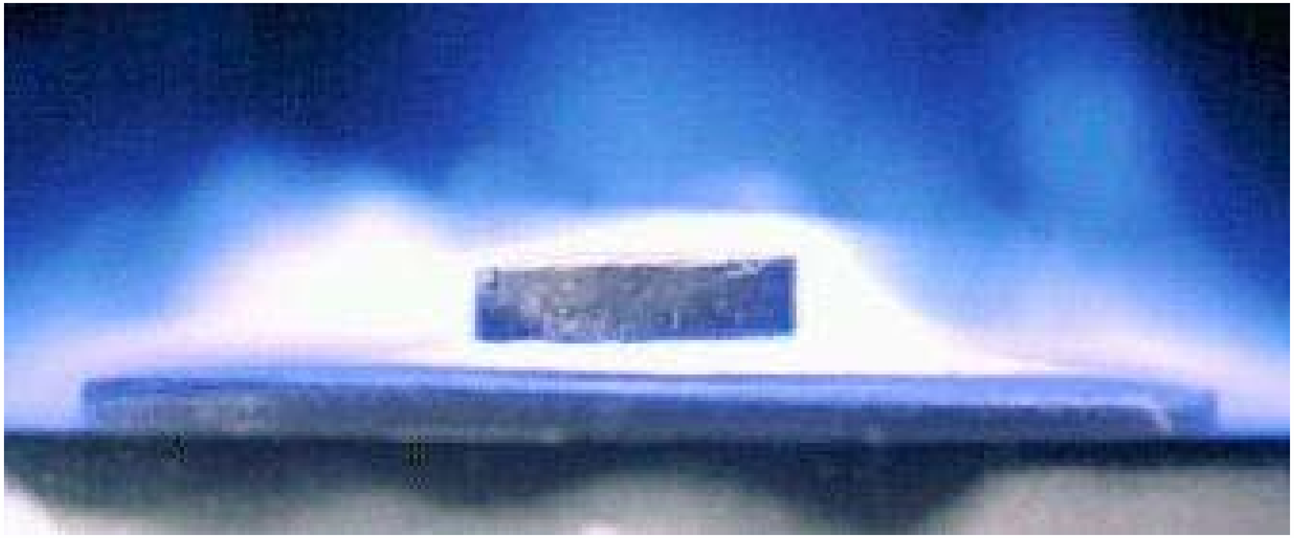


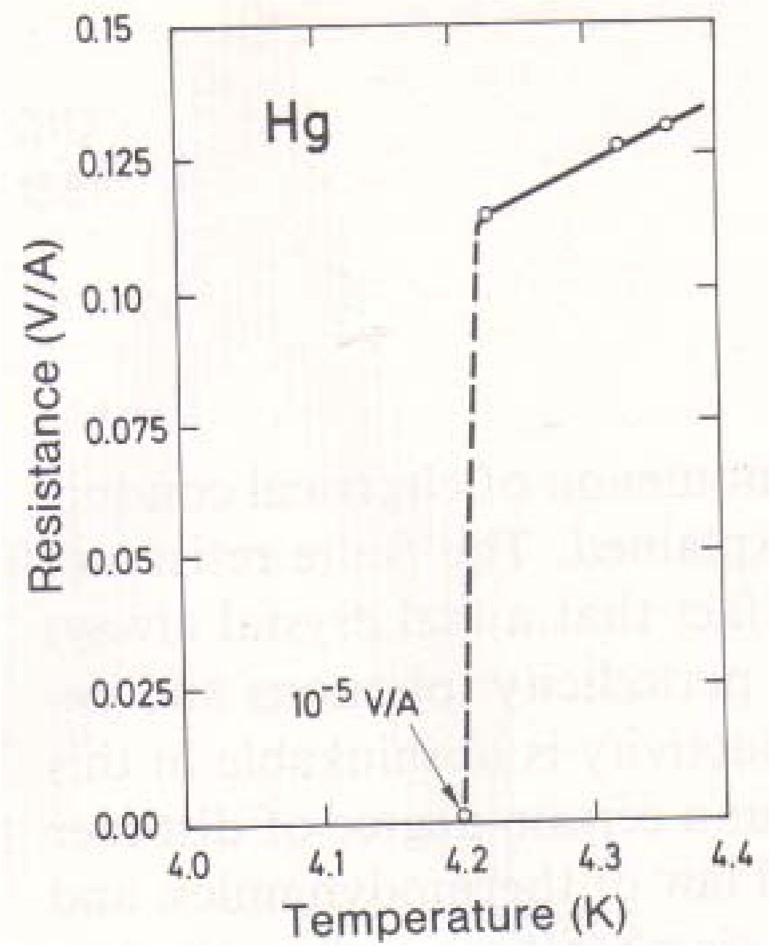
Levitating magnetic bar on a superconducting bed



A Brief Overview on the BCS Theory of Superconductivity

- The basic phenomena
- Cooper pairs
- Origin of the attractive interaction
- The BCS ground state and the superconducting gap
- Thermodynamic properties
- Tunneling
- Penetration depth
- Josephson effects

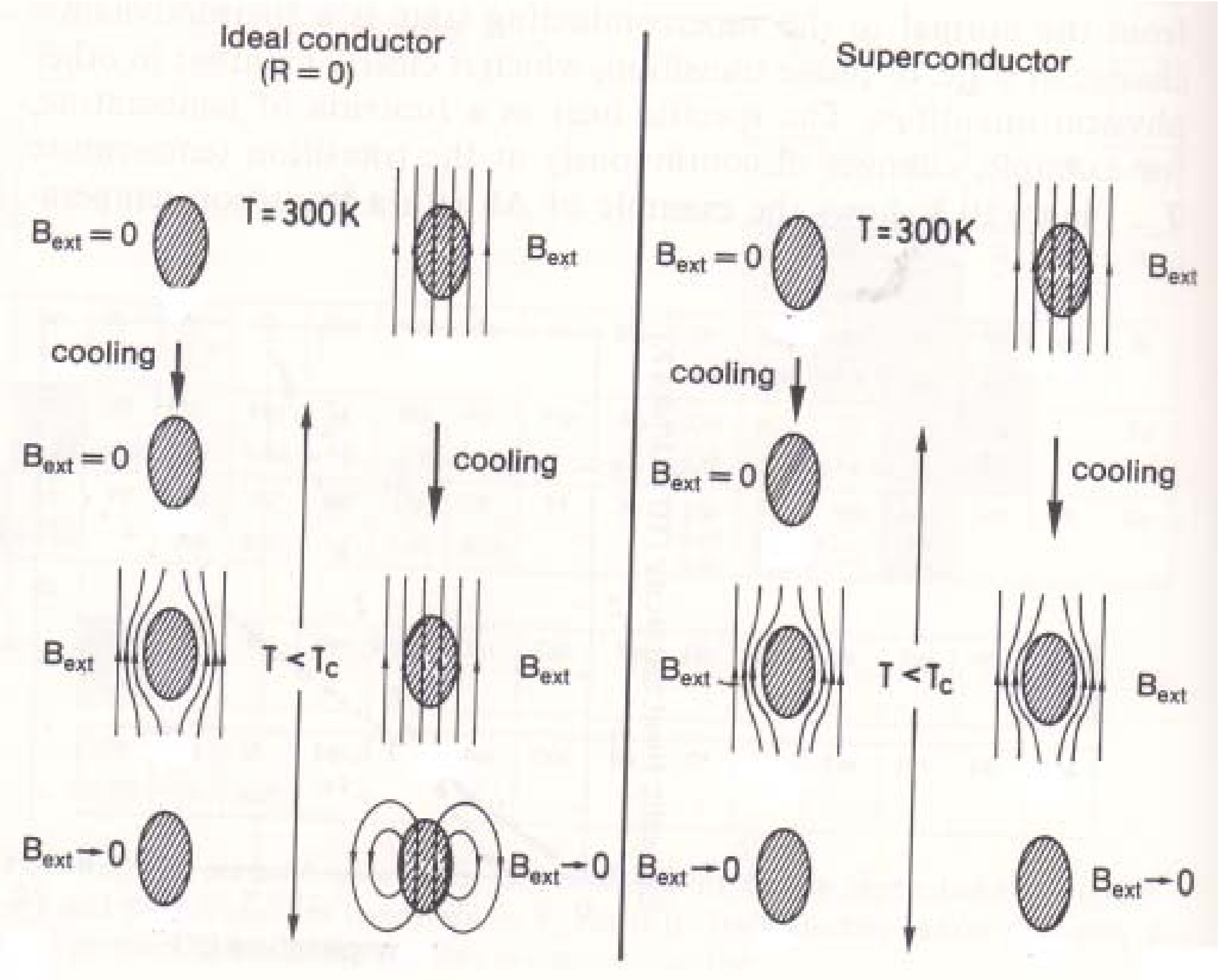
Zero resistivity



H																	He
Li	Be 0.03											B	C	N	O	F	Ne
Na	Mg											Al 1.19	Si 6.7	P 4.6-6.1	S	Cl	Ar
K	Ca	Sc	Ti 0.39	V 5.3	Cr	Mn	Fe	Co	Ni	Cu	Zn 0.9	Ga 1.09	Ge 5.4	As 0.5	Se 6.9	Br	Kr
Rb	Sr	Y 0.5-2.7	Zr 0.55	Nb 9.2	Mo 0.92	Tc 7.8	Ru 0.5	Rh 325 μ	Pd	Ag	Cd 0.55	In 3.4	Sn 3.7;5.3	Sb 3.6	Te 4.5	I	Xe
Cs 1.5	Ba 1.8;5.1	La 4.8;5.9	Hf	Ta 4.4	W 0.01	Re 1.7	Os 0.65	Ir 0.14	Pt	Au	Hg 4.15 3.95	Tl 2.39 1.45	Pb 7.2	Bi 3.9 7.2;8.5	Po	At	Rn
Fr	Ra	Ac															

Ce 1.7	Pr	Nd	Pm	Sm	Eu	Gd	Tb	Dy	Ho	Er	Tm	Yb	Lu 0.1-0.7
Th 1.37	Pa 1.3	U 0.2	Np	Pu	Am	Cm	Bk	Cf	Es	Fm	Md	No	Lw

