Coarsening Measurement References and the Quantum-to-Classical Transition

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We investigate the role of inefficiency in quantum measurements in the quantum-to-classical transition, and consistently observe the quantum-to-classical transition by coarsening the references of the measurements (e.g., when and where to measure). Our result suggests that the definition of measurement precision in quantum theory should include the degree of the observer's ability to precisely control the measurement references.

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Introduction.—Typical quantum phenomena observed on microscopic scales somehow disappear on macroscopic scales. There have been trials to explain the quantum-to-classical transition. Decoherence is one of the well known and successful attempts to explain such a revelation of a classical world out of quantum mechanical rules [1]. There are two crucial elements in the framework of quantum mechanics: one is the *state* of a physical system represented by a wave function, and the other is the *measurement* represented by non-negative operators. The decoherence program focuses on the evolution of the state: it describes a transition of a quantum state to a classical one due to its interactions with environments.

Recently, a different point of view was presented [2], where coarsening of measurements is attributed to the cause of the quantum-to-classical transition. Along this line, it was also pointed out that coarsening of measurements makes it hard to detect micro-macro entanglement in optical systems [3]. However, there exist seemingly contradicting results where even fuzzy measurements allow us to observe severe violations of Bell's inequality [4,5] and also of the Leggett-Garg inequality [6]. It means that fuzziness in measurements does not always result in the quantum-to-classical transition. There is yet another example in which coarsening measurements results in local realism under stronger restrictions [7]. There have been extensive attempts to clarify sophisticated conditions of the quantum-to-classical transition [8–12], and it has been found that the quantum-to-classical transition does not always occur when it is expected [9,11,12]. Indeed, a condition of the measurement process in which the quantum-to-classical transition is definitely forced to occur is yet to be found.

In fact, a complete measurement process is composed of two parts: the one part is to set a measurement reference and control it while the other is the final detection with the corresponding projection operator. The aforementioned works to explain the quantum-to-classical transition have focused on the role of inefficiency in the final detection by coarsening its measuring resolution. On the other hand, the control of the measurement reference is described by an appropriate unitary operator with a reference variable applied to the projection operator. It is worth investigating the role of the measurement reference by coarsening the accuracy of this unitary operation. Such unitary operations are often indispensable when strong quantum effects, incompatible with classical physics, are observed by standard tools such as Bell's inequality [13] and the Leggett-Garg inequality [14].

Does the accuracy of controlling such measurement references play a crucial role in the quantum-to-classical transition? In this Letter, we intensively tackle this question using a generic example of macroscopic entanglement together with specific physical examples. Our study clearly shows that coarsening of the final measurement resolution and that of the measurement reference lead to completely different results. The quantum-to-classical transition is forced to occur when the reference of measurement is coarsened, while it is not the case when only the final projection is coarsened. This aspect of the "accuracy of the measurement reference" has not received proper attention in the context of the quantum-to-classical transition. We believe that our discussion, by clarifying the conditions of the quantum-to-classical transition, sheds light upon the appearance of a classical world from another angle.

Generic study.—We first consider a generic example with an infinite dimensional system together with an orthonormal basis set $\{|o_n\rangle\}$, where n takes integer indexes from the minus to the plus infinities. Let us consider observable $O^k = O^k_+ - O^k_-$, where

$$O^k_+ = \sum_{n=k+1}^{\infty} |o_n\rangle\langle o_n|, \qquad O^k_- = \sum_{n=-\infty}^{k} |o_n\rangle\langle o_n|, \qquad (1)$$

and O^k represents a "sharp" dichotomic measurement with eigenvalues ± 1. A fuzzy version of this dichotomic measurement may be written as

