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EPL, **86** (2009) 58002

A LETTERS JOURNAL EXPLORING THE FRONTIERS OF PHYSICS

June 2009

EPL, **86** (2009) 58002
doi: 10.1209/0295-5075/86/58002

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# Jamming transition in traffic flow under the priority queuing protocol

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received 11 January 2009; accepted in final form 14 May 2009
published online 16 June 2009

PACS 89.75.Hc – Networks and genealogical trees
PACS 89.70.-a – Information and communication theory
PACS 89.20.Ff – Computer science and technology

**Abstract** – Packet traffic in complex networks undergoes the jamming transition from free-flow to congested state as the number of packets in the system increases. Here we study such jamming transition when queues are operated by the priority queuing protocol and packets are guided by the dynamic routing protocol. We introduce a minimal model in which there are two types of packets distinguished by whether priority is assigned. Based on numerical simulations, we show that traffic is improved in the congested region under the priority queuing protocol, and it is worsened in the free-flow region. Also, we find that at the transition point, the waiting-time distribution follows a power law, and the power spectrum of traffic exhibits a crossover between two $1/f^{\alpha}$ behaviors with two different exponents $\alpha < 2$ in low- and high-frequency regime, respectively. This crossover is originated from a characteristic waiting time of packets in the queue.



Information packet transport via the Internet is an important problem in complex systems from the perspectives of both theory and application. Data packets created at certain nodes in the Internet travel to their destinations under transmission control protocols. During the journey, packets interact with other packets as they share a common line or buffer in the network. Accordingly, several types of collective behaviors can emerge in the form of self-similar traffic [1,2] or chaos [3]. To enhance transport efficiency, one would like to design an appropriate protocol to transport as many packets as possible with the lowest cost. With this goal in mind, several packet transport models have been introduced.

When a packet is sent from one node to another in a network, it is usually routed along the shortest path; such a path is undoubtedly the best route when the number of packets in the network is relatively small. However, when many packets are floating around in the network, traffic congestion can occur. This problem can be especially serious in scale-free (SF) networks [4], since hubs are the bottlenecks of traffic flow. To resolve this congestion, many routing protocols have been proposed, including the hub avoidance protocol [5,6] and the optimal routing protocol [7]. These are static routing protocols, so that a path from one node to another is fixed regardless of the traffic level in the system at any given time. In contrast, there is a dynamic routing protocol that guides packets to alternative paths depending on the traffic on the path to each target [8].

When a packet travels through a node (router), it is temporarily stored in the buffer (queue) at the node. There can be many queuing protocols that control the order of packet transmission in the queue. The most common one is the "first-in-first-out" (FIFO) protocol. Alternatively, the "last-in-first-out" (LIFO) protocol can be used [9–11]. The priority queue is a rather different protocol [12]. Each packet is assigned a priority upon its birth. A packet with the highest priority is treated first in the queue, irrespective of its order of arrival. The diffusion process [13] under the priority queuing protocol in complex networks have been studied previously.

In this paper, we study the jamming transition of packet transport on SF networks using the priority queuing protocol and the dynamic routing protocol by adapting Dijkstra's algorithm [14]. At each time step, every node creates a packet with probability $q$ whose destination is chosen randomly. These packets are assigned to be either with or without priority. The fraction of the packets that are priority-assigned is $f$. In the queue, packets with priority are delivered firstly toward their destinations along

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the paths that are updated at every time step determined by Dijkstra's algorithm. Packets without priority are also guided similarly, but after packets with priority. Similar types of packets in the queue are treated following the FIFO protocol. Packets with priority may be regarded as paid packets when downloaded from a certain web site. Note that when $f=0$ or $f=1$, the priority queuing protocol reduces to the standard FIFO protocol. We present a phase diagram for the free-flow and congested phases in the parameter space $(q,f)$, and we show that the priority queuing protocol is efficient when the system is congested.

