THE μ-PROBLEM AND THE STRONG *CP*-PROBLEM

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We investigate a possible connection of a solution of the strong *CP*-problem and the generation of a mass term μ in the low energy Higgs superpotential of supersymmetric models. This possibility comes from the fact that both supersymmetry and the Peccei–Quinn symmetry (to give an acceptable invisible axion) are broken at the same scale.

In this note we want to investigate a possible interrelation of two problems of a type usually called naturalness problems. The first one is the well-known strong *CP*-problem [1]. It can be avoided in the presence of a spontaneously broken anomalous global symmetry (PQ-symmetry) which predicts the presence of a pseudo-Goldstone boson: the axion. Constraints from cosmological considerations, however, require the breakdown scale of the PQ-symmetry to be in the range [2,3]

$$10^9\,\mathrm{GeV} \leqslant M_{\mathrm{PQ}} \leqslant 10^{12}\,\mathrm{GeV}\,. \tag{1}$$

Surprisingly this coincides with the required supersymmetry breakdown scale of $N = 1$ supergravity models which have recently attracted much attention [‡1]. In such models the supersymmetry breakdown at M_{S} induces a gravitino mass $m_{3/2} \sim M_{\mathrm{S}}^2/M$ with

$$M = M_{\mathrm{Planck}}/\sqrt{8\pi} \approx 2.4 \times 10^{18}\,\mathrm{GeV}. \tag{2}$$

A very attractive property of such models is the possible induction of the breakdown scale of the weak interactions M_{W} through the presence of $m_{3/2}$ and one expects M_{W} to be within a few orders of magnitude of $m_{3/2}$.

The actual motivation to discuss supersymmetry in the context of the standard SU(3) × SU(2) × U(1) model was to find a relation between M_{W} and M_{S} and this is achieved in the models mentioned above.

These models, however, face another problem of naturalness which we want to call the "μ-problem" where μ is the coefficient of the $\mathrm{H\bar{H}}$ term in the low energy superpotential and H and $\bar{\mathrm{H}}$ denote the usual Higgs SU(2) doublet chiral superfields. To understand what we really mean by the μ-problem let us first discuss the history of μ in a grand unified model where we choose SU(5) as a gauge group to be specific. The Higgs doublets H and $\bar{\mathrm{H}}$ are imbedded in the fundamental representations of SU(5) H_5 and $\bar{\mathrm{H}}_{\bar{5}}$, which in addition contain color triplets H_3 and $\bar{\mathrm{H}}_{\bar{3}}$. In order not to create problems with proton decay these triplets have to be massive and since the mass scale of the potential is given by the grand unification scale M_{X} (let us choose 10^{16} GeV) we also expect the mass scale of these triplets to be of order of M_{X}. Indeed in many grand unified models one can insert a term $M_{\mathrm{X}}\mathrm{H}_5\bar{\mathrm{H}}_{\bar{5}}$ in the superpotential, consistent with all the symmetries of the theory. This solves the problem of the triplets but now the Higgs doublets also have a mass $\mu \sim M_{\mathrm{X}}$. To avoid this one can introduce additional terms in the superpotential [5] and use the newly introduced parameters to keep μ small com-

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pared to M_X:

$$\mu/M_X \lesssim 10^{-14} . \tag{3}$$

With this prescription any value of μ can be chosen but the whole procedure is considered to be artificial. A natural value for μ (apart from M_X) would be zero provided that one understands why it vanishes. Models have been constructed in which H_3 and $\bar{H}_3$ can obtain a mass and in which H and $\bar{H}$ do not couple to fields with large vacuum expectation values. In these models μ vanishes if the coeffcient of $H_5\bar{H}_{\bar{5}}$ is zero, and in a fully satisfactory model *one would require this term to be forbidden by a symmetry.* Such models can be constructed [6]. If such a symmetry is exact and not spontaneously broken $\mu = 0$ would also be true in any order of perturbation theory. In general, however, such a symmetry is anomalous and could serve as a Peccei–Quinn symmetry to solve the strong *CP*-problem. To avoid a cosmologically unacceptable invisible axion, however, this symmetry cannot be spontaneously broken at the grand unification scale and a new scale in the range of (1) has to be introduced. In this case one could also imagine that starting with $\mu = 0$ a nonvanishing μ could be generated.

