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Traffic states of a model highway with on-ramp ☆

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Abstract

Several distinct traffic states are identified from real highway traffic data (Kerner and Rehborn, Phys. Rev. Lett. 79 (1997) 4030; Kerner and Rehborn, Phys. Rev. E 53 (1996) R4275; Kerner, Phys. Rev. Lett. 81 (1998) 3797; Kerner and Rehborn, Phys. Rev. E 53 (1996) R1297). Influence of on-ramp flux is important in generating the stop-and-go traffic flow near the ramps (Lee et al., Phys. Rev. Lett. 81 (1998) 1130; Lee et al., Phys. Rev. E 59 (1999) 5101). In this work, we study the phase diagram of the continuum traffic flow model equation in the presence of an on-ramp. Using an open boundary condition, traffic states and metastabilities are investigated for several representative values of the upstream boundary flux and for a range of the on-ramp flux. We find several traffic states such as the pinned localized cluster (PLC) state, the oscillating pinned localized cluster (OPLC) or the recurring hump (RH) state, the oscillating congested traffic (OCT) state and the homogeneous congested traffic (HCT) state. The latter two are traffic jam states. The free flow, the OPLC state and the jam can coexist in a certain metastable region where the free flow can undergo phase transitions to either of the two states under fluctuations. Some of these states are related to observed traffic states. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

Traffic flow is an everyday example of many-body systems that everybody faces on roads. Several years ago, a number of physicists initiated an approach which models the traffic flow as a driven non-equilibrium system [1,2]. Advanced technologies,

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with the help of which traffic flow can be measured quite accurately, also allowed reliable tests of theoretical models. Through these combined efforts of theoretical and empirical studies, one came to the understanding of various characteristic properties of traffic flow that appear to be common to free highways in many countries. One famous example is the understanding of the so-called phantom traffic jam [3,4], which occurs spontaneously without any obvious reason such as bottleneck or accidents [5].

On the other hand, traffic flow in inhomogeneous sections of highways, which contain for instance on-ramps or off-ramps, turned out to be a more difficult problem. Recent empirical studies [6–8] indicate that the congested traffic flow in inhomogeneous highways exhibits nontrivial behaviors: Vehicle motions in all lanes are largely synchronized and the notion of a unique density-flow relation, called the fundamental diagram which is a useful description of traffic flow in homogeneous highways, is not applicable. Also, these traffic behaviors are amazingly stable. It is observed that in some cases, they can be maintained for several hours, which suggests that they are not transient properties but related with a stable dynamic phase of traffic flow.

A couple of theoretical models [9,10] that incorporate the effect of on-ramps are proposed. Using these models, it is realized that the presence of on-ramps generates nontrivial traffic phases such as the recurring hump (RH) state [11] and the homogeneous congested traffic (HCT) state [10]. More extensive investigations of the models [12,13] recently revealed that the RH and HCT states are not the only possibilities and other nontrivial traffic states can appear.

The identification of possible phases and the construction of the phase diagram are important parts of understanding physical systems. We review in this paper the present understanding of traffic phases in a highway with an on-ramp, largely based on Refs. [13,12]. In the next section, a traffic flow model proposed in Ref. [9] is introduced and in Section 3, we present the traffic states identified from this model [13]. The results of Ref. [12] are also discussed. In Section 4, we compare the predictions of Refs. [13,12] with a more recent empirical study where four distinct traffic states are identified. Problems in the present traffic models are discussed.

2. Model highway with an on-ramp

A simple one-lane model of the highway traffic flow, which is first introduced in Ref. [3] and later modified to include the effects of an on-ramp [9], reads

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho v)}{\partial x} = q_{\mathrm{in}}(t)\varphi(x)\,, \tag{1}$$

$$\rho\left(\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x}\right) = \frac{\rho}{\tau}[V(\rho) - v] - c_0^2\frac{\partial \rho}{\partial x} + \mu\frac{\partial^2 v}{\partial x^2}\,, \tag{2}$$

where $\rho(x,t)$ is the local vehicle density and $v(x,t)$ the local velocity. $q_{\mathrm{in}}(t)\varphi(x)$ is the source term representing the external flux through an on-ramp. The spatial distribution of the external flux $\varphi(x)$ is localized near $x = 0$ (on-ramp position) and normalized so that $q_{\text{in}}(t)$ denotes the total incoming flux at time t. $V(\rho)$ is the safe velocity that is achieved in the time-independent and homogeneous traffic flow. In Eq. (2), the second term on the right-hand side represents an effective "pressure" gradient on vehicles due to the anticipation driving [3] and the velocity fluctuations [14,15]. The third term takes into account an intrinsic damping effect that smears out sharp shock fronts via statistical fluctuations in actual traffic [16]. Here τ, c_0, μ are appropriate constants. The flux or flow, ρv, is denoted below by either q or f.