Perfect conductor vs. Superconductor



Conductor

$$\mathbf{E} = 0$$

$$\mathbf{J} = \sigma \mathbf{E}$$

; Ohm's law

Superconductor

$$\mathbf{B} = \nabla \times \mathbf{A} = 0$$

$$\mathbf{J} = -\frac{1}{\Lambda c}\mathbf{A}$$

; London equation

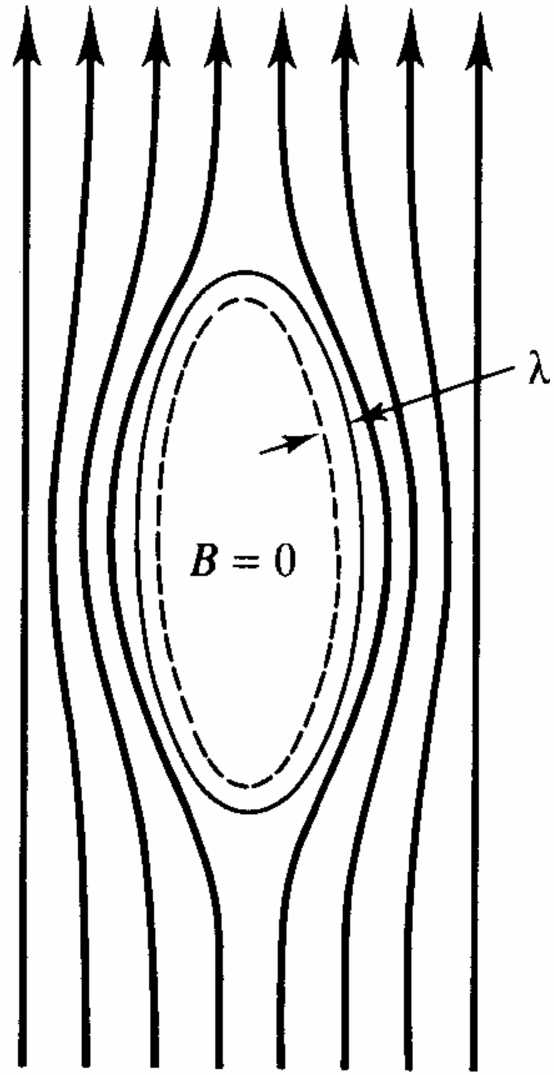
Meissner effect

$$\vec{B} = 0$$

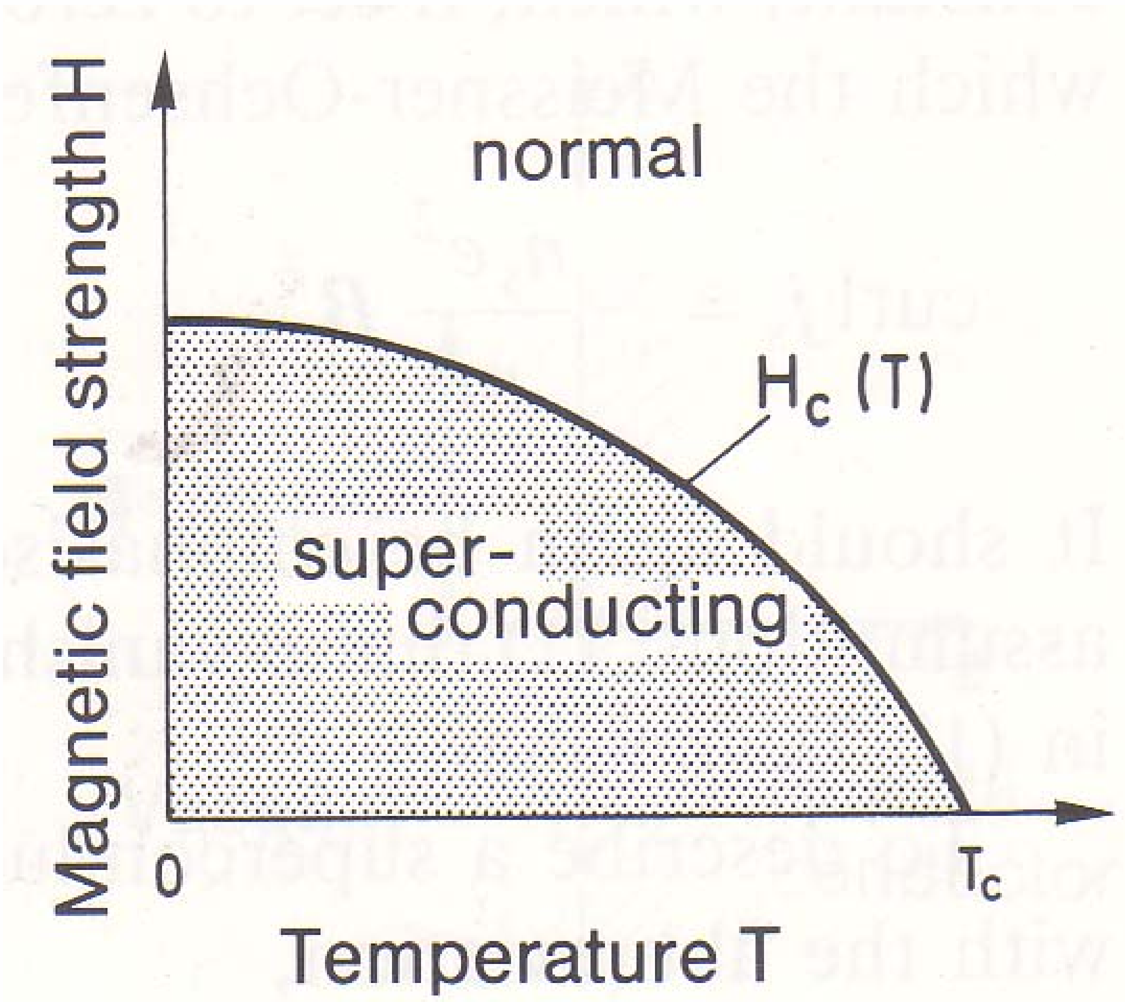
London equation

$$\vec{J}_s = -\frac{1}{\Lambda c}\vec{A}$$

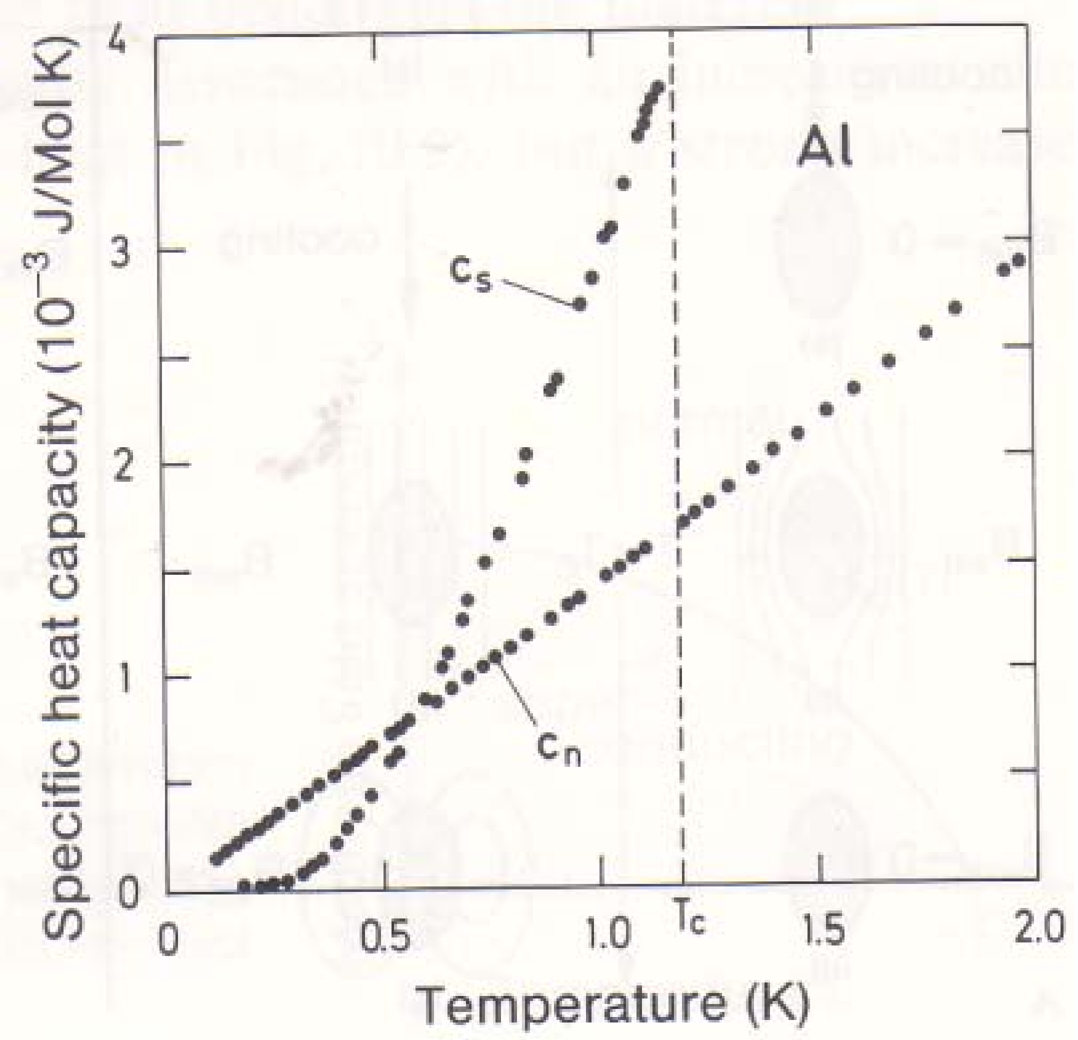
$$\Lambda = \frac{4\pi\lambda^2}{c^2} = \frac{m}{n_s e^2}$$



Critical Magnetic Field $H_c(T)$



Heat Capacity of Normal and Superconducting Al

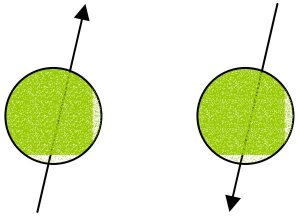


Cooper Pairs

- Macroscopic quantum state $\psi(\vec{x}) = e^{i\varphi} \,|\, \psi \,|$

$$\vec{J}_s = \frac{e^{*}\hbar}{i2m^{*}}(\psi^{*}\nabla\psi - \psi\nabla\psi^{*}) - \frac{e^{*2}}{m^{*}c}|\psi|^{2}\vec{A}$$

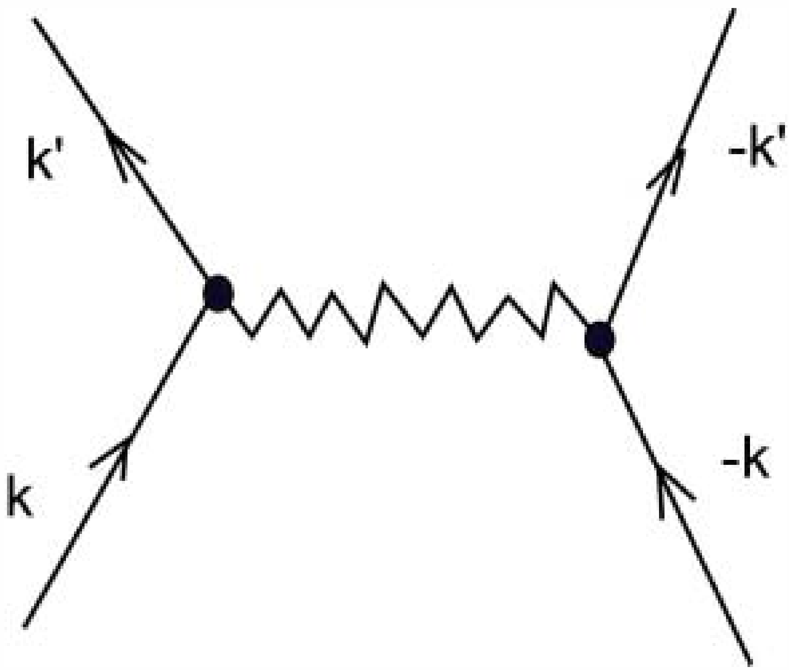
- Boson vs. Fermion



$$\left|\psi_o\right\rangle = \sum_{k>k_F} g_k c^{+}_{k\uparrow} c^{+}_{-k\downarrow} \left|O\right\rangle$$

