SEOUL NATIONAL UNIVERSITY – SCHOOL OF PHYSICS

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SPRING SEMESTER 2004

Solid State Physics II

Chapter 9 Phase Transition

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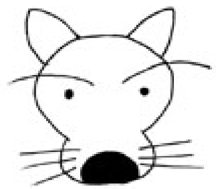
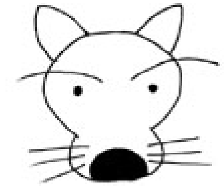


Ginzburg-Landau Theory of Phase Transitions: Mean Field Approach

Reading Assignment:

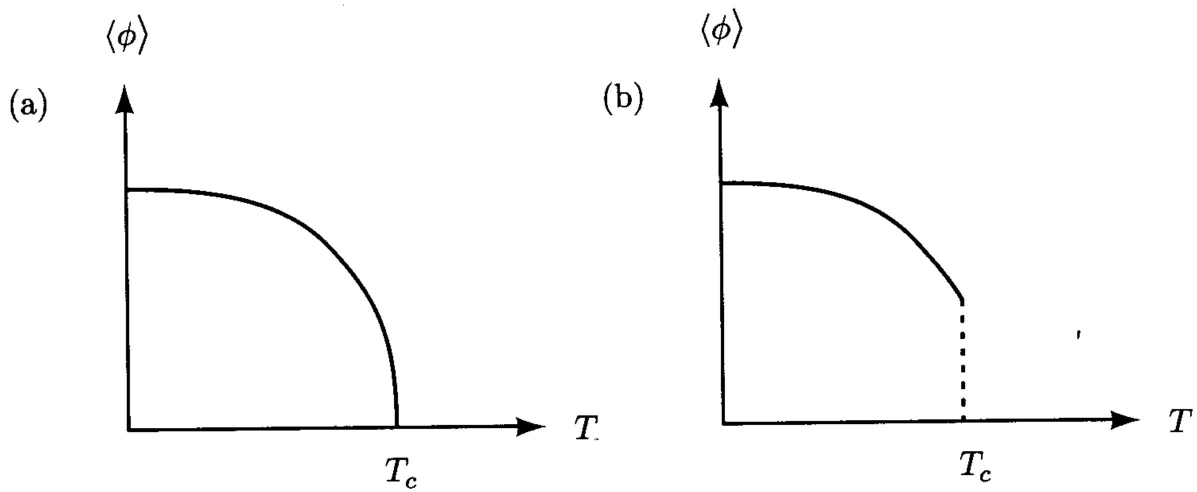
1. Kittel, Chap. 13, Appendix I. [Basic]
2. P.M. Chaikin and T.C. Lubensky, Principles of Condensed Matter Physics, Chap. 4 [Advanced]



Order Parameters: Description of Phase Transition

	$T > T_c$	$T < T_c$	Symmetry
Supercondutor	$\psi_s(\mathbf{r}) = 0$	$\psi_s(\mathbf{r}) \neq 0$	$U(1)$
Magnets	$\mathbf{M}_s(\mathbf{r}) = 0$	$\mathbf{M}_s(\mathbf{r}) \neq 0$	$O(3)$
Ferroelectrics	$\mathbf{P}_s(\mathbf{r}) = 0$	$\mathbf{P}_s(\mathbf{r}) \neq 0$	$O(3)$
Liquid/Gas-Solid	$\rho(\mathbf{G}) = 0$ for $\mathbf{G} \neq 0$	$\rho(\mathbf{G}) \neq 0$ for $\mathbf{G} \neq 0$	$\mathcal{T}_\mathbf{R}$
Order-disorder	$\eta = 0$	$\eta \neq 0$	Z_2

$$\eta = \langle n_A \rangle_A - \langle n_B \rangle_A$$



Bragg-Williams Theory

Consider the Ising model with the spin $\sigma = |\uparrow\rangle$ or $|\downarrow\rangle$. The order parameter $m = \langle\sigma\rangle$ is the average of the spin:

$$m = (N_\uparrow - N_\downarrow)/N$$

- Entropy S:

$$S = \ln C^N_{N_\uparrow} = \ln C^N_{N(1+m)/2}$$

$$\frac{S}{N} = s(m) = \ln 2 - \frac{1}{2}(1+m)\ln(1+m) - \frac{1}{2}(1-m)\ln(1-m)$$