We simulate packet transport under the dynamic rules below on undirected binary scale-free networks generated by the static model [15]. For the network, the total number of nodes is $N=1000$, the average degree of a node is $\langle k\rangle \simeq 4$, and the degree exponent is $\gamma=2.5$. Each type of packet travels along the path that minimizes the quantity

$$L_{s,d}(t)=\ell_x+h\sum_{i\in x}Q_i(t-1), \qquad (1)$$

where $\ell_x$ is the hopping distance along a path $x$ between nodes $s$, $d$, $Q_i(t)$ is the queue length at node $i$ on the path $x$ at time $t$, and $h$ is a traffic control parameter [8]. For packets with (without) priority, $Q_i(t)$ is regarded as the number of priority-assigned (both types of) packets that have accumulated in the queue at node $i$ at time $t$. Hence, the path minimizing $L_{s,d}(t)$ can be the best choice to route the packet at time $t$, since the packet can circumvent congested nodes along its way. Note that we calculate $L_{s,d}(t)$ with $Q_i(t-1)$, the queue lengths at time $t-1$. This path is determined using Dijkstra's algorithm in the simulation. Note that, when $h=0$, this routing protocol reduces to the shortest path routing protocol. We choose other control factors as follows: The queue size is unlimited, and the process rate in each queue is one packet per time step. Thus, if more than one packet arrives at a node per unit time, then the queue length increases. The system is updated in parallel, meaning that all packets move simultaneously with the queue length information of the previous time step. Our simulations are performed for up to $5\times10^4$ time steps.

To characterize the jamming transition, we define an order parameter for each type of packet as the delivery fraction $D_\alpha$, where $\alpha=\mathrm{p}$ for packets with priority, $\alpha=\mathrm{n}$ for packets without priority, and $\alpha=\mathrm{tot}$ for all packets combined. $D_\alpha$ is defined as

$$D_\alpha=\lim_{t\to\infty}\frac{1}{N_\alpha(t-t_0)}\int_{t_0}^{t}\lambda_\alpha(t')\mathrm{d}t', \qquad (2)$$

where $\lambda_\alpha(t)$ is the number of packets of type $\alpha$ arriving at their destinations at time $t$ and $N_\alpha$ is the number of packets of type $\alpha$ generated in a unit time. Here, $N_\alpha$ is $N_\alpha=Nqf$, $Nq(1-f)$, and $Nq$ for $\alpha=\mathrm{p}$, n, and tot, respectively. If the traffic of packets of type $\alpha$ is in the free-flow state, then $D_\alpha$ is close to 1, since all packets of type $\alpha$ arrive successfully at their targets. However, if the traffic is in a congested state, then $0<D_\alpha<1$. When $D_\alpha=0$, the traffic is in the completely congested state.

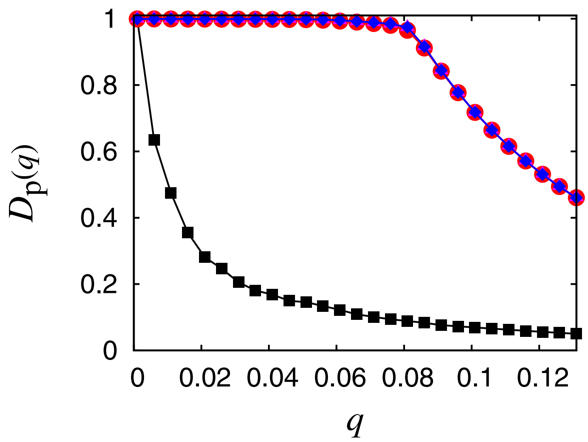


Fig. 1: (Color online) The delivery fraction $D$ for packets with priority *vs.* the packet generation rate $q$ for various $h$'s. The traffic control parameter $h$; $h=0.0$ (□), 0.1 (○), 0.5 (△) and 1.0 (◇). The priority fraction $f$ is chosen as 0.9.

In general, the jamming transition point for packets with priority does not coincide with that for packets without priority, denoted by $q_\mathrm{p}$ and $q_\mathrm{n}$, respectively. Obviously $q_\mathrm{p}>q_\mathrm{n}$. When $f=0$ and $f=1$, all packets are of the same type and the queuing process is governed by the FIFO protocol. In this case, the delivery fraction and the jamming transition point are denoted by $D_0$ and $q_0$, respectively. Technically, we select the transition point as the $q$-value at which $\Delta D$ exhibits a sudden drop for the first time with respect to increasing $q$.

First, we consider the effect of the traffic control parameter $h$. We observe that the traffic is dramatically improved when $h>0$ compared with the case when $h=0$, as shown in fig. 1. However, the traffic seems to be unaffected by changes in $h$ when $h>0$. This result is reasonable because the sum of accumulated packets along the path is far larger than the topological length $\ell_x$ in the congested state. Hence, we will confine our interest to the case when $h=1$ from here on.