- Coherent state: semi-classical quantum state

Origin of Pairing -- electron-phonon interactions



$$V_{eff} = \frac{\left| v_{kq} \right|^2}{\omega - \omega_q}$$

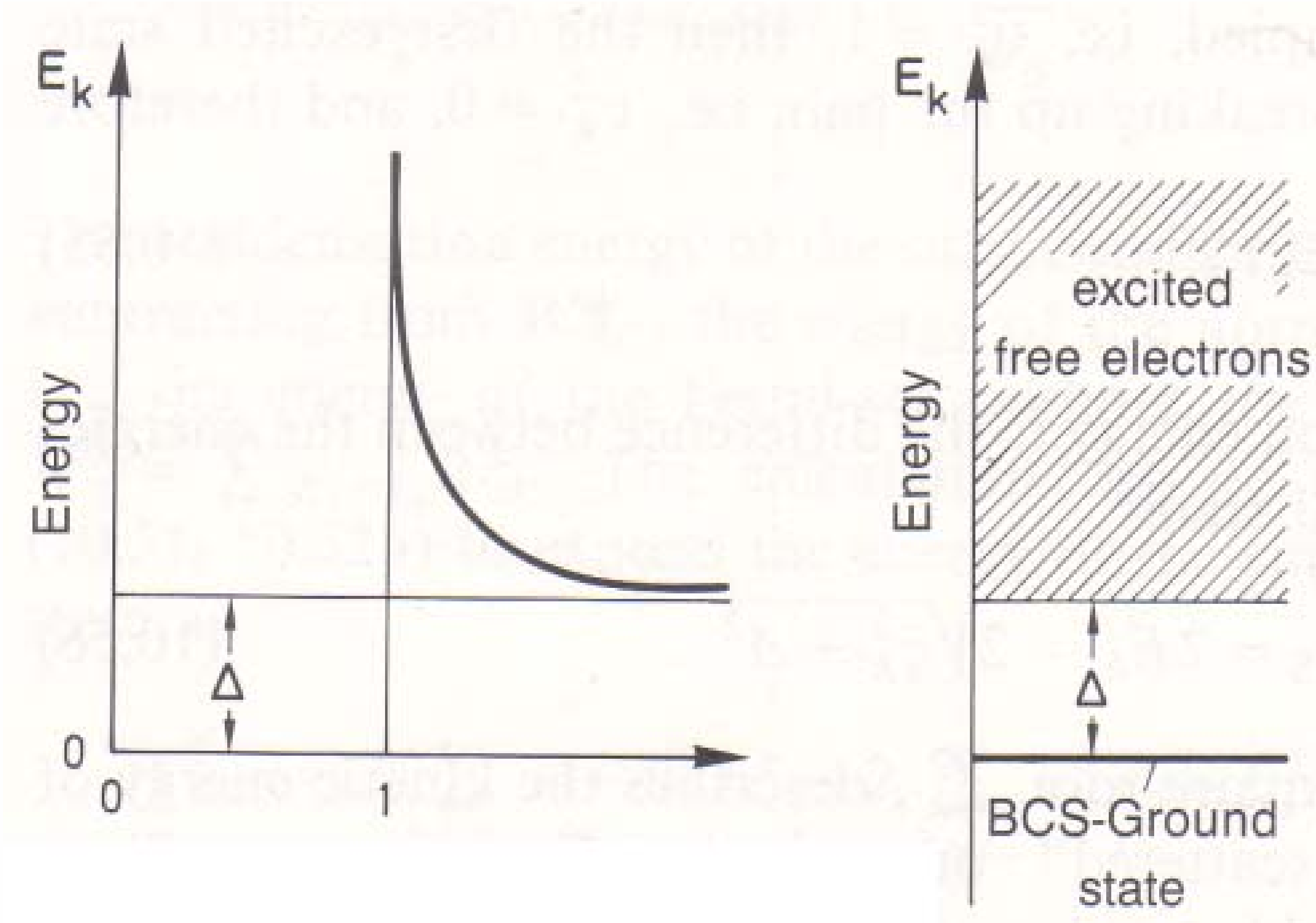
Pairing Potential --- Energy Gap

$$\Delta \approx 2\omega_o e^{-1/N(0)V}$$

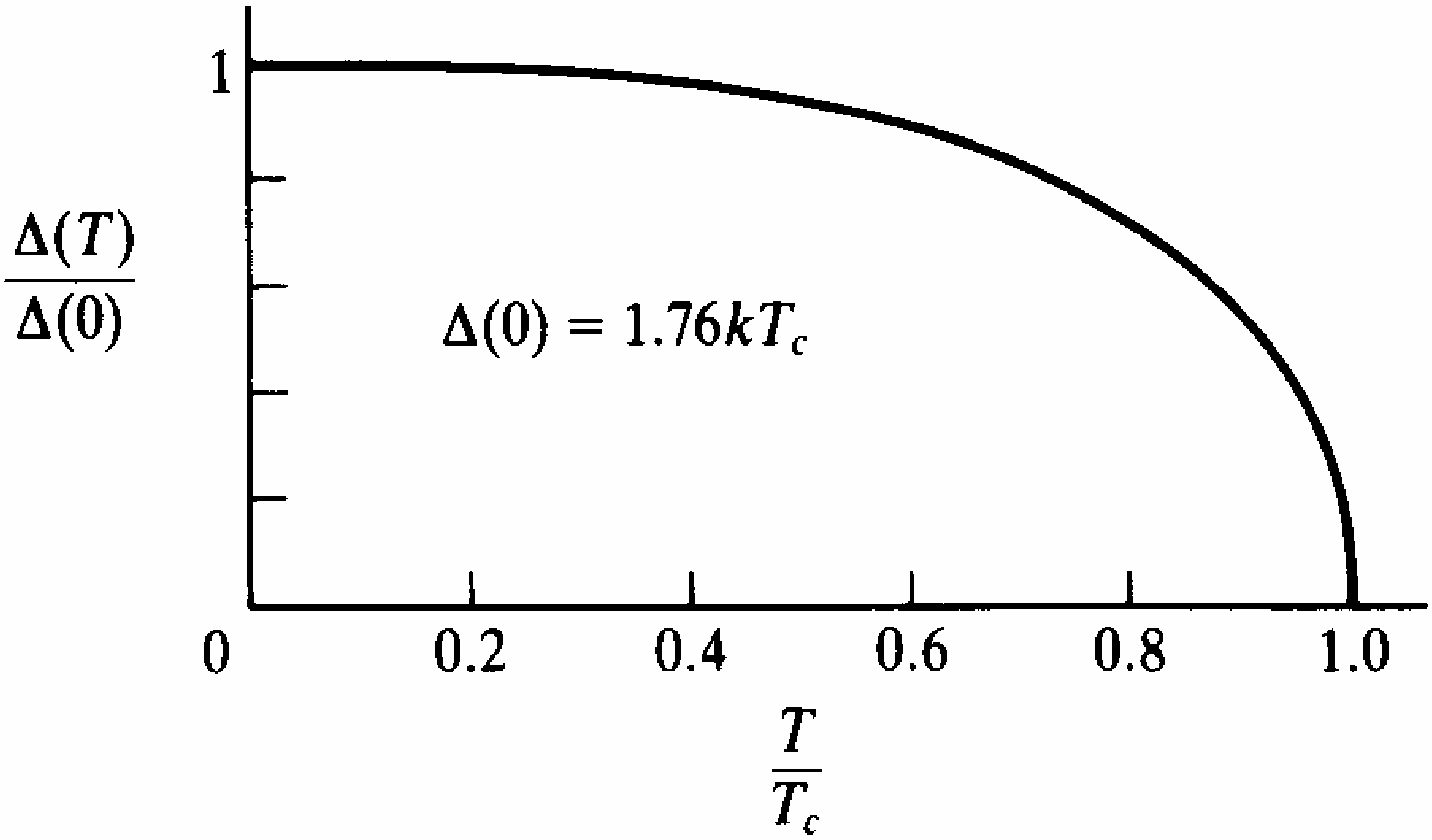
$$\frac{2\Delta}{k_B T_c} = 3.52$$

- Isotope effect: $\omega_o \sim 1/M^{1/2}$

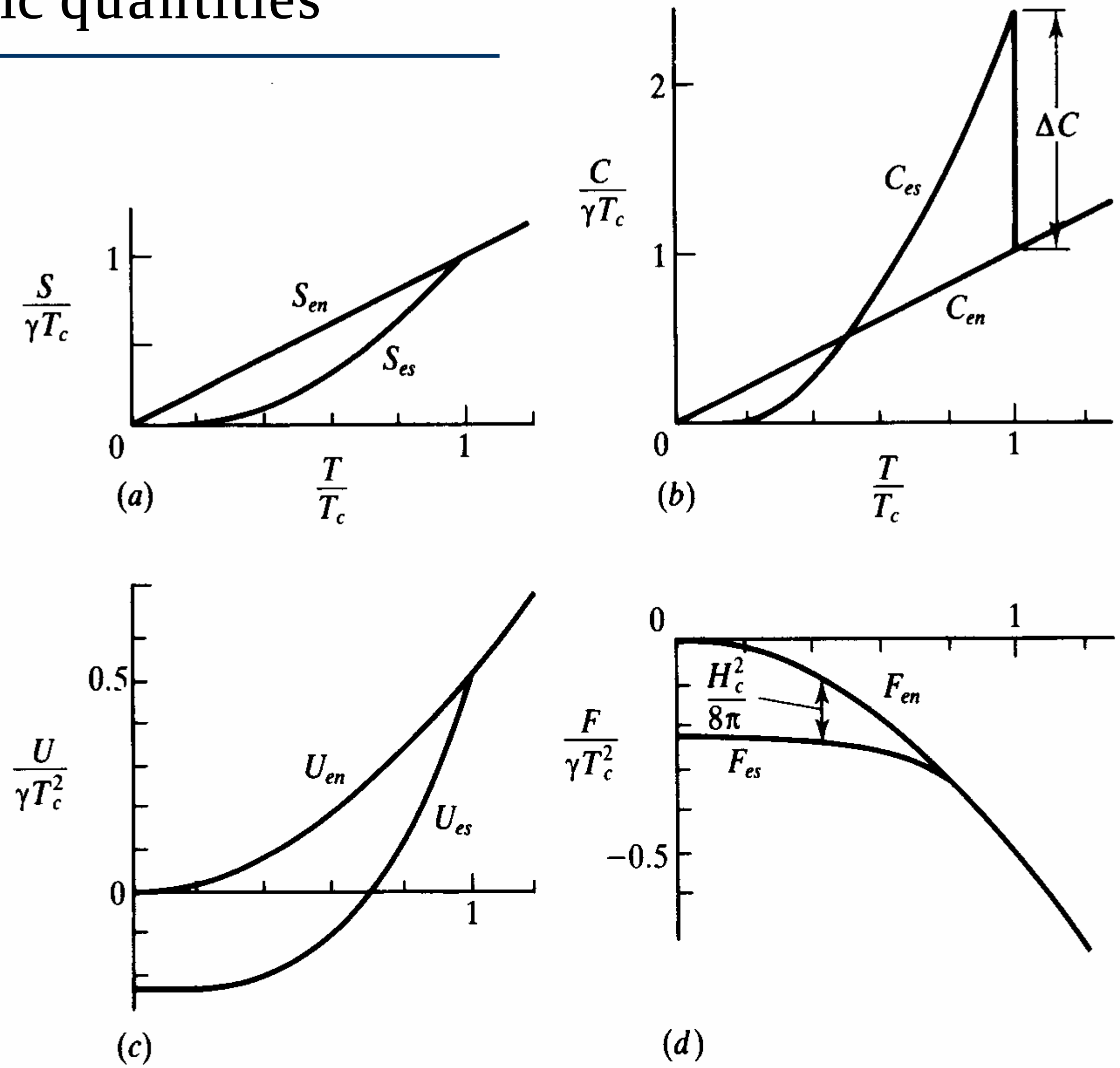
Quasi-particle Density-of-States



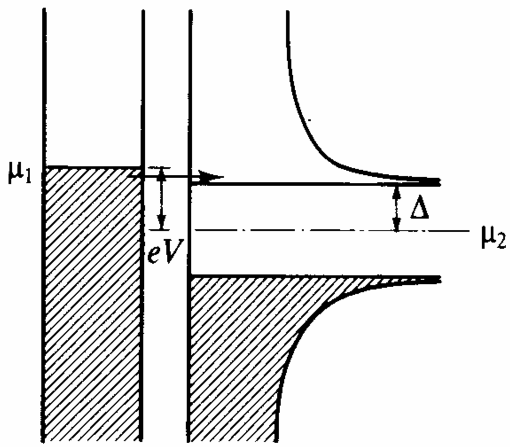