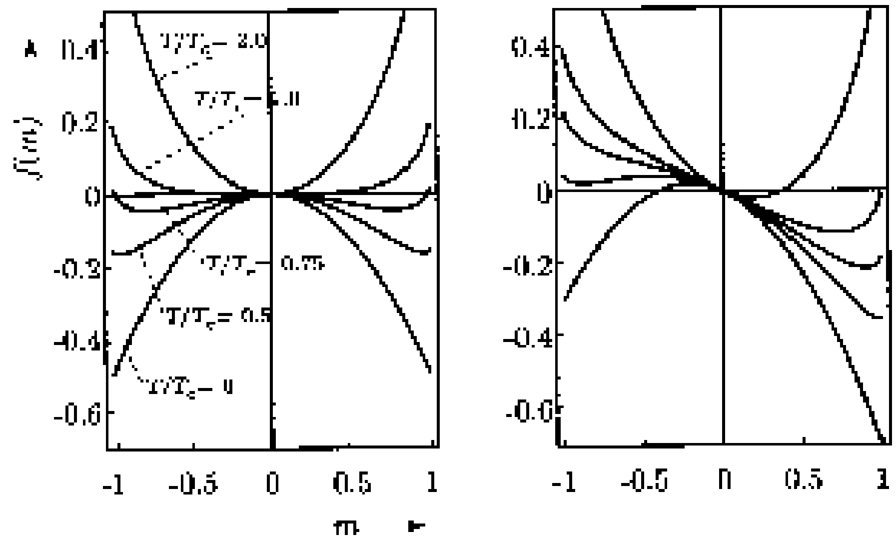
- Average Energy E:

$$E = -J\sum_{\langle ij\rangle} m^2 = -\frac{1}{2}JNzm^2$$

where z is the number of nearest neighbor sites.

- Bragg-Williams free energy $f(T, m)$:

$$f(T, m) = (E - TS)/N = -\frac{1}{2}Jzm^2 + \frac{1}{2}T[(1+m)\ln(1+m) + (1-m)\ln(1-m)] - T\ln 2$$



(a) h=0 (b) h>0

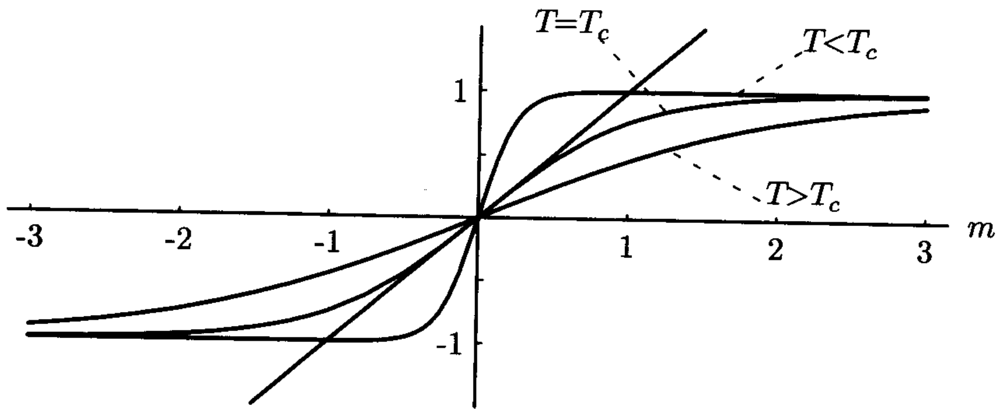
- The equation of state under an external field h:

$$\frac{\partial f}{\partial m} = -zJm + \frac{1}{2}T\ln[(1+m)/(1-m)] = h$$

$$-zJm + T\tanh^{-1} m = h$$

$$\therefore m = \tanh[(h + T_c m)/T]$$

Note that $h_{\text{eff}} = h + T_c m$.