We observe the behavior of the delivery fraction. We consider $D_\mathrm{p}$ as a function of $q$ for various $f$ in fig. 2(a). The preliminary result of this was reported in [16]. For $f=0$ and $f=1$, we obtain $q_0\simeq0.048$, which can change depending on the system size $N$. When $0<f<1$, the jamming transition point $q_\mathrm{p}$ for priority-assigned packets is larger than $q_0$. Since packets without priority do not hamper the traffic of packets with priority, one can obtain the relation

$$q_\mathrm{p}f=q_0. \qquad (3)$$

On the other hand, when $q>q_\mathrm{p}$, a fraction of packets with priority cannot reach their targets during a given time interval. Hence, $D_\mathrm{p}<1$ in such a case.

Second, we examine the behavior of the jamming transition for packets without priority. Since packets with priority delay the traffic of packets without priority under

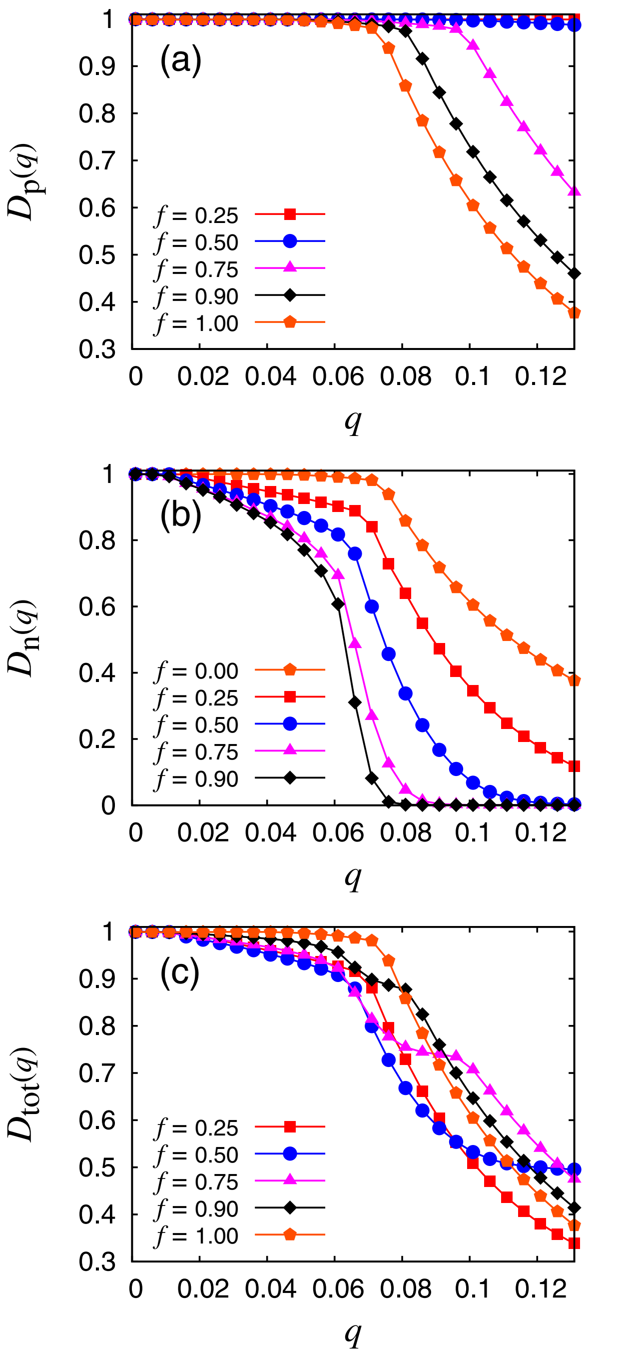


Fig. 2: (Color online) The delivery fraction $D$ for packets with (a) and without (b) priority, and all packets (c) *vs.* the packet generation rate $q$ for various fractions $f$ of the priority assignment.

the priority queuing protocol, the jamming transition for packets without priority occurs at $q_{\mathrm{n}}$ smaller than $q_0$. As $q_{\mathrm{p}} > q_{\mathrm{n}}$ for a given $f$, $D_{\mathrm{p}} = 1$ and $0 < D_{\mathrm{n}} < 1$ for $q_{\mathrm{n}} < q < q_{\mathrm{p}}$. Figure 2(b) shows the behavior of $D_{\mathrm{n}}$ as a function of $q$ for various $f$. For $q > q_{\mathrm{p}}$, the traffic for packets without priority is completely congested, *i.e.*, $D_{\mathrm{n}} \simeq 0$,