Temperature dependence of the energy gap



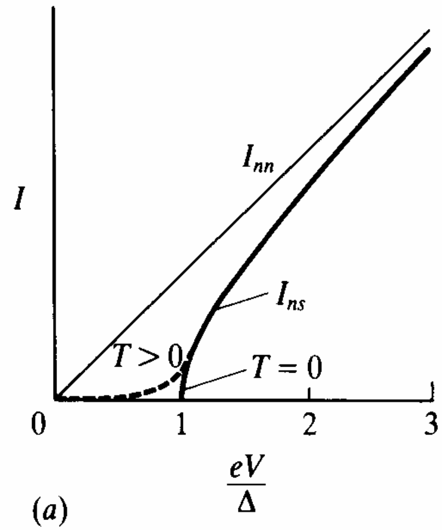
Thermodynamic quantities



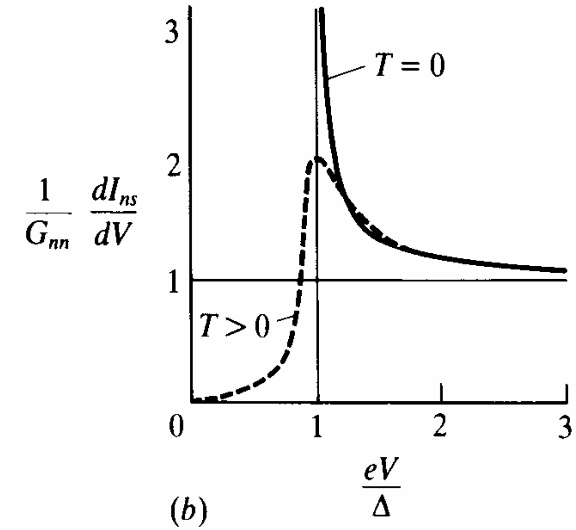
Tunneling properties



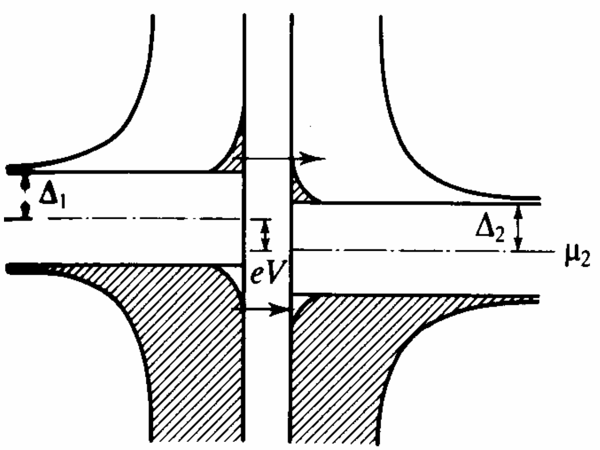
(a)



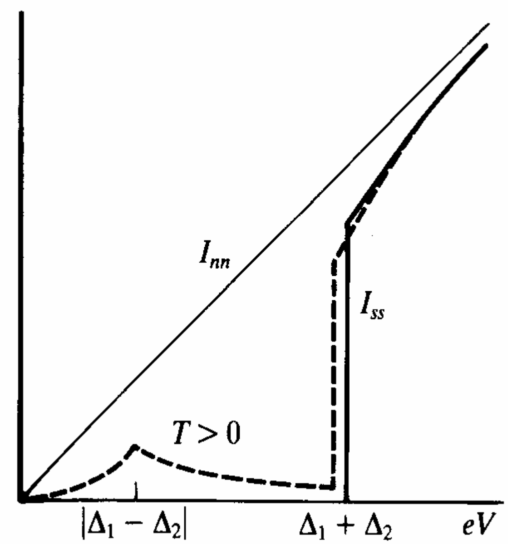
(a)



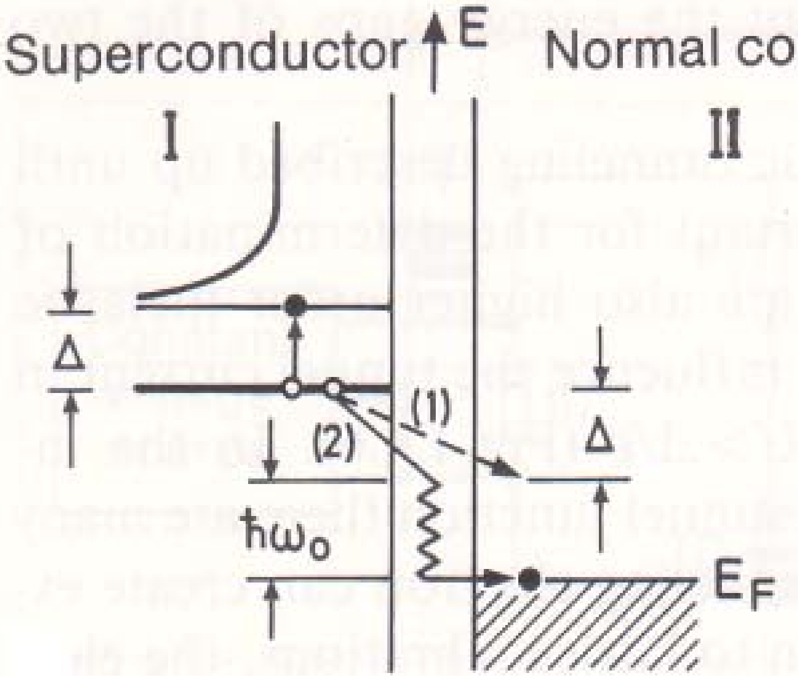
(b)

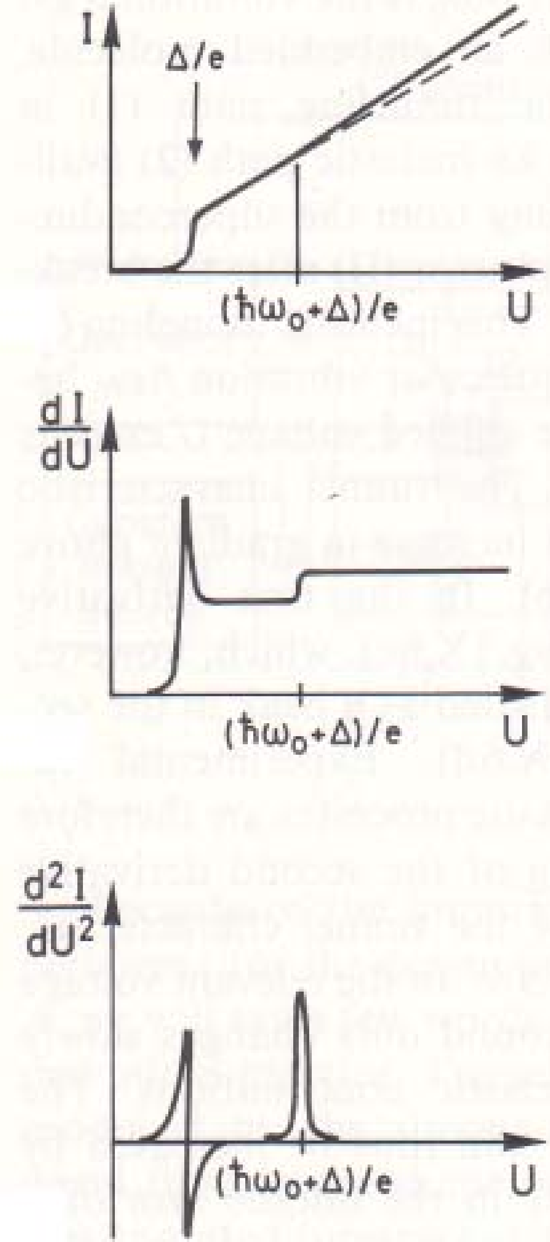


(b)



Tunneling Spectroscopy





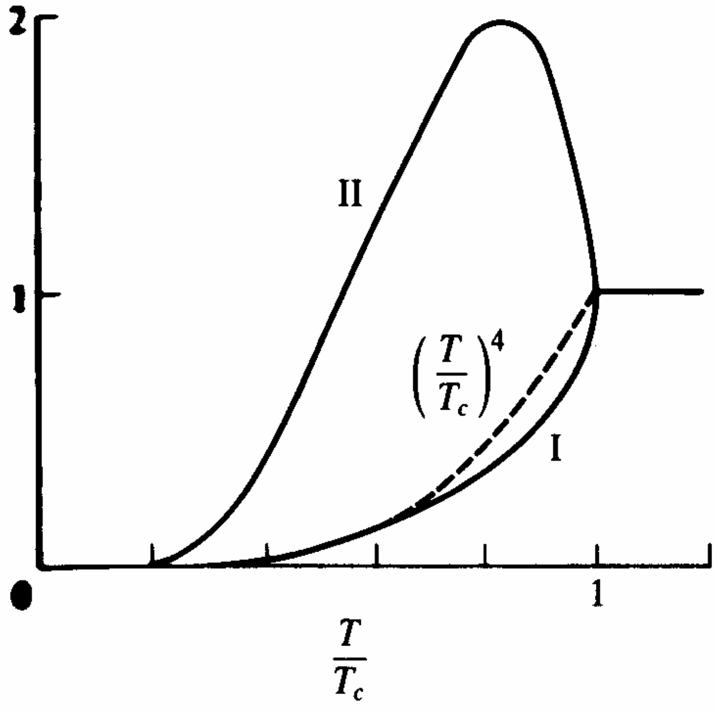
Transition probabilities and coherence effects

