Dynamics of nonlocality for a two-mode squeezed state in a thermal environment

Hyunseok Jeong,[1,*] Jinhyoung Lee,[1,†] and M. S. Kim,[1,2]
[1]*Department of Physics, Sogang University, CPO Box 1142, Seoul 100-611, Korea*
[2]*Department of Applied Mathematics and Theoretical Physics, The Queen's University of Belfast, Belfast BT7 1NN, Northern Ireland*
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We investigate the time evolution of nonlocality for a two-mode squeezed state in a thermal environment. The initial two-mode pure squeezed state is nonlocal with a stronger nonlocality for a larger degree of squeezing. It is found that the larger the degree of initial squeezing, the more rapidly the squeezed state loses its nonlocality. We explain this by the rapid destruction of quantum coherence for the strongly squeezed state.

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I. INTRODUCTION

Quantum nonlocality is one of the most profound features of quantum mechanics [1,2]. It enables current developments of quantum information theory encompassing quantum teleportation [3–8], quantum computation [9], and quantum cryptography [10]. There have been studies on tests of quantum nonlocality versus local realism. Bell suggested an inequality that any local hidden variable theory must obey [2]. Several types of Bell's inequalities have been derived in terms of two-body correlation functions of two measurement variables at distant places [11] to test quantum nonlocality for a spin-1/2 or SU(2) system.

A spin-1/2 system can be utilized as a qubit for quantum computation. Quantum nonlocality of the spin-1/2 system is required as a quantum channel to teleport an unknown qubit state [3,12]. In fact, it is possible to teleport not only a two-dimensional spin-1/2 quantum state but also an N-dimensional state [4] and a continuous-variable state [5]. The type of quantum channel depends on the dimension of the Hilbert space of an unknown quantum state. For teleportation of a continuous-variable state, the quantum channel should be in an entangled continuous-variable state such as the two-mode squeezed state [6]. Recently, practical implementation of quantum teleportation for a continuous-variable state has been realized experimentally using a two-mode squeezed field [7,8]. In quantum teleportation, the most important ingredient is the quantum nonlocality of the channel, which can easily be destroyed in nature. In this paper we are interested in how the thermal environment affects the quantum nonlocality of the two-mode squeezed field.

Quantum nonlocality of an entangled continuous-variable state has been discussed using the Schmidt form for entangled nonorthogonal states [13] and quadrature-phase homodyne measurement [14]. A given state is nonlocal when it violates any Bell's inequality. In fact, a state does not have to violate all the possible Bell's inequalities to be considered quantum nonlocal. A state is quantum nonlocal for the given Bell's inequality that is violated by the measurement of the state. Banaszek and Wódkiewicz defined Bell's inequality based on parity measurement and they found that the two-mode squeezed state violates Bell's inequality, showing quantum nonlocality [15].

In this paper we study the dynamic behavior of the quantum nonlocality based on parity measurement for a two-mode squeezed state in a thermal environment. Measurement of the degree of quantum nonlocality is defined here by the maximal violation of Bell's inequality. The nonlocality is stronger for a squeezed state with a larger degree of squeezing. It is found that the nonlocality disappears more rapidly in the thermal environment as the initial state is squeezed more.

This paper is organized as follows. In Sec. II, Bell's inequality based on parity measurement is discussed. The parity measurement is directly related to the Wigner function. In Sec. III, the two-mode master equation is solved for the dynamics of the Wigner function of the initial two-mode squeezed state. Convolution theory is utilized in the solution [16]. We investigate the dynamic behavior of the quantum nonlocality measured by the maximum violation of Bell's inequality for the two-mode squeezed state in a thermal environment in Sec. IV.