Next, we combine the above two cases and consider the jamming transition for all packets irrespective of priority

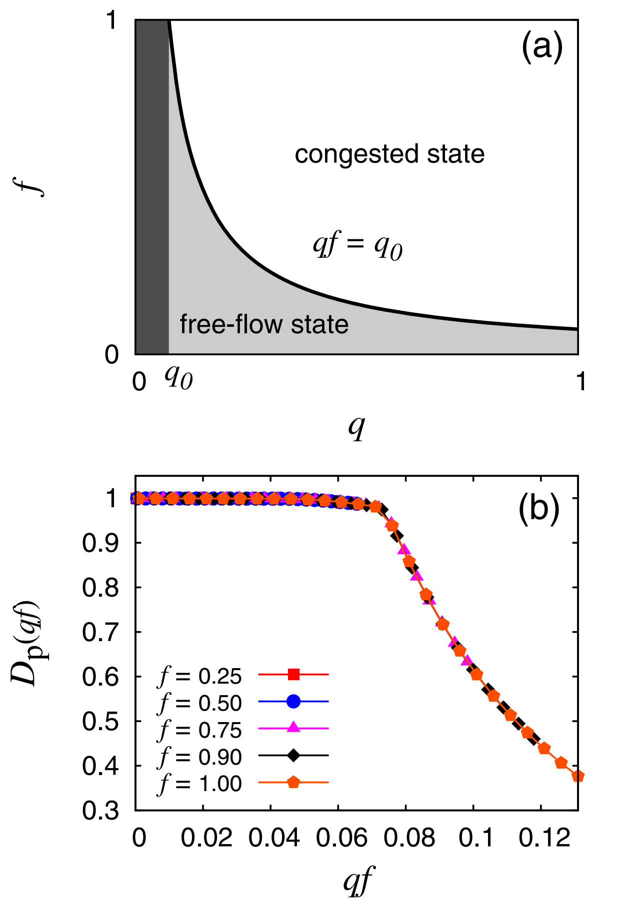


Fig. 3: (Color online) Phase diagram for packet traffic (a) and scaling behavior of the delivery fraction for packets with priority (b). In (a), the dark-grey region represents the free-flow state when the FIFO protocol is used and the light-grey region represents the additional free-flow state for the priority-assigned packets under the priority queuing protocol. (b) shows that $D_{\mathrm{p}}$'s collapse to a single scaling function.

assignment. $D_{\mathrm{tot}}$ is the order parameter for all packets. The behavior of $D_{\mathrm{tot}}$ is shown in fig. 2(c). $D_{\mathrm{tot}}$ satisfies the relationship

$$D_{\mathrm{tot}} = fD_{\mathrm{p}} + (1-f)D_{\mathrm{n}}, \tag{4}$$

where $D_{\mathrm{p}}$ and $D_{\mathrm{n}}$ can change depending on $q$. We summarize the delivery fraction for all packets as

$$D_{\mathrm{tot}} = \begin{cases} 1, & \text{for } q < q_{\mathrm{n}}, \\ f + (1-f)D_{\mathrm{n}}, & \text{for } q_{\mathrm{n}} < q < q_{\mathrm{p}}, \\ fD_{\mathrm{p}}, & \text{for } q > q_{\mathrm{p}}. \end{cases} \tag{5}$$

It is interesting to note that the delivery fraction $D_{\mathrm{tot}}$ under the priority queuing protocol can exceed the $D_0$ obtained from the simple FIFO protocol. This phenomenon can occur in the region of $q > q_{\mathrm{p}}$, as shown in fig. 2(c) (for example, $q > 0.12$ for $f = 0.5$, and $q > 0.096$ for $f = 0.75$). The unexpected improvement in overall transport efficiency is due to the priority queuing protocol,

