Entanglement as the Symmetric Portion of Correlated Coherence

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We show that the symmetric portion of correlated coherence is always a valid quantifier of entanglement, and that this property is independent of the particular choice of coherence measure. This leads to an infinitely large class of coherence based entanglement monotones, which is always computable for pure states if the coherence measure is also computable. It is already known that every entanglement measure can be constructed as a coherence measure. The results presented here show that the converse is also true. The constructions that are presented can also be extended to include more general notions of nonclassical correlations, leading to quantifiers that are related to quantum discord, thus providing an avenue for unifying all such notions of quantum correlations under a single framework.

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Introduction.—Entanglement is perhaps the most well studied form of quantum correlations [1] and forms the basis of many useful quantum protocols, such as quantum cryptography [2], quantum teleportation [3], and superdense coding [4]. Generalized notions of quantum correlations that include but supersede entanglement have also been considered, most prominently in the form quantum discord [5,6], for which there is mounting evidence that nonclassical effects may persist in multipartite scenarios [7–10] even when entanglement is not available.

In a separate development, there is recently a growing amount of interest in the resource theory of coherence [11–13]. Such theories are not limited to the multipartite setting. Nonetheless, there is considerable interest in the study of correlations from the point of view of coherence [14–17]. Other applications include an ever increasing number of physical scenarios, such as quantum macroscopicity [18,19], quantum algorithms [20,21], interferometry [22], and nonclassical light [23–25]. Reference [26] provides a recent overview of the developments to date. Especially relevant are the results in Ref. [14]. There, it was shown that coherence can be faithfully converted into entanglement, and that each entanglement measure corresponds to a coherence measure of the type considered in Ref. [12].

In this Letter, we report a series of constructions that allow nonclassical correlations to be quantified using coherence measures. The arguments do not depend on the particular coherence measure used, and do not even depend on the particular flavor of coherence measure that is employed [11,12,27–29], so long as they satisfy some minimal set of properties. This suggests that entanglement and discord are intrinsically embedded in any reasonable resource theory of coherence. This also establishes that the converse of the relationship in Ref. [14] is true, so that for every coherence measure, there corresponds an entanglement measure. We stress that the arguments are not limited to the coherence resource theory proposed in Ref. [12], as our framework does not depend on the choice of noncoherence producing operations [26]. In addition, we also show that discordlike quantifiers of nonclassical correlations are naturally embedded in any such resource theories of coherence. This operation-free approach contrasts with other approaches considered in Refs. [28,30], which are based on some hybrid set of free operations from both coherence and entanglement theories.

Preliminaries.—We review some elementary concepts concerning coherence measures. Coherence is a basis dependent property of a quantum state. For a given fixed basis $\mathcal{B} = \{|i\rangle\}$, the set of incoherent states $\mathcal{I}$ is the set of quantum states with diagonal density matrices with respect to this basis. States with nonzero off diagonal elements form the set of coherent states that are nonclassical.

The notion of nonclassicality in coherence resource theories is unambiguous, but different resource theories sometimes consider different sets of noncoherence, producing operations in order to justify different coherence measures (see Ref. [26] for a summary). For our purposes, we will not require specific properties of such operations. Resource theories of coherence generally obey several axioms. Let $\mathcal{C}$ be a measure of coherence belonging to some coherence resource theory, then $\mathcal{C}(\rho)$ must satisfy the following: (C1) $\mathcal{C}(\rho) \geq 0$ for any quantum state ρ and equality holds if and only if $\rho \in \mathcal{I}$. (C2) The measure must not increase under a noncoherence producing map Φ, i.e., $\mathcal{C}(\rho) \geq \mathcal{C}[\Phi(\rho)]$. (C3) The measure must be convex, i.e., $\lambda\mathcal{C}(\rho) + (1-\lambda)\mathcal{C}(\sigma) \geq \mathcal{C}[\lambda\rho + (1-\lambda)\sigma]$, for any density matrix ρ and σ with $0 \leq \lambda \leq 1$. An additional property referred to as strong monotonicity is also sometimes considered [12], but strongly monotonic measures that are convex will satisfy the condition (C2), and so does not need to be considered separately.