II. BELL'S INEQUALITY BY PARITY MEASUREMENT

It is important to choose the type of measurement variables when testing nonlocality for a given state. In the original gedanken experiment [1], Einstein, Podolsky, and Rosen (EPR) considered the positions (or the momenta) of two particles as the measurement variables to discuss two-body correlation. Bell [2] argued that the EPR wave function does not exhibit nonlocality because its Wigner function $W(x_1,p_1;x_2,p_2)$ is positive everywhere, allowing its description by a local hidden variable theory. Munro showed that various types of Bell's inequalities are not violated in terms of homodyne measurements of two particles [14,17]. To the contrary, Banaszek and Wódkiewicz [15] examined even and odd parities as the measurement variables and showed that the EPR state and the two-mode squeezed state are nonlocal in the sense that they violate Bell's inequalities such as the Clauser and Horne inequality and the Clauser-Horne-Shimony-Holt inequality.

The even and odd parity operators, $\hat{O}_e$ and $\hat{O}_o$, are the projection operators measuring the probabilities of the field

*Electronic address: jeonghs@quanta.sogang.ac.kr
†Electronic address: hyoung@quanta.sogang.ac.kr

having even and odd numbers of photons, respectively:

$$\hat{O}_e=\sum_{n=0}^{\infty}|2n\rangle\langle 2n|;\quad \hat{O}_0=\sum_{n=0}^{\infty}|2n+1\rangle\langle 2n+1|. \tag{1}$$

The Wigner function at the origin of phase space for a state of density operator $\hat{\rho}$ is proportional to the mean parity [18],

$$W(0)=(2/\pi)\mathrm{Tr}[(\hat{O}_e-\hat{O}_o)\hat{\rho}]. \tag{2}$$

Further, the Wigner function $W(\alpha)$ at the phase point α is the mean parity for the displaced original state,

$$W(\alpha)=(2/\pi)\mathrm{Tr}[(\hat{O}_e-\hat{O}_o)\hat{D}(\alpha)\hat{\rho}\hat{D}^{\dagger}(\alpha)], \tag{3}$$

where $\hat{D}(\alpha)$ is the displacement operator [18].

So far the argument has been confined to parity measurement of a single-mode field. As the quantum nonlocality can be discussed for two-mode fields, we thus define the quantum correlation operator based on joint parity measurements:

$$\begin{aligned}\hat{\Pi}^{ab}(\alpha,\beta)=&\hat{\Pi}_e^a(\alpha)\hat{\Pi}_e^b(\beta)-\hat{\Pi}_e^a(\alpha)\hat{\Pi}_o^b(\beta)\\&-\hat{\Pi}_o^{\alpha}(\alpha)\hat{\Pi}_e^b(\beta)+\hat{\Pi}_o^a(\alpha)\hat{\Pi}_o^b(\beta),\end{aligned} \tag{4}$$

where the superscripts a and b denote the modes and the displaced parity operator $\hat{\Pi}_{e,o}(\alpha)$ is defined as

$$\hat{\Pi}_{e,o}(\alpha)=\hat{D}(\alpha)\hat{O}_{e,o}\hat{D}^{\dagger}(\alpha). \tag{5}$$

The displaced parity operator acts like a rotated spin projection operator in a spin measurement. We can easily derive that the local hidden variable theory imposes the following Bell's inequality [15]:

$$\begin{aligned}|B(\alpha,\beta)|\equiv|\langle&\hat{\Pi}^{ab}(\alpha,\beta)+\hat{\Pi}^{ab}(\alpha,\beta')\\&+\hat{\Pi}^{ab}(\alpha',\beta)-\hat{\Pi}^{ab}(\alpha',\beta')\rangle|\leqslant 2,\end{aligned} \tag{6}$$

where $B(\alpha,\beta)$ is the Bell function.

By a simple extension of the relation (3), the two-mode Wigner function is found to be proportional to the mean of $\hat{\Pi}_{ab}$ such that $W(\alpha,\beta)=(4/\pi^2)\mathrm{Tr}[\hat{\rho}_{ab}\hat{\Pi}^{ab}(\alpha,\beta)]$ for the two-mode state of density operator $\hat{\rho}_{ab}$. The Bell function (6) can then be written in terms of the Wigner functions at different phase-space points,

$$B(\alpha,\beta)=\frac{\pi^2}{4}[W(0,0)+W(\alpha,0)+W(0,\beta)-W(\alpha,\beta)]. \tag{7}$$

The type of Bell's inequality in Eq. (6) was first discussed by Clauser, Horne, Shimony, and Holt [11]. Clauser and Horne later found another type of inequality which can also be expressed in phase space using the quasiprobability Q function [15]. The Q function is related to the probability of the state having no photons. The lower and upper critical values of the Clauser-Horne Bell's inequality are -1 and 0.