In order to investigate the effects of a single on-ramp, we use the open boundary condition. The upstream boundary values of the density and velocity are fixed at $\rho(x = -L/2, t) = \rho_{\text{up}}$ and $v(x = -L/2, t) = V(\rho_{\text{up}})$, respectively. On the other hand, the values at the downstream boundary ($x = L/2$) are linearly extrapolated from their values at neighboring points, $x = L/2 - \Delta x$ and $L/2 - 2\Delta x$ where Δx is the spacing used in the discretization. The numerical simulations are performed using the two-step Lax–Wendroff scheme [17]. We choose the following parameters : $\tau = 0.5$ min, $\mu = 600$ vehicles km/h, $c_0 = 54$ km/h, and $V(\rho) = V_0(1 - \rho/\hat{\rho})/(1 + E(\rho/\hat{\rho})^4)$ where the maximum density $\hat{\rho} = 140$ vehicles/km, $V_0 = 120$ km/h, and $E = 100$. [1]

When these parameters are fixed, traffic states depend on two variables $f_{\text{up}} \equiv \rho_{\text{up}} V(\rho_{\text{up}})$ and $f_{\text{rmp}} \equiv q_{\text{in}}(t)$ representing the strength of the highway "demand", and on the initial conditions $\rho(x, t=0), v(x, t=0)$. As exemplified in the study of homogeneous highways, the initial conditions are important factors since qualitatively different traffic states may be realized depending on the initial conditions. This hysteretic behavior makes the search for possible traffic state quite time consuming and thus we restrict the search to a few representative values of f_{up}. These values are chosen from the knowledge of homogeneous highways ($f_{\text{rmp}} = 0$) where two important critical values exist: [2] Even infinitesimal perturbations in density or velocity grow to mature traffic jams if $f_{\text{up}} > f_c$ (=2249 veh/h for the parameter choice above) while even large perturbations decay out if $f_{\text{up}} < f_b$ (=2047 veh/h). Due to the linear instability above f_c, nontrivial effects of the on-ramp are more likely to appear below f_c. Thus we choose $f_{\text{up}} =$ 2119, 1948, and 1497 veh/h. On the other hand, f_{rmp} is scanned continuously up to $f_{\text{rmp}}^{\max} \equiv f_{\max} - f_{\text{up}}$ where $f_{\max}$ is the maximum flux that time independent and homogeneous traffic flow can support, that is, the maximum of $\rho V(\rho)$.

3. Stable traffic states

For all three investigated values of f_{up}, the traffic flow can remain in the free flow state if $f_{\text{rmp}} < f_{\text{rmp}}^{c} \equiv f_c - f_{\text{up}}$ (Fig. 1). Both in the upstream and the downstream of the on-ramp, the density and velocity are homogeneous and the only effect of the

[1] The form of $V(\rho)$ is chosen so as to be consistent with the available free flow data [6] and also with large density asymptotic behaviors. But its precise form is not important.

[2] We do not consider the very high density region where the steady-state homogeneous solution becomes linearly stable again [3,9].

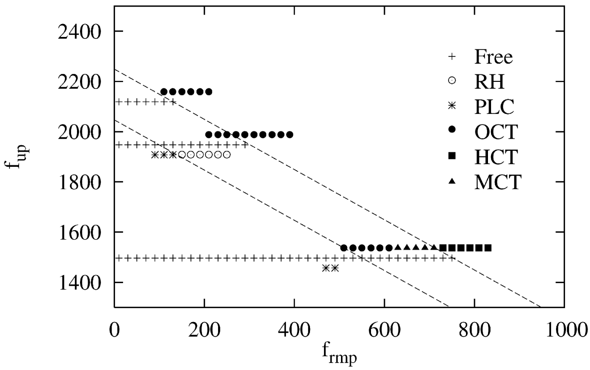

Fig. 1. Stable traffic phases are marked with different symbols in this phase diagram. Some symbols are shifted vertically for clarity. The search for stable traffic phases is focused on three representative values of $f_{\rm up}$, 2119 vehicles/h ($> f_b$), 1948 vehicles/h ($< f_b$), and 1497 vehicles/h. Here $f_b = 2047$ vehicles/h, f_c=2249 vehicles/h, and $f_{\max}$=2336 vehicles/h. The dashed lines represent $f_{\rm rmp}+f_{\rm up}=f_b$ and $f_{\rm rmp}+f_{\rm up}=f_c$, respectively.