- case I: scalar potential (ultrasonic attenuation)

$$\frac{\alpha_s}{\alpha_n} = \lim_{\hbar\omega \to 0} \frac{1}{\hbar\omega} \int \left[f(E) - f(E+\hbar\omega) \right] dE = 2f(\Delta) = \frac{2}{1+e^{\Delta/k_B T}}$$

- case II: electromagnetic ~p·A (nuclear magnetic relaxation)

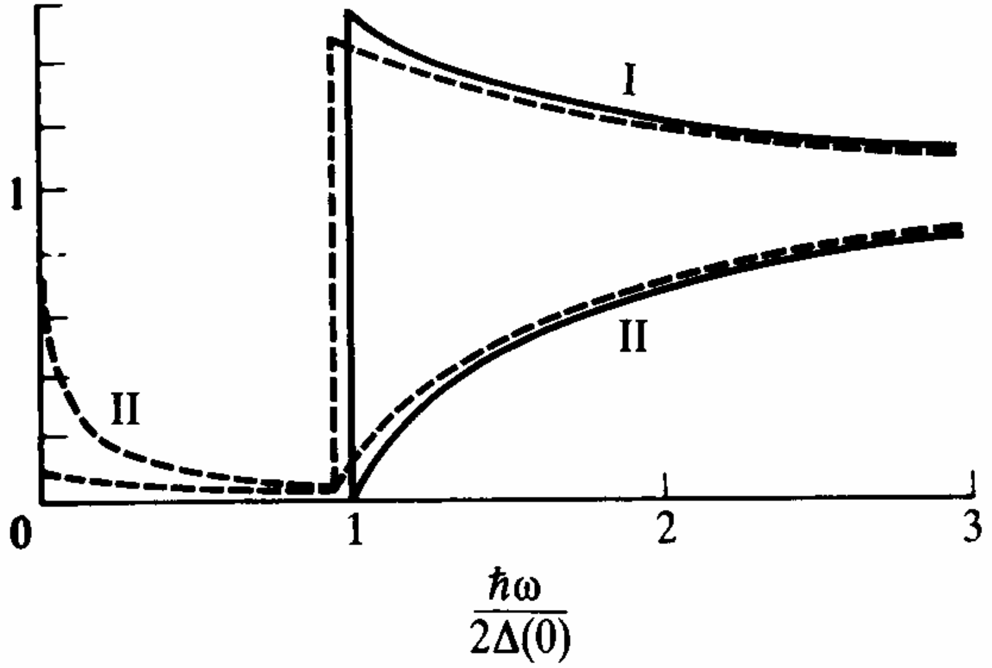
$$\frac{\alpha_s}{\alpha_n} = \lim_{\hbar\omega \to 0} 2\int_{\Delta}^{\infty} \frac{E(E+\hbar\omega)+\Delta^2}{(E^2-\Delta^2)\left[(E+\hbar\omega)^2-\Delta^2\right]^{1/2}} \left(-\frac{\partial f}{\partial E}\right) dE = 2\int \frac{E^2+\Delta^2}{E^2-\Delta^2}\left(-\frac{\partial f}{\partial E}\right) dE$$



Electromagnetic Absorption

$$\left.\frac{\alpha_s}{\alpha_n}\right|_{T=0} = \frac{1}{\hbar\omega}\int_{\Delta-\hbar\omega}^{-\Delta} \frac{\left|E(E+\hbar\omega)+\Delta^2\right|}{(E^2-\Delta^2)^{1/2}\left[(E+\hbar\omega)^2-\Delta^2\right]^{1/2}}dE = \left(1+\frac{2\Delta}{\hbar\omega}\right)E(k) - \frac{4\Delta}{\hbar\omega}K(k)$$

$$k = \frac{\hbar\omega - 2\Delta}{\hbar\omega + 2\Delta}$$



London equation

$$\mathbf{j}_s(\mathbf{r}) = n_s e \mathbf{v}(\mathbf{r})$$

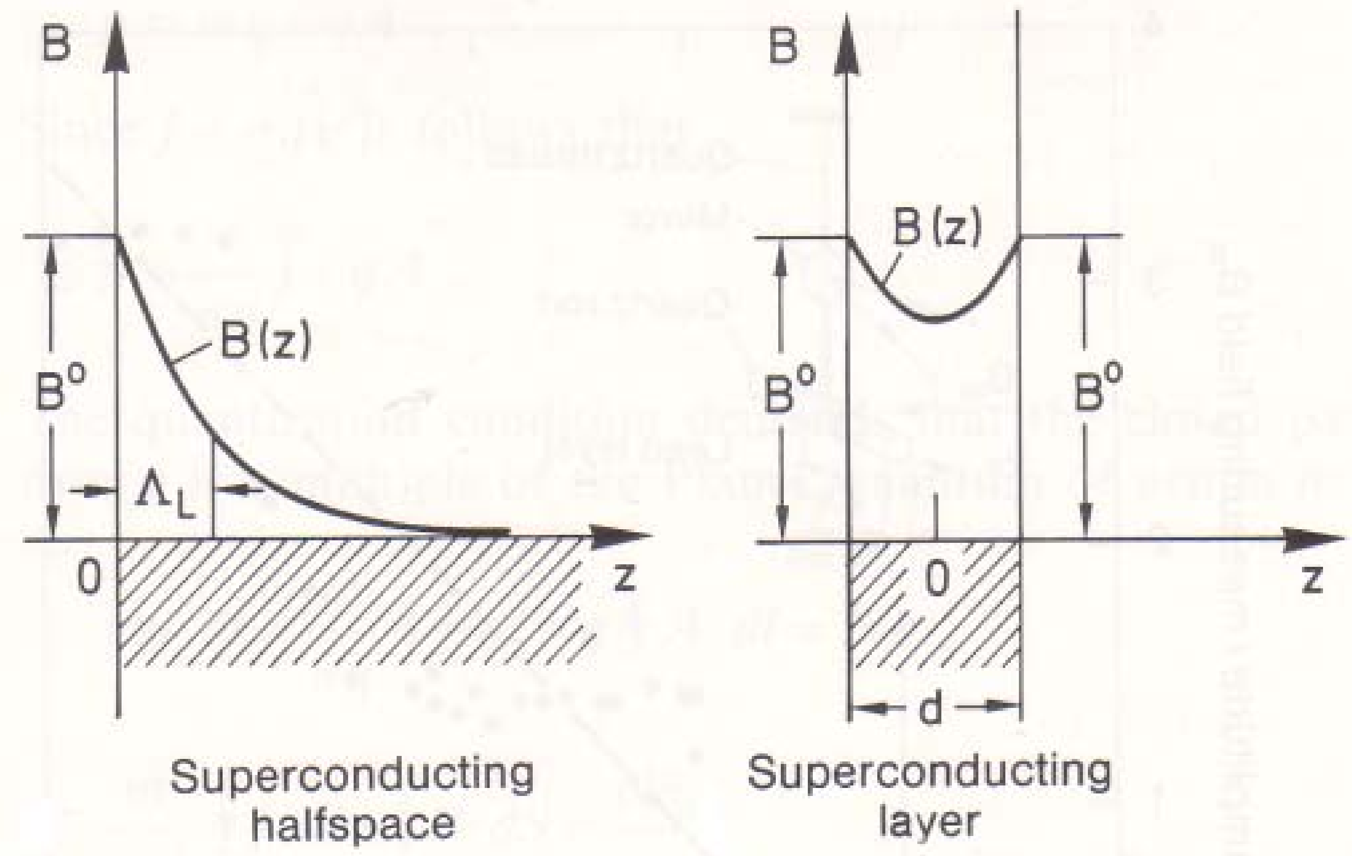
$$\nabla \times \mathbf{h} = \frac{4\pi}{c}\mathbf{j}_s = \left(\frac{4\pi e n_s}{c}\right)\mathbf{v}$$

$$\mathbf{h} + \lambda_L^2 \nabla \times (\nabla \times \mathbf{h}) = 0$$

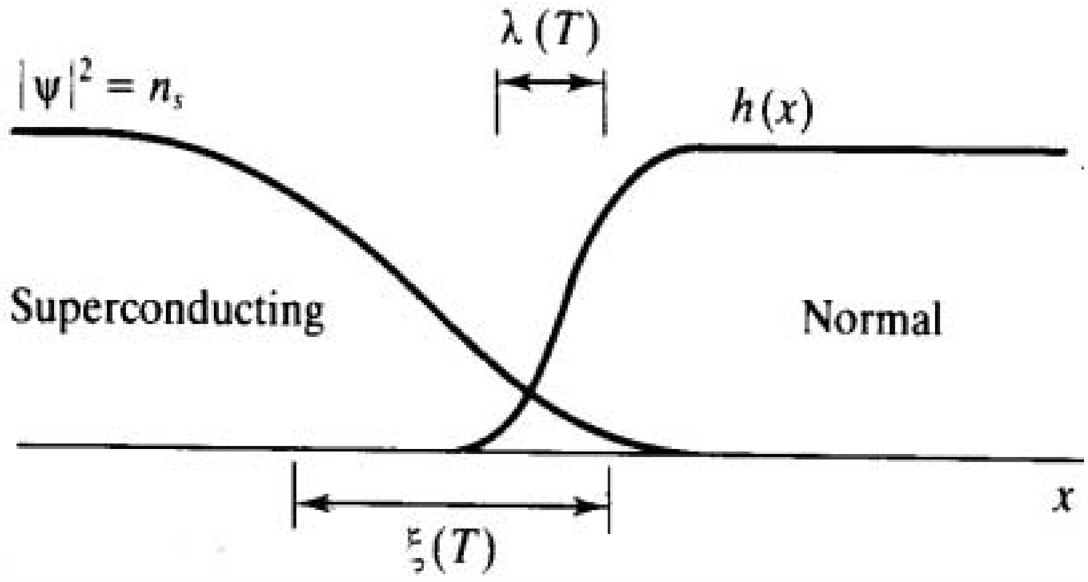
$$\lambda_L = \left(\frac{mc^2}{4\pi n_s e^2}\right)$$; London penetration depth