We have seen that the two-mode Wigner function is useful to test quantum nonlocality of the given field so that, in the next section, we find the evolution of the Wigner function for the initial two-mode squeezed state coupled with a thermal environment.

III. TIME EVOLUTION OF TWO-MODE SQUEEZED STATES IN A THERMAL ENVIRONMENT

A two-mode squeezed state is the correlated state of two field modes a and b that can be generated by a nonlinear $\chi^{(2)}$ medium [19,20]. A two-mode pure squeezed state is obtained by applying the unitary operator on the two-mode vacuum,

$$|\Psi_{ab}(\sigma)\rangle=\exp(-\sigma\hat{a}\hat{b}+\sigma^*\hat{b}^{\dagger}\hat{a}^{\dagger})|0_a,0_b\rangle, \tag{8}$$

where $\sigma=s\exp(-i\varphi)$ and $\hat{a}(\hat{b})$ is an annihilation operator for the mode $a(b)$. The value of s determines the degree of squeezing. The larger s, the more the state is squeezed.

The Wigner function corresponding to the squeezed state is the Fourier transform of its characteristic function $C_W(\zeta,\eta)$ [20],

$$C_W(\zeta,\eta)=\mathrm{Tr}[\hat{\rho}\exp(\zeta\hat{a}^{\dagger}-\zeta^*\hat{a})\exp(\eta\hat{b}^{\dagger}-\eta^*\hat{b})]. \tag{9}$$

For the two-mode squeezed state of density matrix $\hat{\rho}=|\Psi_{ab}(\sigma)\rangle\langle\Psi_{ab}(\sigma)|$, the Wigner function is written as

$$\begin{aligned}W_{ab}(\alpha,\beta)=&\frac{4}{\pi^2}\exp[-2\cosh(2s)(|\alpha|^2+|\beta|^2)\\&+2\sinh(2s)(\alpha\beta+\alpha^*\beta^*)].\end{aligned} \tag{10}$$

The correlated nature of the two-mode squeezed state is exhibited by the $\alpha\beta$ cross term which vanishes when $s=0$.

The Fokker-Planck equation (in Born-Markov approximation) describing the time evolution of the Wigner function in the interaction picture can be written as

$$\begin{aligned}\frac{\partial W_{ab}(\alpha,\beta,\tau)}{\partial\tau}=&\frac{\gamma}{2}\sum_{\alpha_i=\alpha,\beta}\left[\frac{\partial}{\partial\alpha_i}\alpha_i+\frac{\partial}{\partial\alpha_i^*}\alpha_i^*\right.\\&\left.+2\left(\frac{1}{2}+\overline{n}\right)\frac{\partial^2}{\partial\alpha_i\partial\alpha_i^*}\right]W_{ab}(\alpha,\beta,\tau),\end{aligned} \tag{11}$$

where we have assumed that the two modes of the environment are independent of each other and the energy decay rates of the two modes are the same and denoted by γ. The two modes have the same average thermal photon number $\overline{n}$. By solving the Fokker-Planck equation (11), we get the time evolution of the Wigner function at time τ to be given by the convolution of the original function and the thermal environment [16],

$$\begin{aligned}W_{ab}(\alpha,\beta,\tau)=&\frac{1}{t(\tau)^4}\int d^2\zeta\, d^2\eta\, W_a^{\mathrm{th}}(\zeta)W_b^{\mathrm{th}}(\eta)\\&\times W_{ab}\left(\frac{\alpha-r(\tau)\zeta}{t(\tau)},\frac{\beta-r(\tau)\eta}{t(\tau)},\tau=0\right),\end{aligned} \tag{12}$$