$$O_\delta = \sum_{k=-\infty}^{\infty} P_\delta(k) O^k, \qquad (2)$$

where $P_\delta(k) = (1/\delta\sqrt{2\pi})\exp[-(k^2/2\delta^2)]$ is the normalized Gaussian kernel with standard deviation δ. Here, δ defines the degree of fuzziness in the measurement, i.e., the degree of coarsening in the final measurement resolution. We should assume $\delta > 1$ in order to satisfy the normalization condition with the discrete version of the Gaussian function; however, this does not affect any essential aspects of our discussions. We then say that two states are macroscopically distinguishable if they can be distinguished with a small error probability using O_δ with a large value of δ. For example, states $|o_n\rangle$ and $|o_{-n}\rangle$ can be discriminated with the error probability of

$$P_e = 1 - \left[\sum_{k=-\infty}^{\infty} \frac{1}{\delta\sqrt{2\pi}} e^{-(k^2/2\delta^2)} \chi_{n-k}\right]^2, \qquad (3)$$

where χ_j is 1 for $j > 0$ (-1 for $j \leq 0$). Naturally, one can introduce a type of entanglement as follows:

$$|M_n\rangle = \frac{1}{\sqrt{2}}(|o_n\rangle|o_{-n}\rangle + |o_{-n}\rangle|o_n\rangle), \qquad (4)$$

which would become macroscopic entanglement when n is sufficiently large.

We now consider a unitary transform $U(\theta)$, that is the rotation between two states $|o_n\rangle$, $|o_{-n}\rangle$:

$$U(\theta)|o_n\rangle = \cos\theta|o_n\rangle + \sin\theta|o_{-n}\rangle,$$
$$U(\theta)|o_{-n}\rangle = \sin\theta|o_n\rangle - \cos\theta|o_{-n}\rangle. \qquad (5)$$

If one considers measuring a spin-1/2 system or polarization of a photon, the unitary operation simply implies a rotation of the measurement axis. The coarsened version of the unitary operation applied to the projection operator O_δ can be described as

$$O_{\delta,\Delta}(\theta_0) = \int d\theta P_\Delta(\theta - \theta_0)[U^\dagger(\theta) O_\delta U(\theta)], \qquad (6)$$

where $P_\Delta(\theta - \theta_0)$ is the Gaussian kernel centered around θ_0 with standard deviation Δ. In contrast to the value of δ in Eq. (2), Δ in Eq. (6) quantifies the degree of coarsening in the measurement reference.

Now, we study the Bell-Clauser-Horne-Shimony-Holt (Bell-CHSH) inequality [13,15] using the entangled state in Eq. (4). The correlation function is the expectation value of the measurement operators as

$$E_{\delta,\Delta}(\theta_a, \theta_b) = \langle O_{\delta,\Delta}(\theta_a) \otimes O_{\delta,\Delta}(\theta_b)\rangle_{a,b}, \qquad (7)$$

where the average is taken over entangled state $|M_n\rangle_{ab}$. Let us first consider that the unitary transform $U(\theta)$ can be perfectly controlled ($\Delta = 0$) but the final action of measurement is inaccurate ($\delta > 1$). We then obtain an explicit expression of $E_\delta(\theta_a, \theta_b)$ as

$$E_\delta(\theta_a, \theta_b) = \frac{1}{2}[f_\delta(n,\theta_1) f_\delta(-n,\theta_2) + f_\delta(-n,\theta_1) f_\delta(n,\theta_2) + 2 g_\delta(n,\theta_1) g_\delta(n,\theta_2)], \qquad (8)$$

where $f_\delta(n,\theta) = \sum_{k=-\infty}^{\infty} P_\delta(k)(\cos^2\theta\chi_{n-k} + \sin^2\theta\chi_{-n-k})$ and $g_\delta(n,\theta) = \sin\theta\cos\theta\sum_{k=-\infty}^{\infty} P_\delta(k)(\chi_{n-k} - \chi_{-n-k})$. The Bell function can be obtained as

$$B = E_\delta(\theta_a, \theta_b) + E_\delta(\theta'_a, \theta_b) + E_\delta(\theta_a, \theta'_b) - E_\delta(\theta'_a, \theta'_b), \qquad (9)$$

which should satisfy $|B| \leq 2$ by the assumption of local realism [15]. We plot the numerically optimized Bell function in Fig. 1(a) for n and δ. Obviously, an arbitrarily large value of δ can be compensated by increasing n in order to observe violation of the Bell-CHSH inequality.