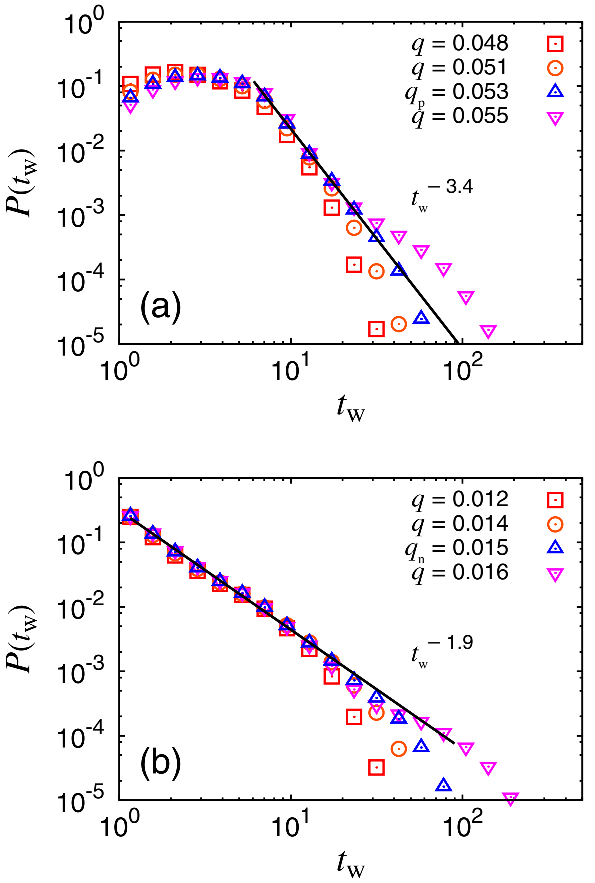


Fig. 4: (Color online) The waiting-time distribution in the free-flow region for packets with (a) and without (b) priority for $f = 0.9$ and $f = 0.25$, respectively. The power law behaviors are obtained at $q_\text{p} \approx 0.053$ and $q_\text{n} \approx 0.015$, which are regarded as the jamming transition points. The solid lines are guidance with slopes $-3.4$ (a) and $-1.9$ (b), respectively.

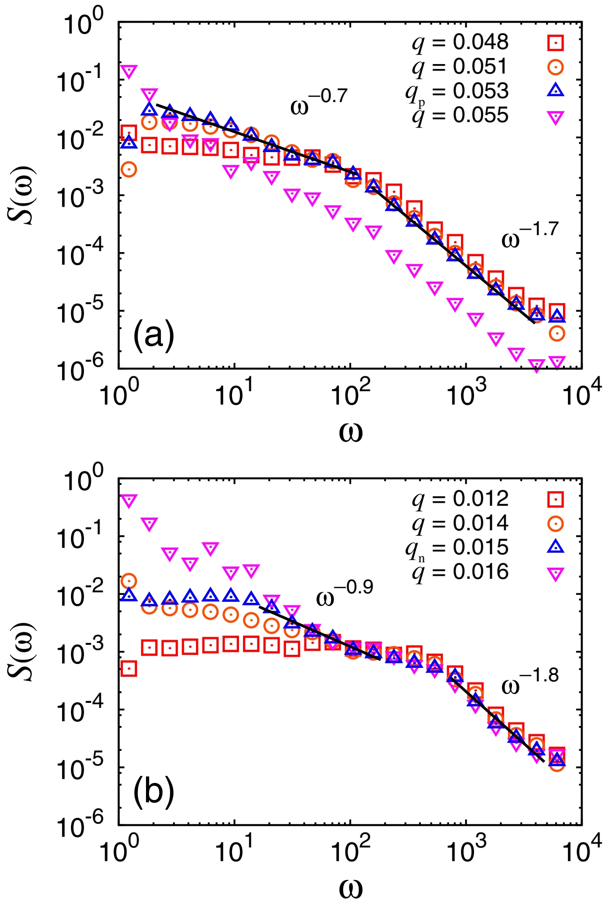


Fig. 5: (Color online) The power spectrum of traffic in the system for packets with (a) and without (b) priority, respectively. We use $f = 0.9$ (a) and $f = 0.25$ (b). For $q_\text{p} = 0.053$ (a) and $q_\text{n} = 0.015$ (b), the power spectrum exhibits a crossover between two power law behaviors. Solid lines are guidelines with slopes $-0.7$ in small $\omega$ regime and $-1.7$ in large $\omega$ regime for (a) and $-0.9$ in small $\omega$ regime and $-1.8$ in large $\omega$ regime for (b).

since it enables us to control the density of packets with priority; these packets remain deliverable packets in the congested state. For the other cases, $D_\text{tot} < D_0$, implying that the overall traffic under the priority queuing protocol is worse in the free-flow state, despite the improvements observed when the system is in the congested state.