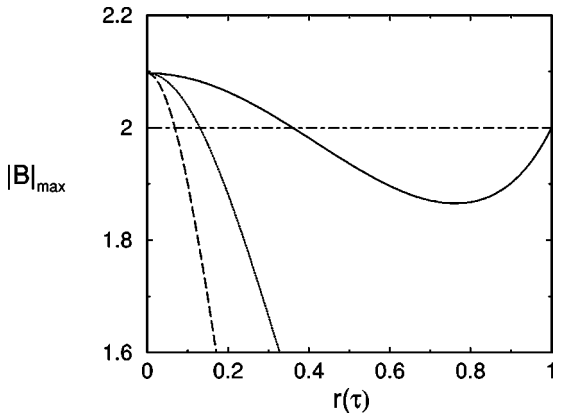

FIG. 1. The time evolution of the maximal value $|B|_{\max}$ of the Bell function versus the dimensionless time $r(\tau)\equiv\sqrt{1-\exp(-\gamma\tau)}$ which is 0 at $\tau=0$ and 1 at $\tau=\infty$. The initial degree of squeezing $s=0.3$ and the average photon number $\overline{n}$ of the thermal environment is $\overline{n}=0$ (solid line), $\overline{n}=0.5$ (dotted line), and $\overline{n}=2$ (dashed line). The larger $\overline{n}$, the more rapidly the nonlocality is lost.

where the parameters $r(\tau)=\sqrt{1-e^{-\gamma\tau}}$ and $t(\tau)=\sqrt{e^{-\gamma\tau}}$. $W^{\rm th}(\zeta)$ is the Wigner function for the thermal state of average thermal photon number $\overline{n}$:

$$W^{\rm th}(\zeta)=\frac{2}{\pi(1+2\overline{n})}\exp\left(-\frac{2|\zeta|^2}{1+2\overline{n}}\right). \tag{13}$$

Performing the integration in Eq. (12), the Wigner function for the initial two-mode squeezed state evolving in the thermal environment is obtained as

$$W_{ab}(\alpha,\beta,\tau)=\mathcal{N}\exp[-E(\tau)(|\alpha|^2+|\beta|^2) +F(\tau)(\alpha\beta+\alpha^*\beta^*)] \tag{14}$$

where

$$E(\tau)=\frac{2r(\tau)^2(1+2\overline{n})+2t(\tau)^2\cosh 2s}{D(\tau)},$$

$$F(\tau)=\frac{2t(\tau)^2\sinh 2s}{D(\tau)},$$

$$D(\tau)=t(\tau)^4+2r(\tau)^2t(\tau)^2(1+2\overline{n})\cosh 2s +r(\tau)^4(1+2\overline{n})^2, \tag{15}$$

and $\mathcal{N}$ is the normalization factor. In the limit of $s=0$, the $\alpha\beta$ cross term vanishes and the state can be represented by the direct product of each mode state such that $W_{ab}(\alpha,\beta,\tau)=W_a(\alpha,\tau)W_b(\beta,\tau)$. It is obvious that the Wigner function (14) exhibits local characteristics in this limit.

The system will eventually assimilate with the environment. This can be seen in the Wigner function, at the limit of $\tau\to\infty$,