The following quantity was considered in Ref. [15] while studying the relationship between coherence and quantum correlations:

$$\mathcal{C}(A:B|\rho_{AB}) \coloneqq \mathcal{C}(\rho_{AB}) - \mathcal{C}(\rho_A) - \mathcal{C}(\rho_B). \tag{1}$$

Where the coherence measure $\mathcal{C}$ was chosen to be the l_1 norm of coherence. It was noted that since it is always possible to choose local bases for the subsystems A and B such that $\mathcal{C}(\rho_A)$ and $\mathcal{C}(\rho_B)$ vanish, the coherence in the system is no longer stored locally, and must exist within the correlations between subsystems A and B.

It was further demonstrated that minimizing this quantity with respect to all possible local bases $\mathcal{B}_A$ and $\mathcal{B}_B$ satisfying $\mathcal{C}(\rho_A) = \mathcal{C}(\rho_B) = 0$ is related to quantum correlations such as discord and entanglement. Formally, they considered:

Definition 1: Correlated coherence.—

$$\mathcal{C}_{\min}(A:B|\rho_{AB}) \coloneqq \min_{\mathcal{B}_{A:B}} \mathcal{C}(A:B|\rho_{AB}), \tag{2}$$

where the minimization is performed over the set of local bases $\mathcal{B}_{A:B} \coloneqq \{(\mathcal{B}_A, \mathcal{B}_B) | \mathcal{C}(\rho_A) = \mathcal{C}(\rho_B) = 0\}$.

We will henceforth refer to Eq. (1) as the generalized correlated coherence and Eq. (2) as just the correlated coherence. The correlated coherence is invariant under local unitary operations, since for any state ρ_{AB} and local basis $\mathcal{B}_{A:B} = \{|i\rangle_A |j\rangle_B\}$, the correlated coherence for the state $U_A \rho_{AB} U_A^\dagger$ and basis $\mathcal{B}_{A:B} = \{U_A |i\rangle_A |j\rangle_B\}$ is identical. Subsequently, entropic versions of the generalized correlated coherence was also studied in Ref. [31] and more recently in Refs. [32,33].

In the next section, we prove that, using $\mathcal{C}_{\min}(A:B|\rho_{AB})$ as our basic building block, every coherence measure can be used to construct a valid entanglement quantifier.

Quantifying entanglement.—We begin with some necessary definitions.

Definition 2: Symmetric extensions.—A symmetric extension of a bipartite state $\rho_{A_1B_1}$ is an extension $\rho_{A_1,\ldots,A_nB_1,\ldots,B_n}$ satisfying $\mathrm{Tr}_{A_2,\ldots,A_nB_2,\ldots,B_n}(\rho_{A_1,\ldots,A_nB_1,\ldots,B_n}) = \rho_{A_1B_1}$ that is, up to local unitaries, invariant under the swap operation $\Phi_{\mathrm{SWAP}}^{A_i \leftrightarrow B_i}$ between any subsystems A_i of Alice and B_i of Bob; i.e., there exists some unitary $U_{A_1,\ldots,A_n}$ such that

$$\Phi_{\mathrm{SWAP}}^{A_i \leftrightarrow B_i}(U_{A_1,\ldots,A_n} \rho_{A_1,\ldots,A_nB_1,\ldots,B_n} U_{A_1,\ldots,A_n}^\dagger) = U_{A_1,\ldots,A_n} \rho_{A_1,\ldots,A_nB_1,\ldots,B_n} U_{A_1,\ldots,A_n}^\dagger \tag{3}$$

Note that this definition is different from the one considered in the separability criterion of Ref. [34], which also considered state extensions that exhibit a different kind of symmetry. Subsequently, for notational compactness, we will use unprimed letters A, B for the system of interest, and primed letters A', B' for the ancillas in the extension. We now consider the correlated coherence of such extensions.

Definition 3: Symmetric correlated coherence.—The symmetric correlated coherence, for any given coherence measure $\mathcal{C}$, is defined to be the following quantity:

$$E_{\mathcal{C}}(\rho_{AB}) = \min_{A'B'} \mathcal{C}_{\min}(AA':BB'|\rho_{AA'BB'}), \tag{4}$$

where the minimization is performed over all possible symmetric extensions of ρ_{AB}. Note that the ancillas A' and B' may, in general, be composite systems.