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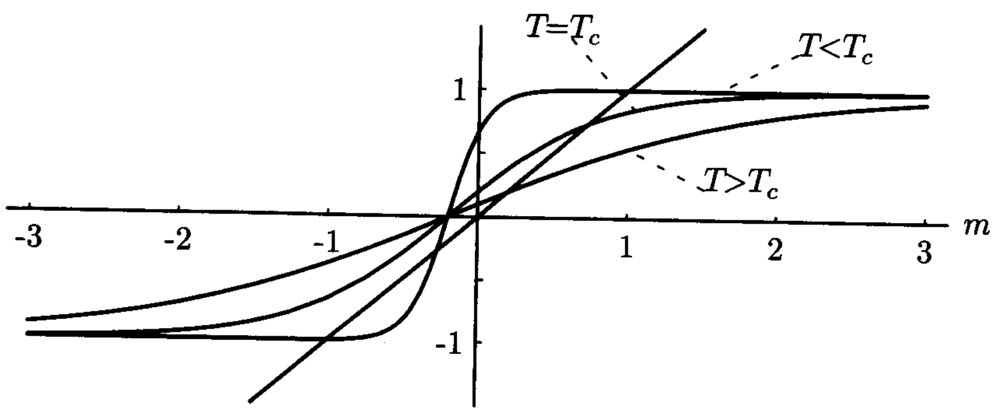


(b)

- Mean field solutions for $h = 0$:
  - near $T \approx 0$:

$$m = \tanh(T_c m/T) \approx 1 - 2e^{-2zJ/T}$$

  - near $T \to T_c^-$:

$$m \approx (T_c/T)m - \frac{1}{3}(T_c/T)^3 m^3 \approx (T_c/T)m - \frac{1}{3}m^3$$

$$m = \pm[3(T_c - T)/T]^{1/2}$$

Ginzburg-Landau Functional

Free energy near T_c

$$s(m) = \ln 2 - \frac{1}{2}m^2 - \frac{1}{12}m^4 + ...$$

$$f(m) = \frac{1}{2}(T - T_c)m^2 + \frac{1}{12}m^4 - T\ln 2 + ...$$

where $T_c = zJ$

Assuming $\phi(\mathbf{r})$ as a local order parameter,

$$F = \int d\mathbf{r} f(T, \phi(\mathbf{r})) + \int d\mathbf{r} \frac{1}{2} c \left|\nabla\phi(\mathbf{r})\right|^2$$

where f can be expanded by

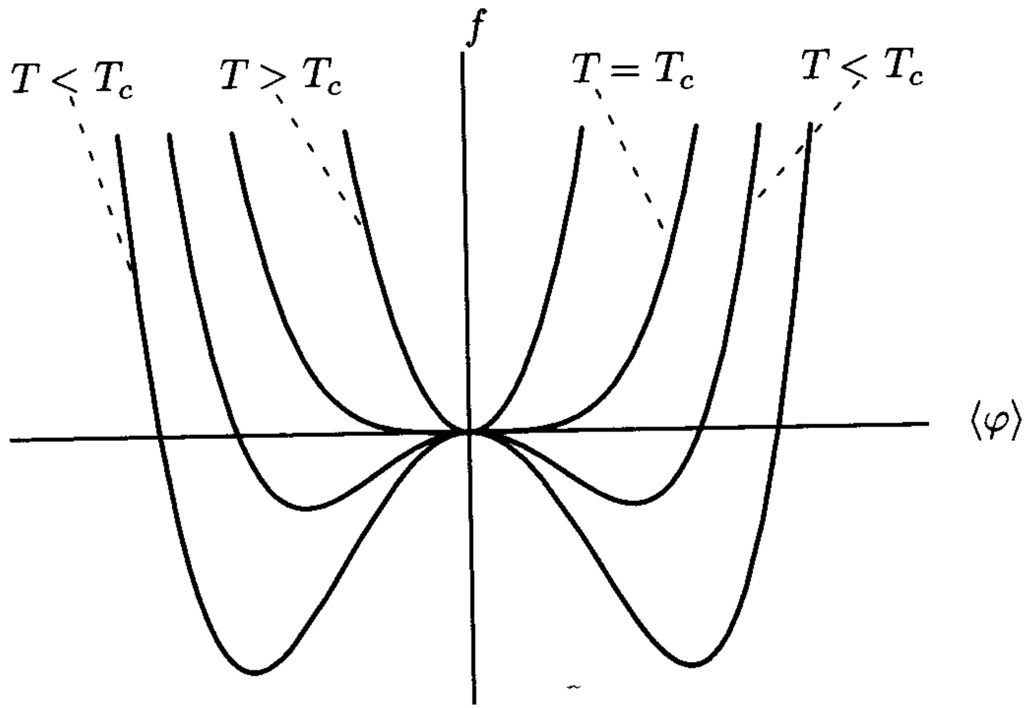
$$f(T, \phi) = \frac{1}{2} r\phi^2 - w\phi^3 + u\phi^4 + ...$$