Figure 3(a) is a phase diagram of the traffic of packets in the space of $(q, f)$. Under the FIFO protocol only, the phase space is divided into two parts by the line $q = q_0$, with the free-flow state appearing in the region $q < q_0$ and the congested state appearing in the region $q > q_0$. However, when the priority queuing protocol is used, the free-flow region can be extended into the region $q < (q_0/f)$. The scaling behavior eq. (3) of the delivery fraction for packets with priority with various $f$'s is checked in fig. 3(b).

We measure the waiting-time distributions $P(t_\text{w})$ for packets with and without priority [9,10]. The waiting time $t_\text{w}$ is defined as the time spent in the queues on the way to the target, excluding the transit time. As shown in fig. 4, the waiting-time distribution follows a power law $P(t_\text{w}) \sim t_\text{w}^{-\delta}$ near the transition points $q_\text{p}$ and $q_\text{n}$, where $\delta \approx 3.4(1)$ for packets with priority and $\delta \approx 1.9(1)$ for packets without priority. In the free-flow regions $q < q_\text{p}$ and $q < q_\text{n}$, the waiting-time distribution behaves as $P(t_\text{w}) \sim e^{-t_\text{w}/\tau}$, where $\tau$ is the mean waiting time and depends on the density of packets in the network. The maximum waiting time of packets in the free-flow region is estimated to be $t_m \approx 70$ for a system of size $N = 10^3$. This means that packets can reach their targets within $t_m$ at the most.

Next, we consider the number of packets with (or without) priority traveling on the network and in queues, called network-load in [10], as a function of time. Then, the power spectrum $S(\omega)$ of the traffic is defined as

$$S(\omega) = \frac{|g(\omega)|^2}{\sum_{\omega=0}^{T/2} |g(\omega)|^2}, \qquad (6)$$

where $g(\omega) = \sum_{t=t_0}^{T-1} F_c(t) \exp(-\mathrm{i}2\pi\omega t/(T-t_0))$ and $F_c(t)$ is the number of packets with $(c=p)$ or without $(c=n)$ priority in the system at time $t$. $t_0$ denotes a certain time in steady state and $T$ is the total simulation time step, taken as $5 \times 10^4$. The power spectrum of the traffic in the system is measured in fig. 5. The behavior of the power spectrum $S(\omega)$ depends on the packet generation rate $q$ as well as the packet type. When the packet generation rate $q$ is near the jamming transition point, the power spectrum exhibits a crossover between two power-law behaviors $S(\omega) \sim \omega^{-\alpha}$ with $\alpha \approx 0.7$ and $\alpha \approx 1.7$ for packets with priority and $\alpha \approx 0.9$ and $\alpha \approx 1.8(1)$ for packets without priority. The crossover behavior occurs roughly at $\omega_c \approx T/2\pi t_m \approx 10^2$. This value corresponds to the maximum waiting time in the system, $t_\mathrm{m}$, roughly estimated in fig. 4 to be $60 \sim 70$ for a system of size $N = 10^3$. The $1/f^\alpha$-type $(\alpha < 2)$ power spectral density suggests that there exists a long time correlation in the transport of both types of packets. The crossover behavior above is distinct feature compared with what were observed in a model system [10] and empirical data [17]. In those systems, the $1/f^\alpha$-type behavior appears in broad range, more than three decades of $\omega$, without showing the crossover behavior. The difference may root in: For our system, we used Dijkstra's algorithm, which reduces the waiting time in queues drastically, so that the maximum waiting time $t_m$ is rather small. Thus, the crossover behavior occurs at finite $\omega_c$. However, in those systems, the maximum time $t_m$ is large, and then the crossover behavior cannot be observed.

In summary, we studied the packet transport problem on SF networks under the priority queuing protocol and the dynamic routing protocol. We showed that total traffic can be improved in the congested state by introducing the priority queuing protocol; however, the overall traffic is worse in the free-flow state and the jamming transition point is reduced, which is reminiscent of the price of anarchy. The jamming transition points for packets with and without priority are different. Near each jamming transition point, the waiting-time distribution follows a power law, and the power spectrum exhibits a crossover between two power-law behaviors. We obtain $1/f^\alpha$-type power spectra in the small $\omega$ regime for both types of packets.

* * *

This work was supported by KOSEF grant Acceleration Research (CNRC) (No. R17- 2007-073-01001-0) and KRCF.

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