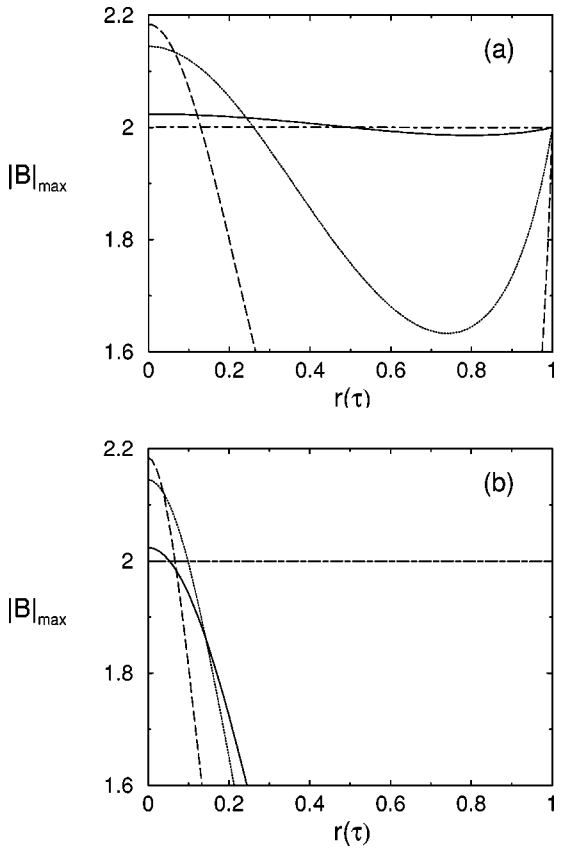

FIG. 2. The time evolution of $|B|_{\max}$ versus $r(\tau)\equiv\sqrt{1-\exp(-\gamma\tau)}$ when the squeezed state is prepared with the initial degree of squeezing $s=0.1$ (solid line), $s=0.5$ (dotted line), and $s=1.0$ (dashed line). The two-mode squeezed state is coupled with the $\overline{n}=0$ vacuum (a) and the $\overline{n}=1$ thermal environment (b). In the vacuum, the larger the degree of squeezing, the more rapidly the nonlocality is lost. In the $\overline{n}=1$ thermal environment, we find that the nonlocality persists longer when the squeezing is $s\sim 0.5$.

$$W_{ab}(\alpha,\beta)=\frac{4}{\pi^2(1+\overline{n})^2}\exp\left(-\frac{2}{(1+2\overline{n})}(|\alpha|^2+|\beta|^2)\right). \tag{16}$$

This is the direct product of two thermal states in modes a and b.

IV. EVOLUTION OF QUANTUM NONLOCALITY

Substituting Eq. (14) into Eq. (7), we find the evolution of nonlocality for the initial two-mode squeezed state in a thermal environment. The Bell function B at time τ is given by

$$B(\alpha,\beta,\tau)=\frac{\pi^2\mathcal{N}}{4}\exp\{1+\exp[-E(\tau)|\alpha|^2] +\exp[-E(\tau)|\beta|^2]-\exp[-E(\tau)(|\alpha|^2+|\beta|^2) +2F(\tau)|\alpha\beta|\cos\theta]\}, \tag{17}$$

where θ_α and θ_β are the phases of α and β and $\theta=\theta_\alpha+\theta_\beta$. When $\cos\theta=-1$, the Bell function $B_m(|\alpha|,|\beta|,\tau)$ is described by the absolute values $|\alpha|$ and $|\beta|$. B_m is symmetric

in exchanging α and β such that $B_m(|\alpha|,|\beta|,\tau) = B_m(|\beta|,|\alpha|,\tau)$. It is straightforward to show that $B \leqslant B_m$ at any instant of time τ. In order to find the evolution of the nonlocality, the maximal value $|B|_{\max}$ of the Bell function B is calculated by the steepest descent method [21] and using the properties of $B_m(|\alpha|,|\beta|,\tau)$. *We say the field is quantum-mechanically nonlocal as* $|B|_{\max}$ *is larger than 2 and the nonlocality is stronger as* $|B|_{\max}$ *gets larger.*