The reason why we prefer such a situation instead of stable $\mu = 0$ comes from considerations in the low energy ($\sim M_W$) sector of the theory. A vanishing μ in this sector has dramatic consequences. If we consider a model with minimal particle content (i.e. just H, $\bar{H}$, quark and lepton superfields as well as the gauge superfields) the superpotential would just consist of the Yukawa couplings that couple H and $\bar{H}$ to quarks and leptons. This superpotential has an additional PQ-symmetry which has to be broken to give masses to all quarks and leptons and implies the existence of an unacceptable Weinberg–Wilczek axion [7]. To avoid this problem one either has to introduce $\mu \neq 0$ to raise the mass of the "axion" or one has to introduce additional fields. The latter possibility has to face its own problems which we cannot discuss here and does not lead to satisfactory models [8].

We thus remain with $\mu \neq 0$ as a necessity. Although only relatively small μ are needed to raise the mass of the "axion" to acceptable values (by small we mean here small compared to $m_{3/2}$) these models have the rather unpleasant property of a large value of the top quark mass [9]

$$m_{top} \gtrsim 55 \text{ GeV} , \tag{4}$$

to allow the radiatively induced breakdown of SU(2) × U(1) at the correct scale. At the moment we do not know the value of m_{top} but one might ask the question whether all of these models are ruled out if m_{top} is found to be smaller than 50 GeV. In the range 20 GeV $\leqslant m_{top} \leqslant$ 50 GeV a fully satisfactory induction of SU(2) × U(1) can only be achieved if μ is comparable to $m_{3/2}$ [10]. We now face the problem how it could happen that μ is so large.

This is what we mean by the μ-problem. In a first step we had to find a mechanism that explains $\mu = 0$ to avoid $\mu \sim M_X$. This, however, seemed to imply that even including radiative corrections μ remained small compared to the value of the gravitino mass $m_{3/2}$. In the remainder of this note we want to investigate whether this is necessarily true.

If $\mu = 0$ is protected by an unbroken exact symmetry there is of course no way to generate it in perturbation theory. If μ is set artificially equal to zero by hand without the protection of a symmetry it, however, can be generated in perturbation theory, provided that supersymmetry is broken. This situation has been investigated both in globally supersymmetric models and in supergravity. For the case we are interested in here this generates

$$\mu \sim (\alpha/\pi) m_{3/2} , \tag{5}$$

where $\alpha = g^2/4\pi$ and g is a coupling constant. In order for (5) to be a reliable estimate of μ, α has to be small compared to one and μ is small compared to $m_{3/2}$. If (5) is generated through gauge interactions this is the case: the grand unified coupling constant at M_X in the minimal model is given by $\alpha_5 \sim 1/25$. The relevant scale to compute (5) is M_X since a satisfactory induction of the SU(2) × U(1) breaking with small top quark masses requires $\mu \sim m_{3/2}$ already at M_X.

A possibility to arrive at $\mu \neq 0$ could also come from the introduction of new fields Y and a term in the superpotential

$$\lambda Y H \bar{H} , \tag{6}$$

independently of the presence of the symmetry. One arranges Y to have a vacuum expectation value such that $\lambda\langle Y\rangle \sim m_{3/2}$. This, however, seems as artificial as putting $\mu H\bar{H}$ by hand in the first place, we do not understand why $\lambda\langle Y\rangle$ is so small compared to M_X.