We also consider the case in which the unitary transform is coarsened while the efficiency of the final measurement is perfect. In this case, we set Δ to be nonzero while $\delta = 0$. The explicit form of the correlation function is then obtained as

$$E_\Delta(\theta_a, \theta_b) = -\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d\phi_a d\phi_b P_\Delta(\phi_a - \theta_a) \times P_\Delta(\phi_b - \theta_b)\cos[2(\phi_a + \phi_b)]. \qquad (10)$$

Obviously, $E_\Delta(\theta_a, \theta_b)$ is *independent* from the value of n, i.e., macroscopicity of entanglement. In Fig. 1(b), it is clear that regardless of the value of n, the increase of Δ will totally destroy violation of Bell's inequality. We have analyzed the Bell-CHSH inequality but the Leggett-Garg inequality may be considered in the same way by considering a time-dependent unitary operation $U(\theta)$ with $\theta = \omega t$. In what follows, we shall investigate specific physical examples both for the Bell-CHSH and Leggett-Garg inequalities.

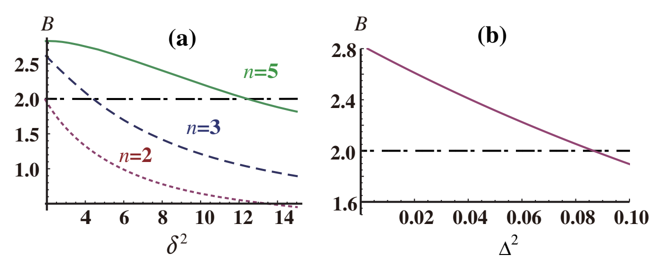

FIG. 1 (color online). Numerically optimized Bell function B of generic entanglement of size factor n against (a) variance δ^2 of the final measurement and (b) variance Δ^2 of the measurement reference. The dot-dashed line indicates the classical limit, 2. As the coarsening degree δ^2 of the final measurement increases in panel (a), the Bell function decreases but this effect can be compensated by increasing the size n of macroscopic entanglement (dotted curve: $n = 2$, dashed curve: $n = 3$, solid curve: $n = 5$). However, panel (b) shows that the Bell function rapidly decreases independent of n when the measurement reference is coarsened.

Bell's inequality with entangled photon number states.—We first consider entangled number state of photons,

$$|\psi_n\rangle = \frac{1}{\sqrt{2}}(|n_H\rangle|n_V\rangle + |n_V\rangle|n_H\rangle), \tag{11}$$

where $|n_H\rangle \equiv |H\rangle^{\otimes n}$ denotes horizontally polarized n photons and $|n_V\rangle \equiv |V\rangle^{\otimes n}$ vertically polarized. If we set $|n_H\rangle \equiv |o_n\rangle$, $|n_V\rangle \equiv |o_{-n}\rangle$, this system is identical to Eq. (4). We then need to find a physical example of a unitary operation such as Eq. (5). We adopt the unitary operation, $U_p(\theta) = \exp[i\theta(|n_H\rangle\langle n_V| + \text{H.c.})]$, a rotation about the x axis of the Bloch sphere of a polarized number-state qubit $\{|n_H\rangle, |n_V\rangle\}$. As this unitary operation depends on the photon number n, it needs the nonlinear Hamiltonian $\hat{H}_n = g(\hat{a}_H^n \hat{a}_V^{\dagger n} e^{i\phi} + \text{H.c.})$ to be realized. One can, in principle, implement this type of highly nonlinear Hamiltonian by decomposing it into a series of Gaussian unitaries and cubic operations [16,17].