The above quantifies the minimum correlated coherence over extensions that possess exchange symmetry between Alice and Bob. Since such a minimization always decreases the correlated coherence, we interpret this quantity as the portion of the correlated coherence that is symmetric.

We now prove several elementary properties. First, we observe that $E_{\mathcal{C}}(\rho_{AB})$ is a convex function of state:

Proposition 1: Convexity.—$E_{\mathcal{C}}(\rho_{AB})$ is a convex function of state, i.e.,

$$\sum_i p_i E_{\mathcal{C}}(\rho_{AB}^i) \geq E_{\mathcal{C}}\left(\sum_i p_i \rho_{AB}^i\right) \tag{5}$$

where p_i defines some probability distribution s.t. $\sum_i p_i = 1$ and ρ_{AB}^i is any normalized quantum state.

Proof.—Let $\rho_{AA'BB'}^{i*}$ be the optimal extension such that $E_{\mathcal{C}}(\rho_{AB}^i) = \mathcal{C}_{\min}(AA':BB'|\rho_{AA'BB'}^{i*})$. We have the following chain of inequalities:

$$\begin{aligned}
&\sum_i p_i E_{\mathcal{C}}(\rho_{AB}^i) \\
&\quad = \sum_i p_i \mathcal{C}_{\min}(AA':BB'|\rho_{AA'BB'}^{i*}) && (6)\\
&\quad = \sum_i p_i \mathcal{C}_{\min}(AA'A'':BB'B''|\rho_{AA'BB'}^{i*} \otimes |i,i\rangle_{A''B''}\langle i,i|) && (7)\\
&\quad \geq \mathcal{C}_{\min}\left(AA'A'':BB'B''|\sum_i p_i \rho_{AA'BB'}^{i*} \otimes |i,i\rangle_{A''B''}\langle i,i|\right) && (8)\\
&\quad \geq E_{\mathcal{C}}\left(\sum_i p_i \rho_{AB}^i\right) && (9)
\end{aligned}$$

The inequality in Eq. (8) occurs because there is at least one local basis that is upper bounded by Eq. (7). To see this, suppose for every i and $\rho_{AA'BB'}^{i*}$, the optimal basis for evaluating $\mathcal{C}_{\min}(AA':BB'|\rho_{AA'BB'}^{i*})$ is $\{|\alpha_{i,j}\rangle_{AA'}|\beta_{i,k}\rangle_{BB'}\}$. Then it is clear that the optimal local basis for $\rho_{AA'BB'}^{i*} \otimes |i,i\rangle_{A''B''}\langle i,i|$ must be $\{|\alpha_{i,j}\rangle_{AA'}|i\rangle_{A''}|\beta_{i,k}\rangle_{BB'}|i\rangle_{B''}\}$ since this was just essentially a relabelling of the basis. Since the coherence measure $\mathcal{C}$ is convex, the classical mixture of quantum states cannot increase the amount of coherence with respect to the basis $\{|\alpha_{i,j}\rangle_{AA'}|i\rangle_{A''}|\beta_{i,k}\rangle_{BB'}|i\rangle_{B''}\}$. Finally, one can verify that the local coherences with respect to this basis is always zero, so this is just one particular local basis that satisfies the necessary constraints. The inequality in Eq. (9) comes from the observation that $\sum_i p_i \rho^*_{AA'BB'} \otimes |i,i\rangle_{A''B''}\langle i,i|$ is a particular symmetric extension of $\sum_i p_i \rho^i_{AB}$. ■

Next, we demonstrate the connection between $E_\mathcal{C}(\rho_{AB})$ and nonseparability, which defines entanglement.

Proposition 2: Faithfulness.—$E_\mathcal{C}(\rho_{AB}) = 0$ iff ρ_{AB} is separable, and strictly positive otherwise.

Proof.—First, note that all coherence measures are nonnegative over valid quantum states, and, as such, $E_\mathcal{C}(\rho_{AB})$, being the coherence of some extension (see Definition 3), must also be nonnegative.