$$\mathbf{j}_s(\mathbf{r}) = -\frac{n_s e^2}{mc}\mathbf{A}(\mathbf{r})$$

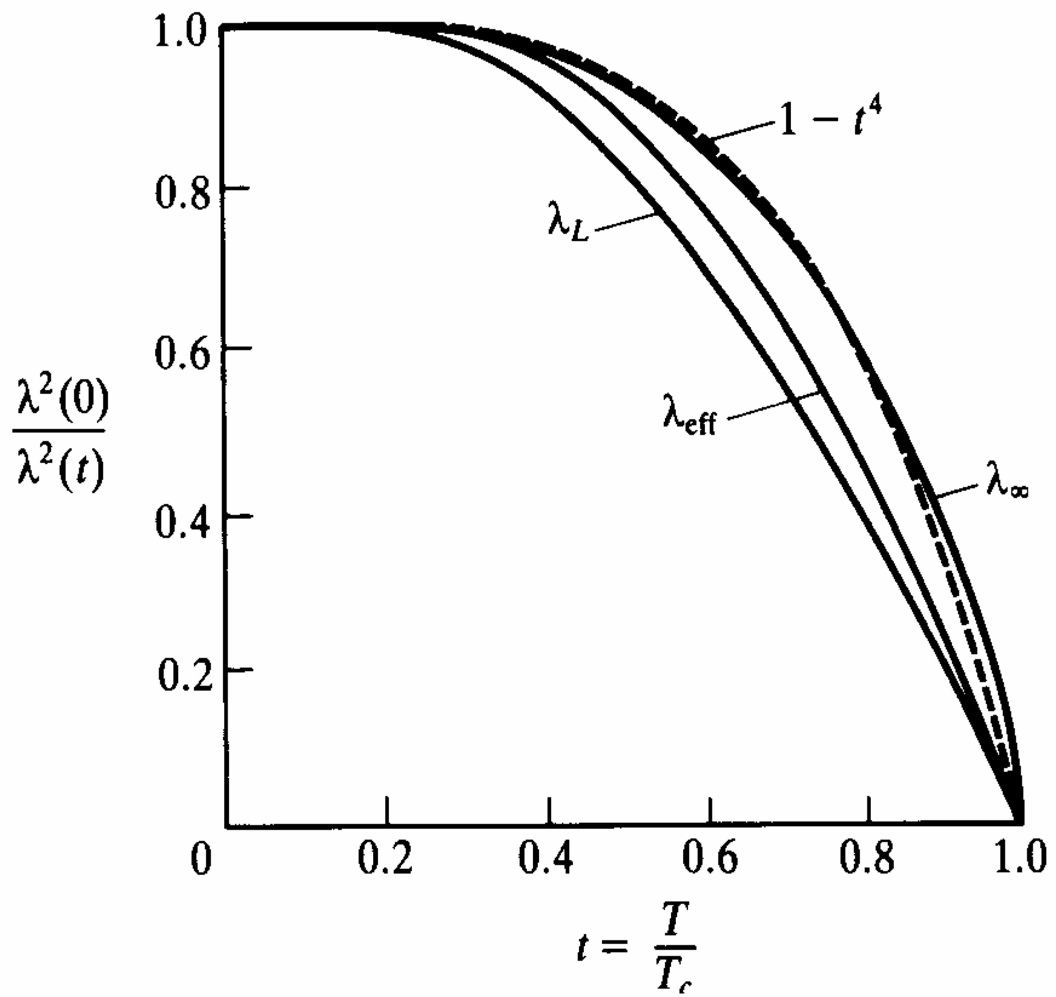
Meissner Effect



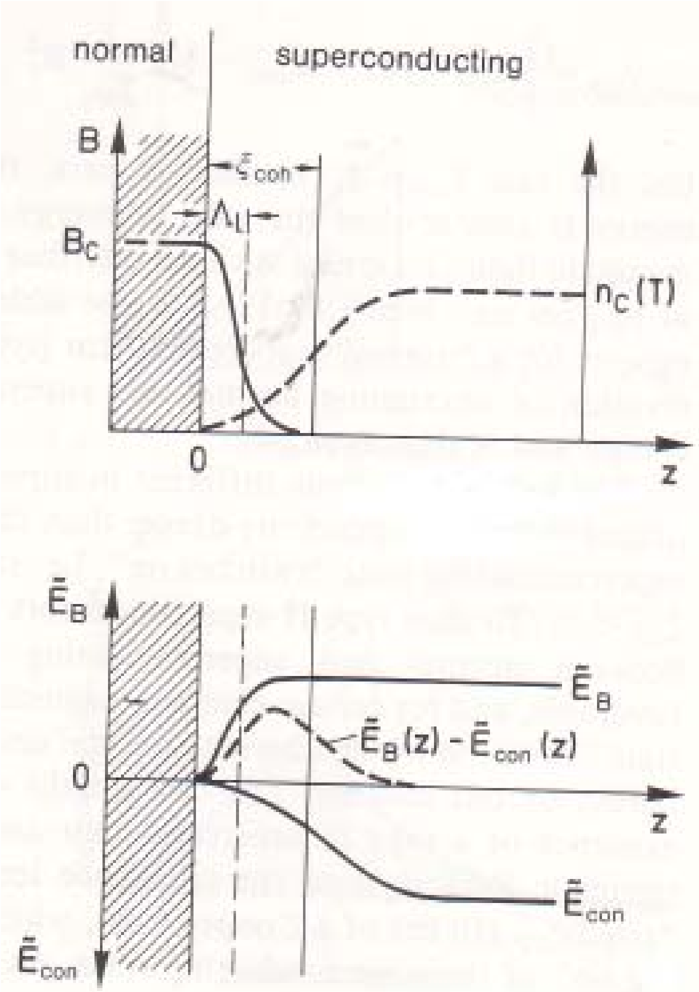
London Penetration depth and Coherence length



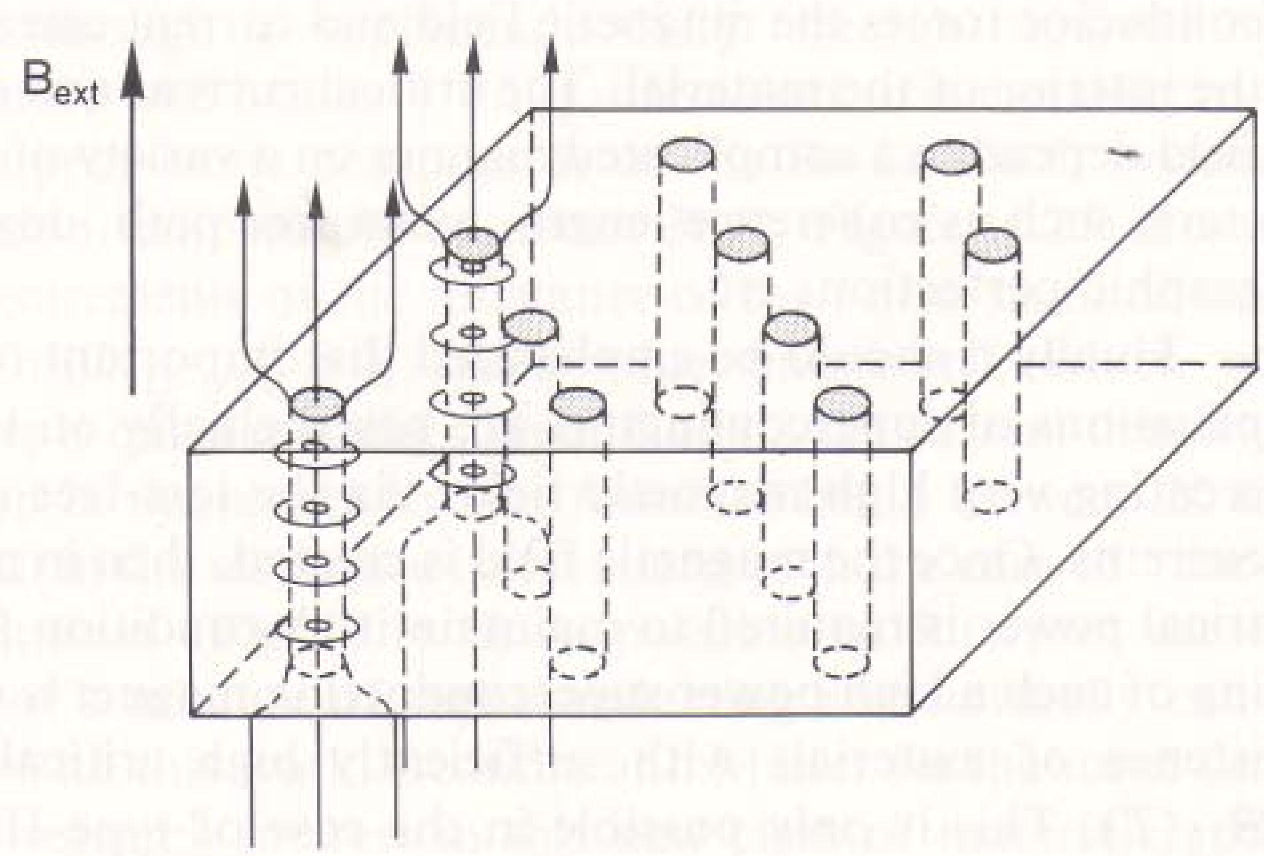
Temperature dependence of penetration depth



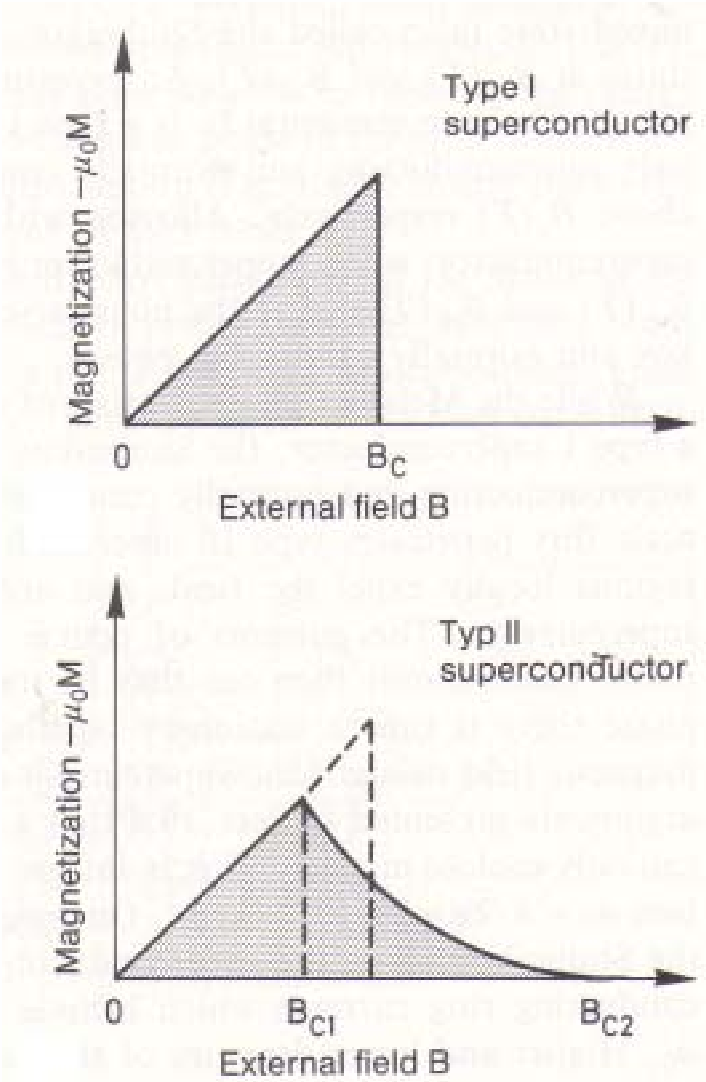
Surface Energy



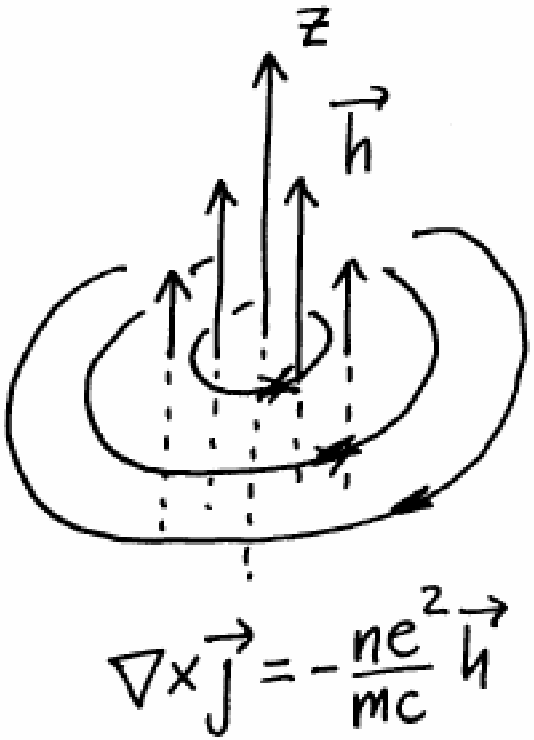
Type-II Superconductor --- negative surface energy