where $r = a(T - T_c)$

Symmetry properties of the free energy functional F!

Second-Order Phase Transition

$$f(T,\phi) = \frac{1}{2} r\phi^2 + u\phi^4$$



The equation of state:

$$r\phi + 4u\phi^3 = h$$

- For $h = 0$,

$$\phi = \begin{cases} 0 & \text{if} \quad T > T_c \\ \pm(-r/4u)^{1/2} & \text{if} \quad T < T_c \end{cases}$$

$$\phi \sim (T_c - T)^\beta$$

where $\beta = 1/2$.

- Susceptibility χ

$$[r + 12u\phi^2]\frac{\partial \phi}{\partial h} = 1$$

$$\chi = \frac{\partial \phi}{\partial h} = \begin{cases} 1/r & \text{if} T > T_c \\ 1/(2|r|) & \text{if} T < T_c \end{cases}$$

$$\chi \sim |T - T_c|^{-\gamma}$$

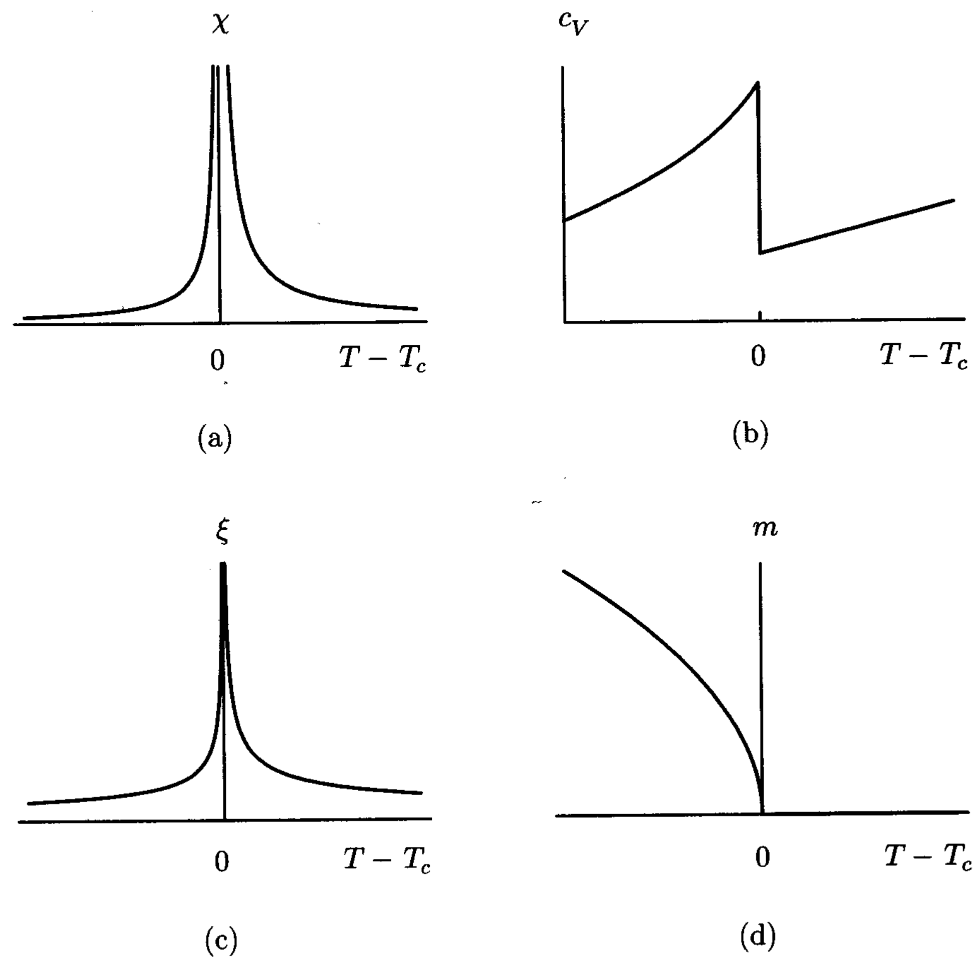
with $\gamma = 1$.