Suppose some bipartite state ρ_{AB} is separable. By definition, a quantum state is separable when it can be written in the form $\rho_{AB} = \sum_i p_i \rho^i_A \otimes \sigma^i_B$. However, one may always write the local states in their diagonal forms $\rho^i_A = \sum_j q_{i,j} |a_{i,j}\rangle_A \langle a_{i,j}|$ and $\sigma^i_B = \sum_k r_{i,k} |b_{i,k}\rangle_B \langle b_{i,k}|$ which, up to a relabelling of variables, is equivalent to a convex sum of separable pure states of the form $\rho_{AB} = \sum_i s_i |a_i\rangle_A \langle a_i| \otimes |b_i\rangle_B \langle b_i|$. Note that $\{|a_i\rangle\}$ and $\{|b_i\rangle\}$ are not necessarily orthonormal sets, and that some of the states $|a_i\rangle$ and/or $|b_i\rangle$ may be identical for different subscripts i. This always permits an extension of the form $\rho_{AA'BB'} = \sum_i s_i |a_i\rangle_A \langle a_i| \otimes |i\rangle_{A'}\langle i| \otimes |b_i\rangle_B \langle b_i| \otimes |i\rangle_{B'}\langle i|$ for some orthonormal set $\{|i\rangle\}$. It can then be directly verified that $\mathcal{C}_{\min}(AA' \colon BB' | \rho_{AA'BB'}) = 0$ so we must have $E_\mathcal{C}(\rho_{AB}) = 0$ for every separable state.

We now prove the converse. Suppose $E_\mathcal{C}(\rho_{AB}) = 0$. Then there must exist some extension for which $\mathcal{C}_{\min}(AA' \colon BB' | \rho_{AA'BB'}) = 0$. This implies that there must exist local bases on AA' and on BB' such that the coherence is zero, so $\rho_{AA'BB'}$ must be diagonal in this basis, i.e., $\rho_{AA'BB'} = \sum_i t_i |\alpha_i\rangle_{AA'} \langle \alpha_i| \otimes |\beta_i\rangle_{BB'} \langle \beta_i|$. Directly tracing out the subsystems A' and B' will lead to a decomposition of the form $\rho_{AB} = \sum_i s_i |a_i\rangle_A \langle a_i| \otimes |b_i\rangle_B \langle b_i|$, so ρ_{AB} must be a separable state.

We then observe that since $E_\mathcal{C}(\rho_{AB})$ must be nonnegative, and it is zero iff ρ_{AB} is separable, then it must be strictly positive for every entangled state. □

Next, we show that $E_\mathcal{C}(\rho_{AB})$ always decreases under local operations and classical communications (LOCC) type operations.

Proposition 3: Monotonicity.—For any LOCC protocol represented by a quantum map Φ_{LOCC}, we have

$$E_\mathcal{C}(\rho_{AB}) \geq E_\mathcal{C}[\Phi_{\mathrm{LOCC}}(\rho_{AB})].$$

A sketch of the proof is as follows: first we observe that $E_\mathcal{C}$ is invariant under local unitary operations and the addition of a pure state ancilla. Since any local operation may be achieved by adding a pure state ancilla, performing a unitary, then tracing out the ancilla, the quantity $E_\mathcal{C}$ cannot increase under local quantum operations. Furthermore, the exchange symmetry of the state extension between Alice and Bob ensures that $E_\mathcal{C}$ is nonincreasing under the copying of classical information. Every LOCC operation may be decomposed as a local operation, classical communication, followed by another local operation, so $E_\mathcal{C}$ is necessarily nonincreasing under LOCC operations. A full proof is presented in Ref. [35].

An entanglement monotone is one that satisfies the conditions of convexity, faithfulness, and monotonicity (See Propositions 1–3), which leads to our main result.

Theorem 1: Entanglement monotone.—$E_\mathcal{C}$ is a valid entanglement monotone for every choice of coherence measure $\mathcal{C}$.