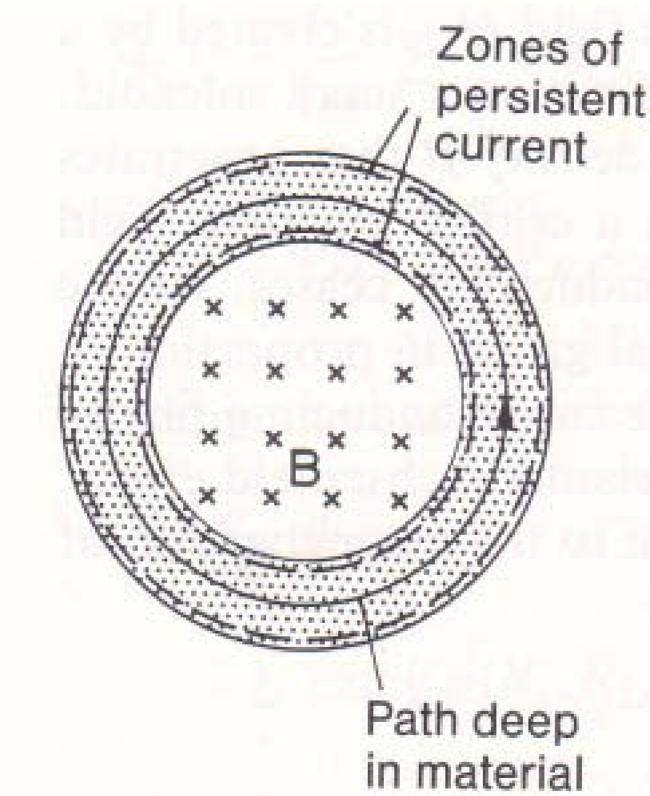


Magnetization Curves --- type-I vs. type-II superconductors



Vortex States --- Flux quantization





- Macroscopic quantum state $\psi(\vec{x}) = e^{i\varphi} \mid \psi \mid$

Along the path deep inside the superconductor: $\mathbf{J}_s = 0$

$$\vec{J}_s = \frac{e^* \hbar}{i2m^*}(\psi^* \nabla \psi - \psi \nabla \psi^*) - \frac{e^{*2}}{m^* c} |\psi|^2 \vec{A}$$

$$\oint \mathbf{J}_s \cdot d\mathbf{l} = 0$$

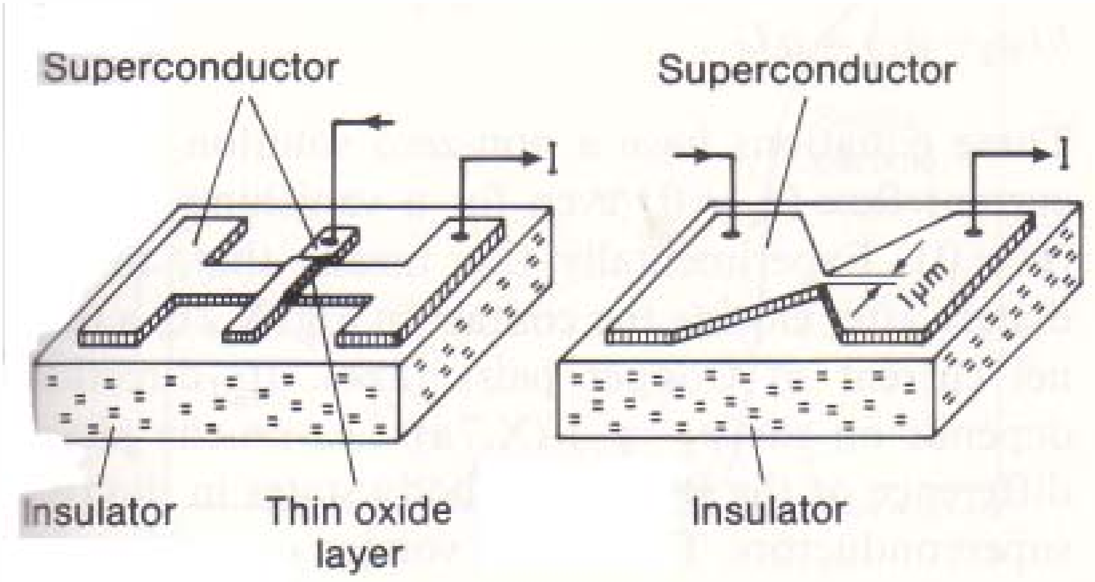
$$\int_0^{2\pi n} d\phi \frac{\hbar c}{2e} - \oint \mathbf{A} \cdot d\mathbf{l} = 0$$

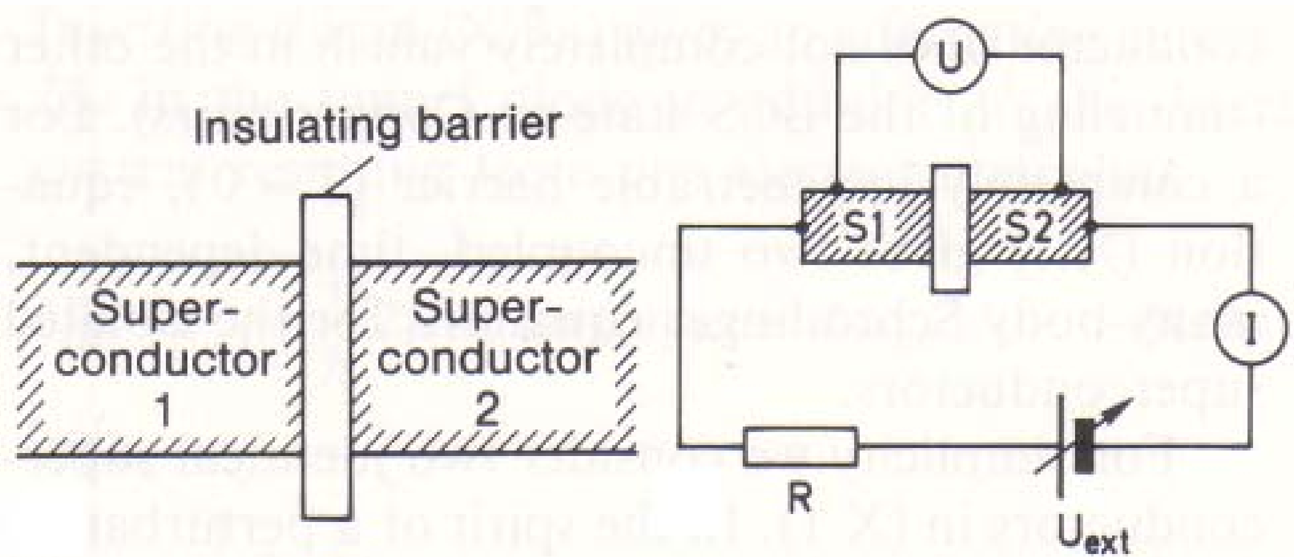
$$\Phi_B = \int \mathbf{B} \cdot d\mathbf{a} = \left(\frac{hc}{2e}\right) n = \phi_o n$$