- Free energy density f

$$f = \begin{cases} 0 & \text{if} T > T_c \\ -r^2/(16u) & \text{if} T < T_c \end{cases}$$

- Specific heat c_v

$$c_v = -T\frac{\partial^2 f}{\partial T^2} = \begin{cases} 0 & \text{if} T > T_c \\ Ta^2/(8u) & \text{if} T < T_c \end{cases}$$



χ
0
$T - T_c$
(a)
c_V
0
$T - T_c$
(b)
ξ
0
$T - T_c$
(c)
m
0
$T - T_c$
(d)

correlation length

$$\chi^{-1}(\mathbf{r},\mathbf{r}') = \frac{\delta^2 F}{\delta\phi(\mathbf{r})\delta\phi(\mathbf{r}')} = (r + 12u\phi^2 - c\nabla^2)\delta(\mathbf{r}-\mathbf{r}')$$

$$\chi(\mathbf{q}) = \frac{1}{r + 12u\phi^2 + cq^2}$$

$$\chi(\mathbf{q}) = \frac{\chi}{1+(q\xi)^2} = \frac{1}{c}\frac{\xi^2}{1+(q\xi)^2}$$

where

$$\xi = c^{1/2}[r + 12u\phi^2]^{-1/2} = \begin{cases} (c/r)^{1/2} & \text{if} T > T_c \\ c^{1/2}/(-2r)^{1/2} & \text{if} T < T_c \end{cases}$$

and the correlation length $\xi \sim |T - T_c|^{-\nu}$ with $\nu = 1/2$.

$$\xi_0 = \left(\frac{c}{r(T=0)}\right)^{1/2} = \left(\frac{c}{aT_c}\right)^{1/2}$$

First-order phase transition

Nematic-to-Isotropic Transition in Liquid Crystal

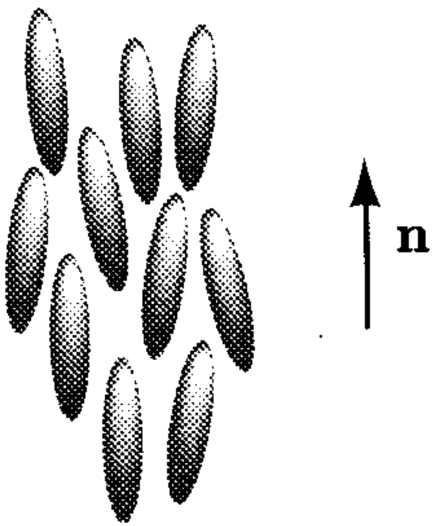
Let $\mathbf{v}_\alpha$ be the long-axis direction of the α-th molecule. The order parameter should be the quadrupole momemnt Q:

$$Q_{ij}(\mathbf{r}) = \frac{V}{N}\sum_\alpha (v_{i\alpha}v_{j\alpha} - \frac{1}{3}\delta_{ij})\delta(\mathbf{r}-\mathbf{r}_\alpha)$$

$$\langle \mathsf{Q} \rangle = \begin{pmatrix} \frac{2}{3}S & 0 & 0 \\ 0 & -\frac{1}{3}S+\eta & 0 \\ 0 & 0 & -\frac{1}{3}S-\eta \end{pmatrix}$$