Considering a realistic condition of photon loss, we use a dichotomic measurement operator

$$\mathcal{O}^p = \sum_{k=1}^{n}(|k_H\rangle\langle k_H| - |k_V\rangle\langle k_V|) + |0\rangle\langle 0| \tag{12}$$

and model an inefficient measurement using a beam splitter before the final photodetector. With its efficiency η, a photodetector for field mode a is described by a perfect detector after a beam splitter $\hat{\mathcal{B}}_{aa'}(\eta) = e^{\zeta(\hat{a}^\dagger \hat{a}' - \hat{a}\hat{a}'^\dagger)/2}$, where $\eta = (\cos\zeta)^2$ and a' represents the vacuum mode. The beam splitter parameter ζ, which determines the transmission ratio η, represents the degree of coarsening in the final detection. It was shown [5] that the Bell-CHSH inequality is violated even by highly inefficient detectors, i.e., η very small as n can be made sufficiently large. In other words, inefficiency of the final detector can be compensated by increasing number n of the entangled photon-number state. However, when the fuzziness of the unitary operation $U_p(\theta)$ is considered with a Gaussian noise as in Eq. (6) without coarsening the final detection, it is straightforward to show that the correlation function is exactly the same as Eq. (10).

It would be interesting to consider both the unitary transform and the final detection being coarsened in order to investigate more realistic scenarios. The correlation function can be obtained as

$$E_{\eta,\Delta}(\theta_a, \theta_b) = \langle \mathcal{O}^p_{\eta,\Delta}(\theta_a) \otimes \mathcal{O}^p_{\eta,\Delta}(\theta_b)\rangle_{a,b}, \tag{13}$$

where

$$\mathcal{O}^p_{\eta,\Delta}(\theta_a) = \int d\theta P_\Delta(\theta - \theta_a)[U^\dagger(\theta)\hat{\mathcal{B}}^\dagger_{aa'}\mathcal{O}^p\hat{\mathcal{B}}_{aa'}U(\theta)] \tag{14}$$

for mode a, and $\mathcal{O}^p_{\eta,\Delta}(\theta_b)$ is likewise defined. We then numerically calculate the optimized Bell function B for several values of n and plot the results in Fig. 2. It shows that the quantum-to-classical transition quickly occurs as fuzziness Δ of the unitary transform increases. The figure also shows that the decreasing rate of the Bell function caused by coarsening the unitary transform does not depend on the value of η.

Bell's inequality with entangled coherent states.—An entangled coherent state [18,19] $|\psi_\alpha\rangle \propto |\alpha,\alpha\rangle + |-\alpha,-\alpha\rangle$, where $|\pm\alpha\rangle$ are coherent states of amplitudes $\pm\alpha$, is considered as a macroscopic quantum state when α becomes large [20]. It is known [21] that effective rotations $U_\alpha(\theta)$ in the space spanned by the basis $\{|\alpha\rangle, |-\alpha\rangle\}$, required for a Bell's inequality test, can be performed using single-mode Kerr nonlinearities and displacement operations. A Bell test can then be performed using dichotomized homodyne measurements, where an eigenvalue $+1$ (-1) is assigned for any positive (negative) outcomes [4,5]. With homodyne efficiency η and Gaussian reference coarsening of θ with standard deviation Δ, the correlation function can be obtained in the same way described above using the measurement operator $\mathcal{O}^h = \int_0^\infty |x\rangle\langle x|dx - \int_{-\infty}^0 |x\rangle\langle x|dx$ that replaces $\mathcal{O}^p$ in Eqs. (13) and (14). Our numerical results [22] presented in Fig. 3 confirm that coarsening of the measurement reference cannot be made up by increasing macroscopicity α [Fig. 3(b)], while it can be made so when the measurement efficiency is coarsened [Fig. 3(a)]. Here, we can consider another interesting case where the angle of the homodyne detection, which should also be controlled precisely as a measurement reference, is coarsened, which leads to qualitatively the same conclusion [22].

