Comment on "Solution of Classical Stochastic One-Dimensional Many-Body Systems"

In a recent Letter, Bares and Mobilia [1] proposed the method to find solutions of the stochastic evolution operator $H = H_0 + \frac{\gamma}{L} H_1$ with a nontrivial quartic term H_1. They claim, "Because of the conservation of probability, an analog of the Wick theorem applies and all multipoint correlation functions can be computed." Using the Wick theorem, they expressed the density correlation functions as solutions of a closed set of integrodifferential equations.

In this Comment, however, we show that applicability of the Wick theorem is restricted to the case $\gamma = 0$ only. To discuss this point, let us consider the generating function of correlation functions $Z[\xi; t] \equiv \langle \tilde{\chi} | \exp[\sum_q \xi_q a_q] e^{-Ht} | \rho \rangle$ introduced by Santos *et al.* [2]. Here a_q ($a_q^\dagger$) is a q-mode annihilation (creation) operator and ξ_q is a Grassmann number with anticommutation relations $\{\xi_q, a_p^\dagger\} = \{\xi_q, a_p\} = \{\xi_q, \xi_p\} = 0$ for all momentum indices p, q. $\langle \tilde{\chi} |$ and $| \rho \rangle$ are the left vacuum and the initial state, respectively, belonging to the even sector of Hilbert space [2]. For the initial state considered in Ref. [1], the generating function Z is

$$Z[\xi; 0] = \exp\left[\sum_{q>0} \frac{\mu^2 \cot(\frac{q}{2})}{1 + \mu^2 \cot^2(\frac{q}{2})} \xi_q \xi_{-q} \right] \tag{1}$$

with $\mu \equiv \rho/(1 - \rho)$ [2]. As mentioned in Ref. [2], the Wick theorem at $t = 0$ follows from the Gaussian form of the generating function. The applicability of the Wick theorem for later time can be tested by checking whether the generating function is a Gaussian for later time or not.

Now let us suppose that Z is a Gaussian with the form $Z[\xi; t_0] = \exp[\sum_{q>0} f(q, t_0) \xi_q \xi_{-q}]$ at time $t = t_0$. For simplicity we assume that f is an odd function of q as in Eq. (1). After an infinitesimal time increment dt, the generating function becomes $Z[\xi; t_0 + dt] = Z[\xi; t_0] + dt \langle \tilde{\chi} | [H, e^{\sum_q \xi_q a_q}] e^{-Ht_0} | \rho \rangle$, where $[,]$ denotes the usual commutation relation and we use the fact that $\langle \tilde{\chi} | H = 0$. Using the property of the left vacuum [$\langle \tilde{\chi} | a_q^\dagger = \cot(\frac{q}{2}) \langle \tilde{\chi} | a_{-q}$, see Ref. [2]] and the commutation relations $[a_q^\dagger, \zeta] = -\xi_q \zeta$, $[a_q, \zeta] = 0$, and ($\zeta \equiv \exp[\sum_q \xi_q a_q]$), we obtain the time evolution of the generating function. In the absence of the quartic term (i.e., $\gamma = 0$), the generating function evolves as

$$Z[\xi; t_0 + dt] = \exp\left\{ \sum_{q>0} \xi_q \xi_{-q} [f(q, t_0) - dt(\nu(q) + \bar{\nu}(q)) f(q, t_0) + 2\epsilon dt \sin q] \right\}, \tag{2}$$

where $\nu(q) = \omega(q) + 2\epsilon \sin q \cot(\frac{q}{2})$ [1]. This implies that Z remains a Gaussian at later time, if $Z[\xi; 0]$ is a Gaussian [see Eq. (1)]. As a byproduct, we obtain a differential equation (DE) for f. Since the DE for f preserves the parity in q, f is an odd function for all t. In the presence of the quartic term (i.e., $\gamma \neq 0$), however, the situation should be modified significantly. After performing the commutation relation between quartic term and ζ, we find

$$\begin{aligned} Z[\xi; t_0]^{-1} \langle \tilde{\chi} | [H_1, \zeta] e^{-Ht_0} | \rho \rangle = {} & 2 \sum_{p,q} (\cos q \cos p - 1) \cot\left(\frac{p}{2}\right) \xi_q \xi_{-q} f(p, t_0) f(q, t_0) \\ & + 2 \sum_{p,q} \sin q \sin p \cot\left(\frac{q}{2}\right) \xi_q \xi_{-q} f(q, t_0) f(p, t_0) - \left[\sum_p \sin p f(p, t_0) \right] \left[\sum_q \sin q \xi_q \xi_{-q} \right] \\ & + \sum_{q_1, q_2, q_3, q_4} \cos(q_1 + q_3) \delta(q_1 + q_2 + q_3 + q_4) f(q_3, t_0) f(q_4, t_0) \xi_{q_1} \xi_{q_2} \xi_{q_3} \xi_{q_4} \\ & + \sum_{q_1, q_2, q_3, q_4} [\cos(q_2 + q_3) - \cos(q_1 + q_3)] \delta(q_1 + q_2 + q_3 + q_4) \\ & \times \cot\left(\frac{q_1}{2}\right) f(q_1, t_0) f(q_3, t_0) f(q_4, t_0) \xi_{q_1} \xi_{q_2} \xi_{q_3} \xi_{q_4} . \end{aligned} \tag{3}$$

Although $Z[\xi; t_0]$ is a Gaussian initially, the quartic terms in Eq. (3) prevent it from remaining a Gaussian at later time, thus breaking the Wick theorem.

If we ignore the quartic term in the generating function, we indeed find the integrodifferential equation of $g(q, t) [\equiv f(q, t) \cot(\frac{q}{2})]$ as given by Eq. (7) in Ref. [1]. Therefore, it may be considered as a Hartree-Fock-type approximation [3]. Since the quartic term is irrelevant in the model considered, Eq. (7) in Ref. [1] predicts the correct (leading) long-time behavior of the density and the correlation function. However, for the systems with the relevant nonquadratic terms in the evolution operator, the validity of the method proposed in Ref. [1] is doubtful.

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