For the case of uniaxial ordering, we have $\eta = 0$ and $S \neq 0$ such that

$$Q_{ij} = S(n_i n_j - \frac{1}{3}\delta_{ij})$$



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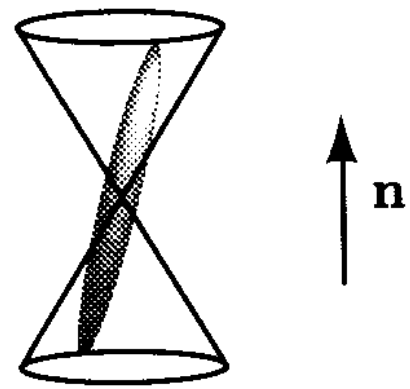

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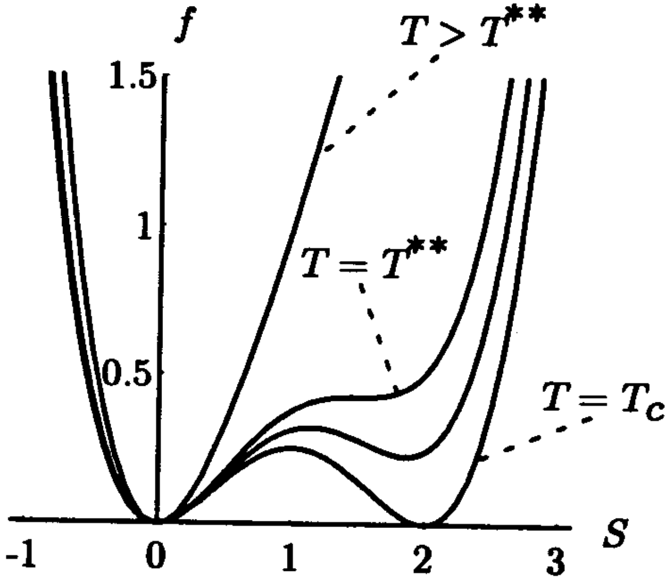
- free energy density f:

$$f = \frac{1}{2}r(\frac{3}{2}\mathrm{TrQ}^2) - w(\frac{9}{2}\mathrm{TrQ}^3) + u(\frac{3}{2}\mathrm{TrQ}^2)^2$$

$$f = \frac{1}{2}rS^2 - wS^3 + uS^4$$

- Choosing $r = a(T - T^*)$, from $\partial f/\partial S = 0$ and $f = 0$,

$$S_c = \frac{w}{2u}, \quad r_c = a(T_c - T^*) = \frac{w^2}{2u}$$



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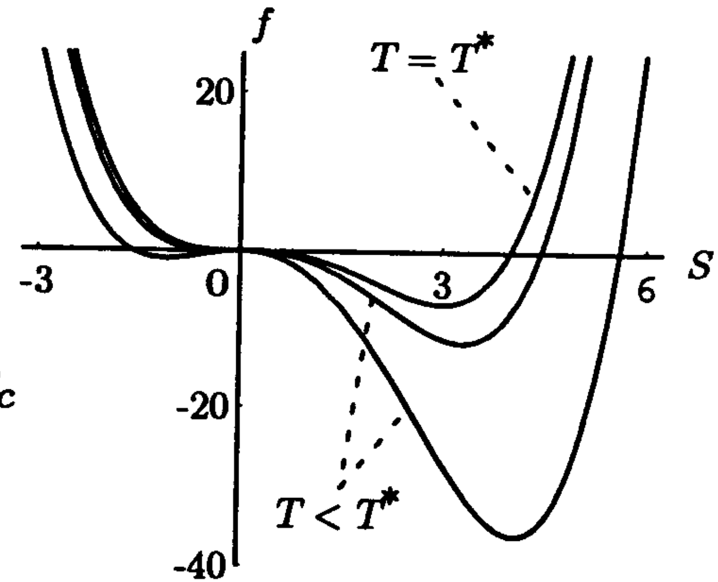

(b)