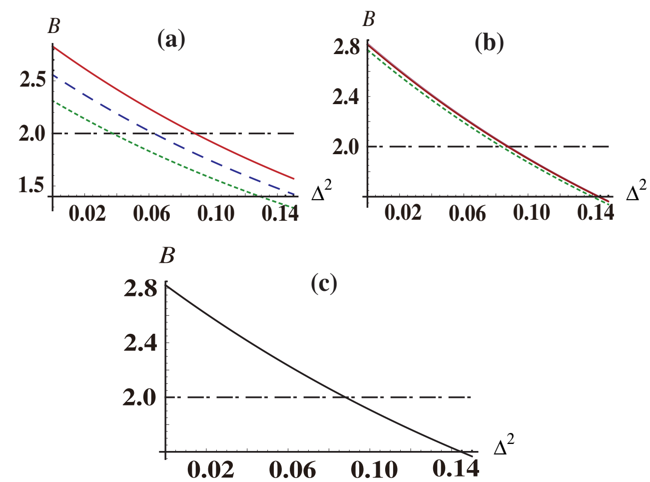

FIG. 2 (color online). Optimized Bell-CHSH function B for entangled photon number states with number n against variance Δ^2 of the Gaussian coarsening angle for different values of detection efficiency (solid curve: $\eta = 1$, dashed curve: $\eta = 0.95$, dotted curve: $\eta = 0.9$) for (a) $n = 1$, (b) $n = 2$, (c) $n = 3$. In the case of $n = 3$, all three curves virtually overlap.

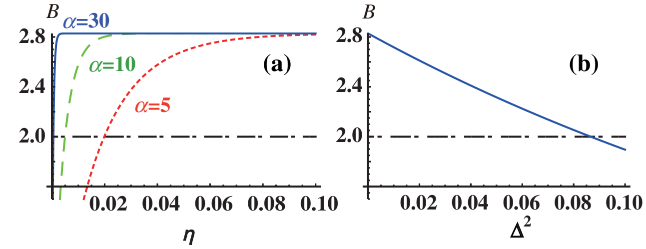

FIG. 3 (color online). Optimized Bell function B for entangled coherent states against (a) homodyne detector efficiency η and (b) variance Δ^2 of Gaussian coarsening of measurement reference for $\alpha = 5$ (dotted curve), $\alpha = 10$ (dashed curve) and $\alpha = 30$ (solid curve). While the decrease of the measurement efficiency can be made up by increasing α [panel (a)], coarsening of the measurement reference causes virtually the same decrease of the Bell function regardless of the values of α [panel (b)].

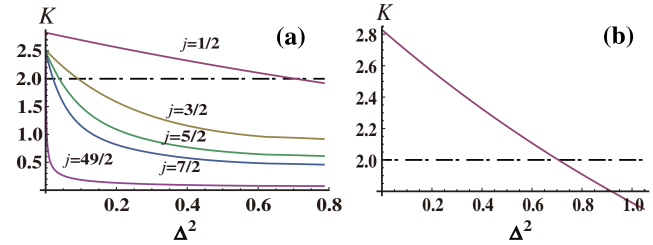

FIG. 4 (color online). (a) Optimized Leggett-Garg function K against variance Δ^2 of Gaussian coarsening of the measurement reference for different spin j states. For the larger values of j, the faster decrease of the Leggett-Garg function is observed. (b) Optimized Leggett-Garg fuction K against variance Δ^2 of Gaussian coarsening time under the nonclassical Hamiltonian. As the temporal reference of the measurement is coarsened by the increase of Δ^2, violation of the Leggett-Garg inequality disappears regardless of j.

Leggett-Garg inequality with spin systems.—The temporal correlation function $C_{ab} \equiv \langle Q(t_a)Q(t_b)\rangle$ between t_a and t_b for a dichotomic measurement operator Q forms the Leggett-Garg inequality