The next try involves the introduction of non-renormalizable terms in the superpotential like

$$(1/M)^{n-1} Y^n H\bar{H} . \tag{7}$$

With vacuum expectation values $\langle Y \rangle \sim M_X \sim 10^{16}$ GeV and $M_{PL} \sim 10^{19}$ GeV one then has to choose n to be 5 or 6 to arrive at $\mu \sim m_{3/2}$. This would be fine if we could find a reason why the terms $Y^m H\bar{H}$ with $m < n$ are forbidden since otherwise μ is expected to be larger. The reason could be a symmetry under which e.g. H, $\bar{H}$ transform with charge 3 and Y transforms with charge −1 and the only allowed term in (7) is the one with $n = 6$. Such a symmetry has to be imposed on the complete superpotential and it usually turns out that such a symmetry is a Peccei–Quinn symmetry i.e. it has an SU(3) anomaly. Y has to receive a vacuum expectation value to generate μ and this would break the symmetry. $\langle Y \rangle \sim M_X$ then would lead to an unacceptable "invisible" axion [3]. The allowed vacuum expectation values are given in (1). They are of the order of the supersymmetry breakdown scale $M_S \sim 10^{11}$ GeV.

To solve the strong *CP* problem a mass scale in the range of (1) has to be inserted in the model which is comparable to the supersymmetry breaking scale M_S and we have $M_{PQ} \sim M_S$ [11]. Let us now see which influence this can have on a possible generation of μ and consider again (7). We of course want to consider a case where $H\bar{H}$ is forbidden by the Peccei–Quinn symmetry. Terms with higher n are allowed if Y transforms nontrivially under this symmetry and a vacuum expectation value of Y breaks $U(1)_{PQ}$ and we want to have $\langle Y \rangle \sim 10^{11}$ GeV. If the term with $n = 2$ is present we would obtain an effective $\mu H\bar{H}$ in the low energy theory with

$$\mu = \langle Y \rangle^2 / M \sim M_S^2 / M \sim m_{3/2} , \tag{8}$$

provided that the term with $n = 1$ is forbidden which however is usually the case if $n = 2$ is allowed. In such a case a μ of the desired magnitude would be generated.

The question remains whether it is possible to construct a model which fulfills the requirements given above. We will show that such a construction is possible. It requires the introduction of new fields and mass parameters in the 10^{11} GeV range, but this is required in any model that solves the strong *CP*-problem with an invisible axion independent of the question of a generation of μ and as such is not an additional complication inserted to solve the μ problem.

Let us start with the hidden sector which is responsible for the breakdown of supersymmetry. We introduce two singlet superfields Z and Z′ and two fields A, A′ in the 75 [‡2] representation of SU(5) with superpotential [11]

$$g_H = (\lambda \,\mathrm{Tr}(AA') + m^2) Z + (\lambda' \,\mathrm{Tr}(AA') + m'^2) Z' + \beta , \tag{9}$$

where β is a constant used to fine tune the cosmological constant. The equations $\partial g/\partial Z = \partial g/\partial Z' = 0$ have no common solution provided that $\lambda m'^2 \neq \lambda' m^2$ and supersymmetry is broken. If we choose m and m' in the range of 10^{11} GeV this will also correspond to the supersymmetry breaking scale. For a wide range of parameters also A and A′ will receive a VEV of this order of magnitude. This is important since A and A′ appear only in the combination AA′ in (9) and could therefore have nontrivial transformation properties under a Peccei–Quinn symmetry. A VEV of A and A′ would break this symmetry at the desired scale and we would have a relation between M_{PQ} and $M_S \sim 10^{11}$ GeV.

Next we introduce the superpotential for the matter fields

$$\begin{aligned} g_M = \sum_{a,b=1}^{3} & (f_{ab} 10_a \bar{5}_b \bar{H}_{\bar{5}} + \tilde{f}_{ab} 10_a 10_b H_5) \\ & + \tilde{h}\, \overline{10}' A\, 10' + \tilde{h}'\, \overline{10}'\, \overline{10}' \bar{H}_{\bar{5}} \\ & + \sum_{a=1}^{3} (g_a 10' \bar{5}_a \bar{H}_{\bar{5}} + \tilde{g}_a 10' 10_a H_5) + \tilde{g}' 10' 10' H_5 , \end{aligned} \tag{10}$$

where $a, b = 1, ..., 3$ label the three generations of quarks and leptons and $\overline{10}'$ and $10'$ are newly introduced superfields that become heavy through the third term in (10) after A has received a VEV. Superpotential (10) has a $U(1)_{PQ} \times U(1)$ invariance one of which is anomalous. The charges are as shown in table 1. Actually the superpotential in (10) is the most general one that is consistent with these symmetries. It contains the usual Yukawa couplings and also the "mass term" and interactions of the newly introduced $10'$ and $\overline{10}'$. Sufficient $\Delta I_W = 0$ masses for $10'$ and $\overline{10}'$ greater than TeV do not lead to phenomenological problems related to flavor changing processes [12].