The initial two-mode squeezed state is always nonlocal as $|B|_{\max} > 2$ for $s > 0$. $|B|_{\max}$ increases monotonically as the degree s of squeezing increases. The state becomes maximally nonlocal with $|B|_{\max} \sim 2.190\,55$ as $s \to \infty$ [15]. At an intermediate time $0 < \tau < \infty$, the pure squeezed state evolves to a two-mode mixed squeezed state and nonlocality is lost at a certain time of evolution. Figures 1 and 2 show $|B|_{\max}$ versus the dimensionless time $r(\tau)$ defined in Eq. (12). We find that the nonlocality initially prepared persists until a characteristic time $\tau_c(s,\bar{n})$ depending on the temperature of the thermal environment and the initial squeezing. In Fig. 1 it is found that, when the environment is the vacuum, $|B|_{\max}$ decreases as time proceeds. After reaching a minimum value, $|B|_{\max}$ increases to 2, which is the value of $|B|_{\max}$ for the vacuum. Even though it is not clearly seen in the figure due to the scale, for any $\bar{n} \neq 0$ thermal environment, $|B|_{\max}$ increases to its value for the thermal field after decreasing to a minimum. In Fig. 1, as $\bar{n}$ gets larger $|B|_{\max}$ decreases much faster and further.

In Fig. 2, we identify an interesting phenomenon: the larger the initial degree of squeezing the more rapidly $|B|_{\max}$ decreases. We analyze the reason why $|B|_{\max}$ decreases more rapidly for larger initial squeezing as follows.

The two-mode squeezed state (8) can be represented by the continuous superposition of two-mode coherent states (a similar analysis has been done for a single-mode squeezed state [22]),

$$|\Psi_{ab}(\sigma)\rangle = \int d^2\alpha\, G(\alpha,\sigma)|\alpha,\alpha^* e^{i\varphi}\rangle, \tag{18}$$

where the Gaussian weight function

$$G(\alpha,\sigma) = (\pi \sinh s)^{-1} \exp\left[-\left(\frac{1-\tanh s}{\tanh s}\right)|\alpha|^2\right]. \tag{19}$$

As s gets larger, the weight of a large-α state is greater, so that the contribution of $|\alpha,\alpha^* e^{i\varphi}\rangle$ for a large α becomes more important in the continuous superposition (18).

The quantum interference between coherent component states is the key to the quantum nature of the field. The quantum interference is destroyed by the environment. The speed of destruction depends on the distance between the coherent component states and the average thermal energy of the environment [23]. This is the reason why a macroscopic quantum superposition state is not easily seen in nature. In the continuous superposition (18) we find that as the degree of squeezing is larger, the superposition extends further, so that the quantum interference can be destroyed more easily. Quantum nonlocality in a two-mode squeezed state also originates from quantum interference between the coherent component states, which can be destroyed easily as the contribution of the large-amplitude coherent state becomes important.

In fact, the uncertainty increases to its maximum and then decreases to the value of the environment when a single-mode squeezed state is influenced by a thermal environment [24]. The uncertainty increases faster as the degree of squeezing is larger. This can be explained using the same argument as the loss of quantum nonlocality.

In Fig. 2(a), when the environment is the vacuum, it is found that the characteristic time $\tau_c(s,\bar{n})$ to lose the quantum nonlocality is shorter as the initial degree of squeezing is larger. In Fig. 2(b), with a nonzero temperature thermal environment ($\bar{n} \neq 0$), we find that a larger degree of squeezing does not necessarily result in a shorter characteristic time $\tau_c(s,\bar{n})$. This clearly shows that the characteristic time is a function of the average number of thermal photons as well as the degree of squeezing. However, it is still true that $|B|_{\max}$ decreases faster (the slope of its curve is steeper) when s is larger. It is also found that $|B|_{\max}$ decreases faster for $\bar{n} \neq 0$ than for $\bar{n} = 0$.

We have studied the dynamic behavior of nonlocality for a two-mode squeezed state in a thermal environment. A two-mode squeezed state can be used for the quantum channel in quantum teleportation of a continuous-variable state. The two-mode squeezed state is found to be a nonlocal state regardless of its degree of squeezing, and a higher degree of squeezing brings about a larger quantum nonlocality. As the squeezed state is influenced by the thermal environment, the nonlocality is lost. The rapidity of the loss of nonlocality depends on the initial degree of squeezing and on the average thermal energy of the environment. The more strongly the initial field is squeezed, the more rapidly the maximum nonlocality decreases. This has been analyzed extensively.

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