$$K \equiv C_{12} + C_{23} + C_{34} - C_{14} \leq 2, \tag{15}$$

which is forced by macroscopic realism [14]. In the case of the Leggett-Garg inequality that utilizes time sequential measurements, it is natural to consider coarsening of the temporal references. We shall consider coarsening of two types of unitary operations for spin-j systems considered in Refs. [2,6] with the dichotomized parity measurement $Q = \sum_{m=-j}^{j} (-1)^{j-m}|m\rangle\langle m|$, where $|m\rangle$ is a spin eigenstate of the spin-j operator $\hat{J}_z$. The first unitary operation to be considered is $U_j(\theta) = e^{-i\theta\hat{J}_x}$ with $\theta = \omega t$ and the initial state is assumed to be the maximally mixed spin-j system $\sum_{m=-j}^{j} |m\rangle\langle m|/(2j+1)$. We again consider Gaussian coarsening of the unitary operation applied to the measurement operator Q as $Q_\Delta(\theta_0) = \int d\theta P_\Delta(\theta - \theta_0)[U_j^\dagger(\theta) Q U_j(\theta)]$. The temporal correlation function between t_a and t_b can be obtained as $C_{ab} = p_{+_a+_b} + p_{-_a-_b} - p_{+_a-_b} - p_{-_a+_b}$, where $p_{+_a+_b}$ is the probability for measuring $+$ at t_a and then $+$ at t_b, and so on. After some calculation, we obtain

$$C_{ab} = \sum_{m=-j}^{j} \int_{-\infty}^{\infty} d\theta' P_\Delta(\theta' - \theta_{b-a}) e^{2im\theta'}/(2j+1), \tag{16}$$

where $\theta_{b-a} = \omega(t_b - t_a)$. We plot the numerically optimized Leggett-Garg function in Fig. 4(a) and observe the decrease of the Leggett-Garg function for any value of j by increasing the coarsening degree of the measurement reference. We note that the larger value of j leads to more rapid destruction of the Leggett-Garg violation by coarsening the measurement reference.

It was shown [6] that under a unitary operation that can generate a macroscopic superposition, the Leggett-Garg inequality is violated even with a coarsened measurement. The corresponding unitary operation is $U(\theta) = \exp[i\theta(|+j\rangle \times \langle -j| + \text{H.c.})]$ with $\theta = \omega t$ and this is identical to the unitary operation in Eq. (5) if $|\pm j\rangle$ are replaced with $|o_{\pm n}\rangle$. The nonclassical Hamiltonian associated with such a unitary operation is

$$\hat{H} = i\omega(|-j\rangle\langle +j| - |+j\rangle\langle -j|). \tag{17}$$

Assuming an initial state $|+j\rangle$, we can calculate the temporal correlation function C_{ab} by the same procedure described above, and it is found to be $C_{ab} = e^{-\Delta^2/2}\cos[\omega(t_b - t_a)]$ by applying the same Gaussian coarsening of the measurement reference. The temporal correlation function C_{ab} is obviously independent of j and the Leggett-Garg violation disappear by coarsening of the unitary operation as plotted in Fig. 4(b). Thus, the results for the spin system with the Leggett-Garg inequality are consistent with the previous ones.

Remarks.—There have been studies to explain the quantum-to-classical transition: they have focused on either the evolution of the state or the accuracy of the final measurement resolution. However, the accuracy of the measurement reference has not been properly considered in this context. Our study consistently shows that when a measurement reference such as the timing and the axis angle of the measurement is coarsened, it cannot be compensated by increasing "macroscopicity" of the quantum state or by using an interaction to generate such macroscopic quantum states. This is obviously not the case when only the measurement resolution is coarsened. Our investigation covers a wide range of physical systems from discrete to continuous variable systems using various degrees of freedom such as spins, polarizations, photon numbers, and quadrature variables. Even though our discussions mainly adopt terminologies in optics, they can be generalized to various physical systems such as atomic and mechanical systems [22].

Our result provides new insight into the quantum-to-classical transition from a different angle by revealing the importance of the observer's ability in controlling the measurement reference, and, more generally, the importance of preciseness in quantum operations.

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Note added.—At the completion of our work, we became aware of Ref. [23]. They considered superpositions of coherent states and suggested a conjecture that outcome precision or control precision has to increase in order to observe quantum effects. While the system considered in Ref. [23] is different from ours, the results in their work are consistent with our conclusions in emphasizing the importance of the "control precision" of quantum measurements.

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