Entanglement for pure states.—Choosing the coherence measure to be the relative entropy of coherence, which is defined as $\mathcal{C}(\rho_{AB}) = \mathcal{S}[\Delta(\rho_{AB})] - \mathcal{S}(\rho_{AB})$ where $\Delta(\rho_{AB})$ is the completely dephased state [12], then for pure states, the measure $E_\mathcal{C}$ exactly coincides with the well-known entropy of entanglement. This is because pure quantum states only have trivial extensions and always permit the Schmidt decomposition $|\psi\rangle_{AB} = \sum_i \sqrt{\lambda_i}\, |i,i\rangle_{AB}$ where we observe that the local bases $\{|i\rangle_A\}$ and $\{|i\rangle_B\}$ satisfy the condition $\mathcal{C}(\rho_A) = \mathcal{C}(\rho_B) = 0$. In this basis, $\mathcal{S}[\Delta(\rho_{AB})] = \mathcal{S}(\sum_i \lambda_i |i,i\rangle_{AB}\langle i,i|) = \mathcal{S}[\mathrm{Tr}_B(|\psi\rangle_{AB}\langle\psi|)]$, which is the expression for the entropy of entanglement. This shows that the entropy of entanglement is at least an upper bound of $E_\mathcal{C}$. It remains to be proven that the local bases $\{|i\rangle_A\}$ and $\{|i\rangle_B\}$ achieve the required minimization in $E_\mathcal{C}$. This is in fact a property of all continuous coherence measures.

Theorem 2: $E_\mathcal{C}$ for pure states.—For any continuous coherence measure $\mathcal{C}$ and pure state $|\psi\rangle_{AB}$ with Schmidt decomposition $|\psi\rangle_{AB} = \sum_i \sqrt{\lambda_i}\, |i,i\rangle_{AB}$, $E_\mathcal{C}(|\psi\rangle_{AB}) = \mathcal{C}(|\psi\rangle_{AB})$, where the coherence is measured with respect to the local bases $\{|i\rangle_A\}$ and $\{|i\rangle_B\}$ specified by the Schmidt decomposition.

We sketch the proof here. First, we observe that from the Schmidt decomposition, $|\psi\rangle_{AB} = \sum_i \sqrt{\lambda_i}\, |i,i\rangle_{AB}$, if the coefficients λ_i are nondegenerate, then the local bases $\{|i\rangle_A\}$ and $\{|i\rangle_B\}$ are the only bases, up to overall phase factors, that satisfy the condition $\mathcal{C}(\rho_A) = \mathcal{C}(\rho_B) = 0$, which proves the theorem for the nondegenerate case. This can be extended to the degenerate case, by slightly perturbing the state and then applying Nielsen's Theorem [36]. A continuity argument then proves Theorem 2. A more complete discussion may be found in Ref. [35].

Theorem 2 reveals that, for every coherence measure and pure bipartite state, there is always a basis where the coherence exactly quantifies the entanglement. It also shows that for every coherence measure that is computable, there corresponds a computable entanglement measure over pure states. We have already seen that the relative entropy of coherence corresponds to the entropy of entanglement, and the same outcome is obtained if we were to choose the entanglement of formation [37]. For the l_1 norm of coherence, we get the simple closed form formula $E_{\mathcal{C}}(|\psi\rangle_{AB}) = \sum_{i\neq j}\sqrt{\lambda_i\lambda_j}$ where $|\psi\rangle_{AB} = \sum_i\sqrt{\lambda_i}|i,i\rangle_{AB}$. This turns out to be the concurrence for a 2 qubit pure state [38]. Other coherence measures with known closed form formulas include the affinity of coherence [39] and the geometric coherence [14]. See Fig. 1 for a comparison of these measures. In general, there exists an infinite number of computable coherence measures [40]. We also note that once one has an entanglement monotone for pure states, then it is possible to generalize it to mixed states via a convex roof construction [41], which provides yet another avenue for generating new entanglement measures from coherence measures.

Generalizations to quantum discord.—In the previous section, the symmetric portion of the correlated coherence was considered, in which case it was found to directly address entanglement. We now show that dropping the requirement of symmetry naturally leads to discordlike measures of correlation.