Magnetic flux quantum: $$\phi_o = \frac{hc}{2e}$$

Josephson Effect

- Macroscopic quantum state $\psi(\vec{x}) = e^{i\varphi} \,|\, \psi \,|$

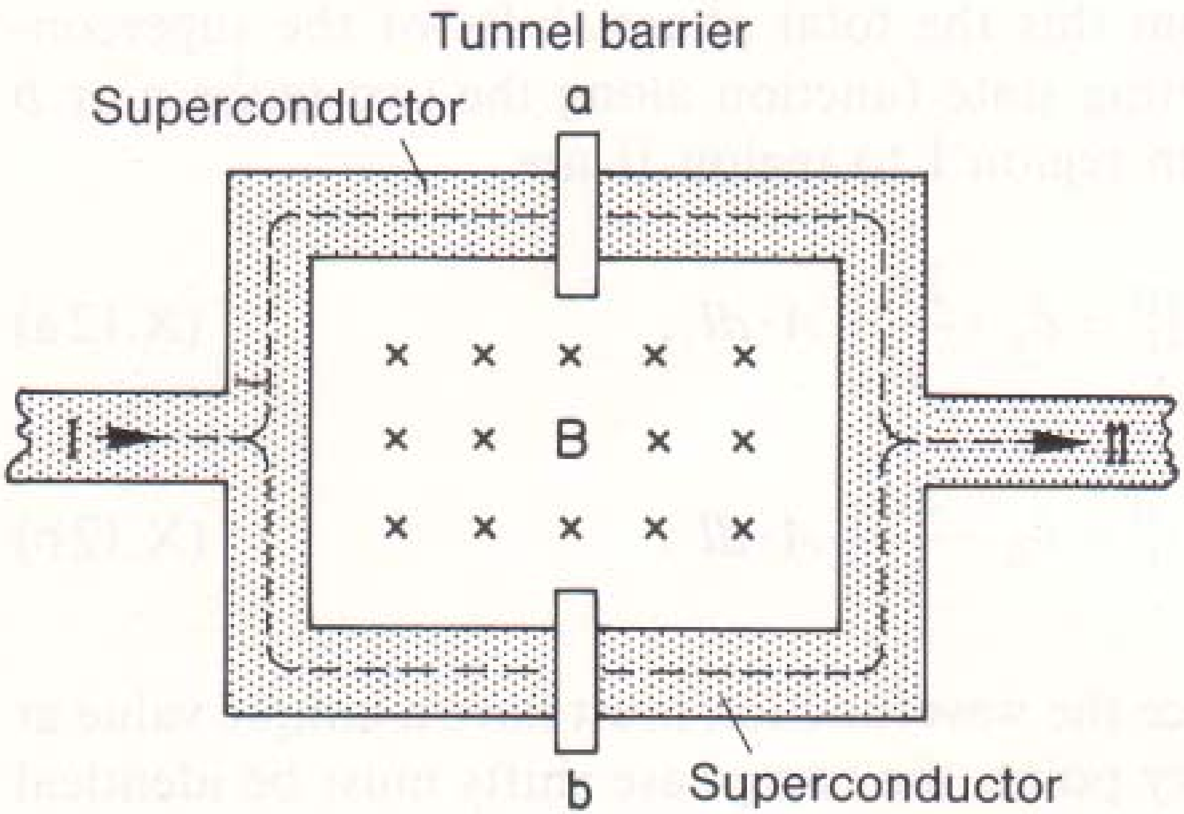


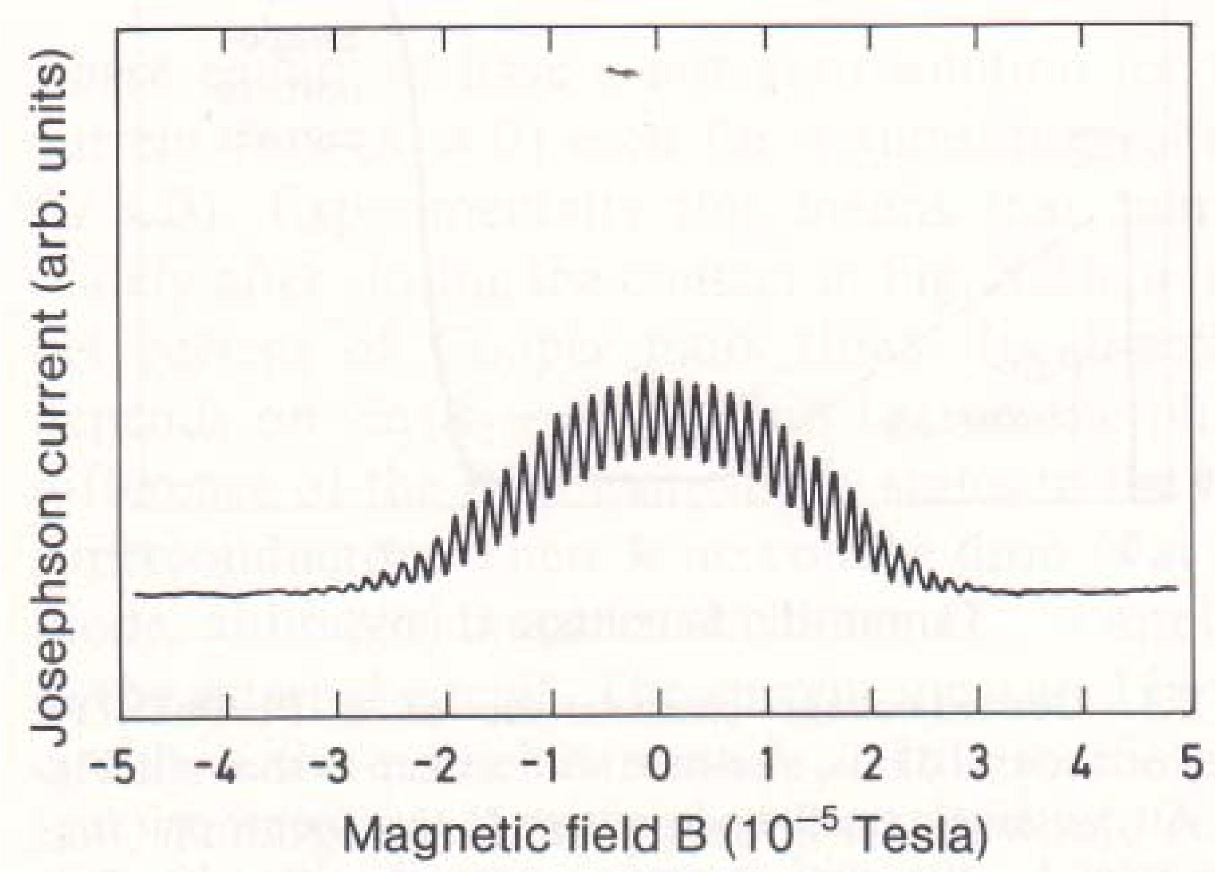


$$\psi = |\psi| e^{i\phi}$$

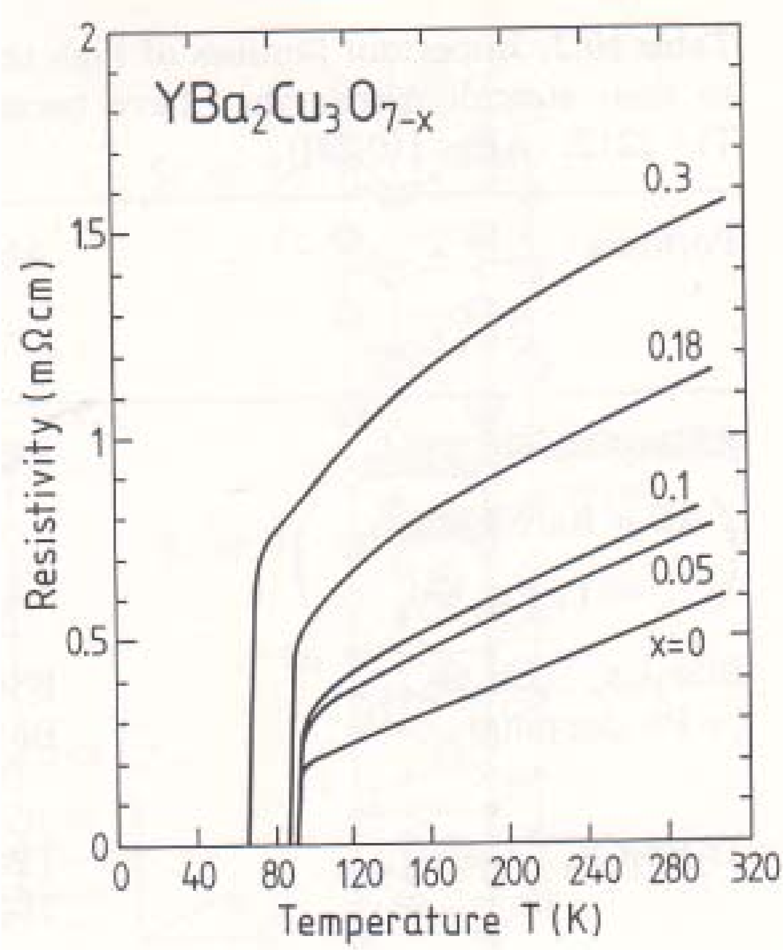
$$I_x = \frac{2e\hbar}{M_{12}m} |\psi_+||\psi_-| \sin(\phi_+ - \phi_-) = I_m \sin(\phi_+ - \phi_-)$$

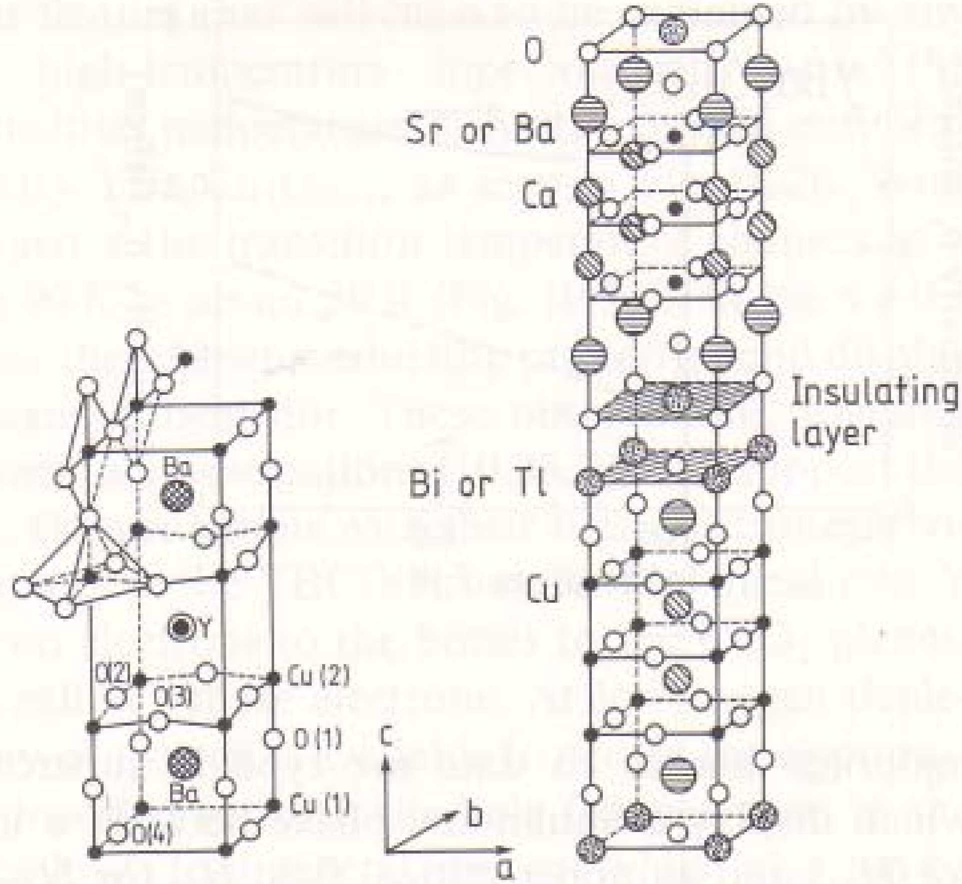
SQUID (Superconducting Quantum Interferometer Device)




Josephson current (arb. units)
-5
-4
-3
-2
-1
0
1
2
3
4
5
Magnetic field B (10^{-5} Tesla)

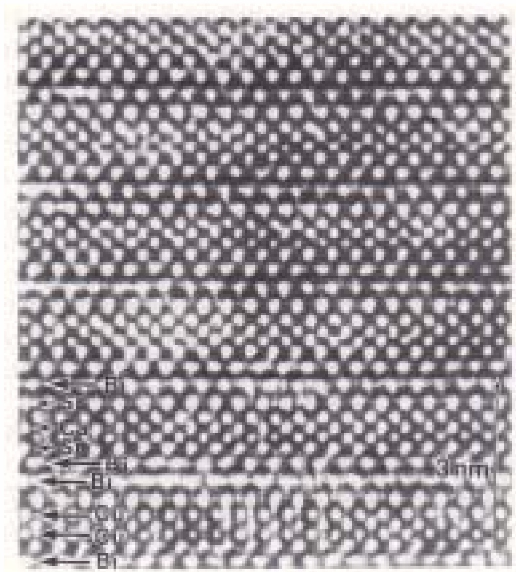
High T_c Superconductors





$YBa_2Cu_3O_7$

$Bi_2Sr_2Ca_2Cu_3O_{10}$
or
$Tl_2Ba_2Ca_2Cu_3O_{10}$



Bibliography

- Kittel, *Introduction to Solid State Physics*, Chap. 12 (entry level)
- Ibach and Luth, *Solid-State Physics*, Chap. 10 (entry level)
- de Gennes, *Superconductivity of Metals and Alloys* (advanced level)
- Tinkham, *Introduction to Superconductivity* (advanced level)