HOME WORK #5
(due date: Monday, 19 November 2007)

1. In analogy with the 1D chain system discussed in the class, let us consider a linear 1D chain with lattice constant a and two different atoms of masses m_1 and m_2 within the unit cell. Suppose all spring constants of the springs connecting two neighboring atoms are the same, i.e., k_0.

(a) Show that the Hamiltonian of this two-atoms-per-unit-cell chain can be represented by

$$\mathcal{H} = \sum_{n=0}^{N}\sum_{\alpha=1}^{2}\left[\frac{p_{n,\alpha}^2}{2m_\alpha}\right] + \sum_{n=0}^{N}\left[\frac{1}{2}k_0(u_{n,1}-u_{n,2})^2 + \frac{1}{2}k_0(u_{n,2}-u_{n+1,1})^2\right].$$

(b) Obtain the equation of motion for $u_{n,\alpha}(t)$.

(c) Assuming a normal mode solution for $u_{n,\alpha}(t) = u_{k\alpha}^0 \exp[i(kna - \omega_k t)]$, find the eigen-modes $\xi_k(t)$ and the corresponding eigen-frequencies ω_k.

(d) What happens to ω_k when $k \to 0$? Also discuss whether this result is equivalent to the one-atom-per-unit-cell results when we take $m_1 = m_2$.

Ref.: QM, Cohen-Tannoudji, Chap. V; Solid State Physics, Kittel, Chap. 4.