For quantum discord, the set of states that has zero discord and are thus "classical," are the set of classical-quantum states that can be written in the form $\rho_{AB} = \sum_i p_i|i\rangle_A\langle i|\otimes\rho_B^i$. One may readily address this set of classical-quantum states by considering extensions without the requirement that it is symmetric. Let us consider the following:

Definition 4: Asymmetric discord of coherence.—The asymmetric discord of coherence, for any given coherence measure $\mathcal{C}$, is defined to be the following quantity:

$$D_{\mathcal{C}}(\rho_{AB}) = \min_{B'}\mathcal{C}_{\min}(A:BB'|\rho_{ABB'}) \tag{10}$$

where the minimization is performed over all possible extensions satisfying $\mathrm{Tr}_{B'}(\rho_{ABB'}) = \rho_{AB}$.

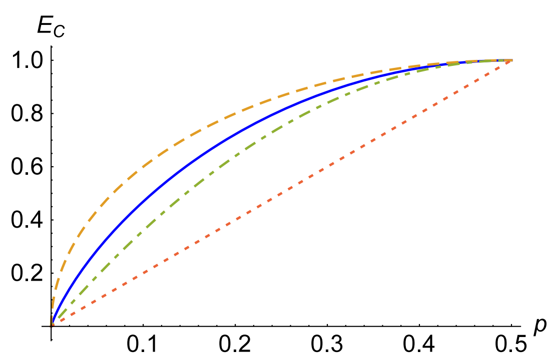

FIG. 1. A comparison of $E_{\mathcal{C}}$ for the state $\sqrt{p}|0,0\rangle + \sqrt{1-p}|1,1\rangle$ using different choices of the coherence measure $\mathcal{C}$. The measures compared are the coherence of formation (solid), the relative entropy of coherence (solid), the l_1 norm of coherence (dashed), the affinity of coherence (dot dashed) and the geometric coherence (dotted). Measures are normalized so they coincide at $p = 0.5$.

We then observe that this always defines a discordlike quantifier for every coherence measure $\mathcal{C}$.

Theorem 3: Asymmetric Discord.—$D_{\mathcal{C}}(\rho_{AB}) = 0$ iff ρ_{AB} is classical quantum; i.e., the state can be written as $\rho_{AB} = \sum_i p_i|i\rangle_A\langle i|\otimes\rho_B^i$ where $\{|i\rangle_A\}$ is some orthonormal set. It is strictly positive otherwise.

Proof.—First, suppose $\rho_{AB}=\sum_i p_i|i\rangle_A\langle i|\otimes\rho_B^i$. Writing ρ_B^i in terms of its pure state decomposition, we have

$$\rho_{AB} = \sum_i p_i|i\rangle_A\langle i|\otimes\sum_j q_{ij}|\beta_{i,j}\rangle_B\langle\beta_{i,j}|.$$

This state always permits an extension on Bob's side of the form

$$\rho_{ABB'} = \sum_i p_i|i\rangle_A\langle i|\otimes\sum_j q_{ij}|\beta_{i,j}\rangle_B\langle\beta_{i,j}|\otimes|i,j\rangle_{B'}\langle i,j|$$

for which $\mathcal{C}_{\min}(A:BB'|\rho_{ABB'}) = 0$ and so $D_{\mathcal{C}}(\rho_{AB}) = 0$.

Conversely, if $D_{\mathcal{C}}(\rho_{AB}) = 0$, then this implies that we can write $\rho_{ABB'} = \sum_i p_i|i\rangle_A\langle i|\otimes|\beta_i\rangle_{BB'}\langle\beta_i|$. This is a classical-quantum state and will remain classical quantum even if we trace out the subsystem B'. This proves the converse statement, so we must have $D_{\mathcal{C}}(\rho_{AB}) = 0$ iff ρ_{AB} is classical quantum.