[‡2] For reasons which will become clear later we choose A and A′ here to transform as 75-representations of SU(5) although at this stage we could equally well have used singlets or adjoint representations.

Table 1

	10	$\bar{5}$	A	10′	$\overline{10'}$	$\bar{H}_{\bar{5}}$	H_5
$U(1)_{PQ}$	−1	−1	2	−1	−1	2	2
U(1)	1	−3	0	1	−1	2	−2

We have not yet given mass to the color triplets of H_5 and $\bar{H}_{\bar{5}}$ and we also need the breakdown of SU(5) at the grand unification scale M_X. We introduce (50,0) and ($\overline{50}$, 0) representations of SU(5) $\times$ $U(1)_{PQ}$ to allow masses for the Higgs triplets via $H_5 A'\overline{50}$ and $\bar{H}_{\bar{5}}A'50$ and they will receive a mass of order M_{PQ}. The breakdown of SU(5) at $M_X \sim 10^{16}$ GeV can be achieved with an additional adjoint representation, (Σ) which also should have PQ-charge zero in order not to break $U(1)_{PQ}$ at M_X. This model is then similar to the one proposed by Grinstein [6] with the difference that in our case the two 75's receive only a VEV of order M_{PQ} instead of M_X.

Supersymmetry is broken at $M_S \sim 10^{11}$ GeV and $U(1)_{PQ}$ breaks at $M_{PQ} \sim 10^{11}$ GeV due to the VEVs of A and A′ and the model has an acceptable invisible axion. A solution of the strong *CP*-problem and the absence of an $H_5\bar{H}_{\bar{5}}$ requires these complications. Unfortunately up to now nobody has found an easier way to achieve this.

Nonrenormalizable terms like the ones in (7) will now generate a nonvanishing μ. Since $H\bar{H}$ has $U(1)_{PQ}$ charge 4 and only A, A′ and Σ receive vacuum expectation values there is just one such term that can induce μ [A′ has PQ-charge (−2) compare eq. (9)]:

$$M^{-1} A'A'H\bar{H}\,, \tag{11}$$

and we obtain $\mu \sim M_{PQ}^2/M \sim m_{3/2}$. The reason for this "equality" of μ and $m_{3/2}$ is that M_S coincides with M_{PQ}, a relation suggested by the standard arguments given earlier. It seems that this mass scale plays a central role in supersymmetric models. Apart from $M_S \sim M_{PQ} \sim 10^{11}$ GeV this mass scale also appears in cosmological scenarios related to the grand unified phase transition and the creation of the baryon asymmetry in the context of supersymmetric models.

In conclusion, the above example should be viewed as an existence proof for a model that solves the strong *CP*-problem and the μ-problem simultaneously. It is certainly aesthetically not very appealing and also involves nonrenormalizable terms in the framework of $N = 1$ supergravity. The complicated superpotential arises from the requirements to get an acceptable invisible axion and to understand why $\mu \ll M_X$. It would be simpler if we would not include grand unification in our discussion. The nonrenormalizable terms then generate $\mu \sim m_{3/2}$. We do not know whether there exist more elegant ways to solve these problems. Maybe these exist but it might also be that this example shows us again how hard it is to obtain $\mu = m_{3/2}$ once one has understood why $\mu \lll M_X$. In the above model at least one can explain $\mu \sim m_{3/2}$ through the mechanism that solves the strong *CP*-problem and not through artificial adjustments of parameters, chosen just for the reason to obtain $\mu \sim m_{3/2}$.

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