Since $\mathcal{C}$ is a coherence measure and so is nonnegative, and $D_{\mathcal{C}}(\rho_{AB}) = 0$ iff ρ_{AB} is classical quantum, we must have that for any non-classical-quantum state, $D_{\mathcal{C}}(\rho_{AB}) > 0$. This completes the proof. □

The most general notion of nonclassical correlation is one where the set of classical states is the set of classical-classical states, or completely classical states. These are quantum states that can always be written in the form $\rho_{AB} = \sum_{i,j}p_{ij}|i\rangle_A\langle i|\otimes|j\rangle_B\langle j|$. This can be directly addressed via the correlated coherence itself, without consideration of any extensions of states, which is the natural end point of the relaxation of constraints that were previously considered in the transition from $E_{\mathcal{C}}$ to $D_{\mathcal{C}}$

Theorem 4: Symmetric Discord.—$\mathcal{C}_{\min}(A:B|\rho_{AB}) = 0$ iff ρ_{AB} is classical classical; i.e., the state can be written as $\rho_{AB} = \sum_{i,j}p_{ij}|i\rangle_A\langle i|\otimes|j\rangle_B\langle j|$, where $\{|i\rangle_A\}$ and $\{|j\rangle_B\}$ are some orthonormal sets. It is strictly positive otherwise.

Proof.—First, suppose $\rho_{AB} = \sum_{i,j}p_{ij}|i\rangle_A\langle i|\otimes|j\rangle_B\langle j|$. It is then immediate clear by considering the basis $\{|i\rangle_A|j\rangle_B\}$ that $\mathcal{C}_{\min}(A:B|\rho_{AB}) = 0$.

Conversely, if $\mathcal{C}_{\min}(A:B|\rho_{AB}) = 0$, then this implies that we can write $\rho_{AB} = \sum_{i,j}p_{ij}|i\rangle_A\langle i|\otimes|j\rangle_B\langle j|$ since there must be some local basis $\{|i\rangle_A\}$ and $\{|j\rangle_B\}$ for which ρ_{AB} is diagonal. This proves the converse statement so we must have $\mathcal{C}_{\min}(A:B|\rho_{AB}) = 0$ iff ρ_{AB} is classical classical.

Since $\mathcal{C}_{\min}(A:B|\rho_{AB})$ is a coherence measure and so is nonnegative, and $\mathcal{C}_{\min}(A:B|\rho_{AB}) = 0$ iff ρ_{AB} is classical classical, we must have that for any non-classical-classical state, $\mathcal{C}_{\min}(A:B|\rho_{AB}) > 0$. This completes the proof. □

We also observe that, for pure bipartite states, $\mathcal{C}_{\min}(A:B||\psi\rangle_{AB}) = D_{\mathcal{C}}(|\psi\rangle_{AB}) = E_{\mathcal{C}}(|\psi\rangle_{AB})$, so the discord quantifiers converge with entanglement over pure states, which is a known property of quantum discord.

Conclusion.—We have presented a coherence based construction that is always a valid quantifier of entanglement. The construction is also generalizable to include larger classes of quantum correlations, leading to discord-like quantifiers of nonclassicality. The arguments are independent of not only the type of coherence measure used, they are also independent of the kind of noncoherence producing operation that is being considered. Such entanglement measures must therefore exist for any convex coherence quantifier that shares a common notion of classicality. This suggests that notions of entanglement and discord must exist in every reasonable resource theory of coherence.

In Ref. [14], it was demonstrated that, for every entanglement measure, there exists a corresponding coherence measure. This was achieved by considering the entanglement of the state after performing some preprocessing in the form of an incoherent operation. We ask the converse question: does every coherence measure correspond to some entanglement measure? The results discussed in this Letter prove this in the affirmative. Therefore, the number of possible entanglement measures must be exactly equal to the number of coherence measures.

The fact that entanglement can always be defined as the symmetric portion of correlated coherence also further illuminates the role that is being played by incoherent operations in Ref. [14]. Since coherence can always be faithfully converted into entanglement, we see that incoherent operations may always faithfully shift local coherences into the portion of the correlated coherence that is symmetric.

As there is no bound coherence, one may always distill pure coherent states from coherence in the asymptotic regime [37]. On the other hand, entanglement is famously a bound resource [42,43], so there exist entangled states from which pure entangled states cannot be distilled using LOCC. Despite this, Theorem 1 shows that coherence and entanglement resource theories can still be bridged. In any local basis, however, an entangled or discorded state is always coherent, so pure coherent states can always be distilled from them via incoherent operations.

We hope that the discussion presented here will inspire further research into the interplay between coherence and quantum correlations.

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