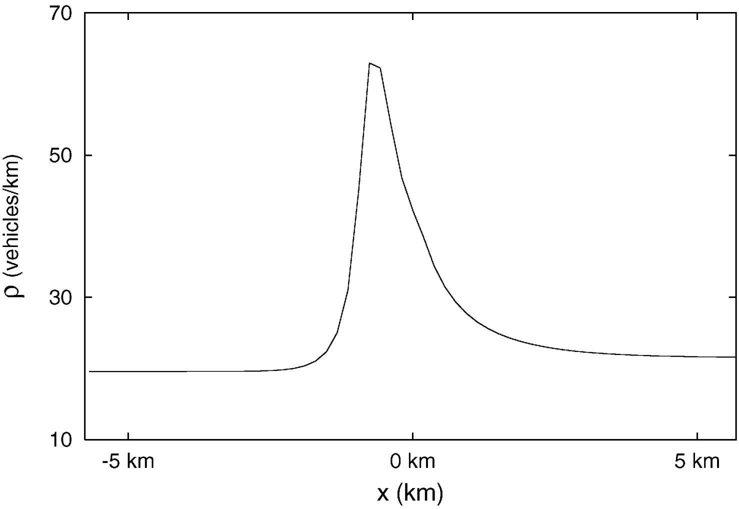

Fig. 2. The spatial density profile of the PLC state for $f_{\rm up} = 1948$ vehicles/h and $f_{\rm rmp} = 121$ vehicles/h. The on-ramp is at $x = 0$ km. The profile does not change with time. After Ref. [13].

on-ramp is the formation of a transition layer, i.e., a smooth density increase and a velocity decrease near the on-ramp.

For the intermediate (1948 veh/h) and the small (1497 veh/h) $f_{\rm up}$, a nontrivial traffic state appears if $f_{\rm rmp}$ is sufficiently large. Fig. 2 shows the pinned localized cluster (PLC) state where the density profile in the congested region near the on-ramp forms a large cluster (a different name, standing localized cluster (SLC) state, is used in Ref. [13]). The cluster does not expand or move with time and thus both density

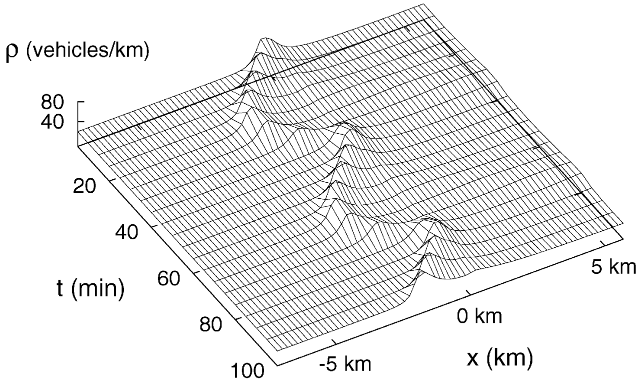

Fig. 3. The evolution of the RH state for $f_{\mathrm{up}} = 1948$ vehicles/h and $f_{\mathrm{rmp}} = 222$ vehicles/h. After Ref. [13].

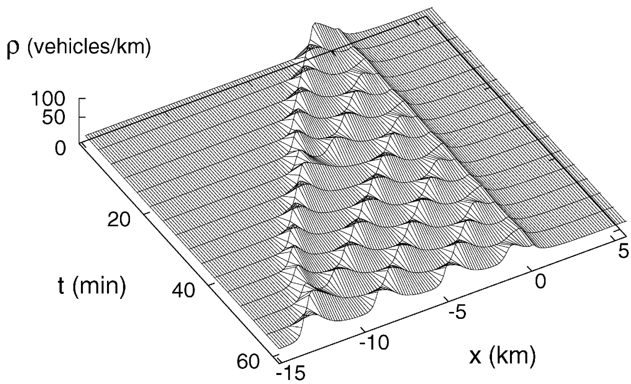

Fig. 4. The spatiotemporal evolution of the density of the OCT state for $f_{\mathrm{up}} = 1948$ vehicles/h and $f_{\mathrm{rmp}} = 381$ vehicles/h. After Ref. [13].

and velocity profiles remain stationary. An identical state is also predicted in Ref. [12] using a different traffic flow model.

For the intermediate f_{up} and larger values of f_{rmp}, the RH state appears (Fig. 3). Clusters (or humps) are generated recurrently near the on-ramp and move backwards. After some time since its birth, each cluster changes its movement direction and decays. Thus, the congested region contains moving clusters and an oscillating pattern may appear like in Fig. 3 (for this reason, a different name, oscillating pinned localized cluster (OPLC) state, is also proposed [18]). Note that the humps move only within a finite region. The RH state is not found in Ref. [12].

For the intermediate f_{up}, further increase of f_{rmp} leads to a different kind of traffic state, which also appears for the small f_{up} when f_{rmp} is sufficiently large. Fig. 4 shows the oscillating congested traffic (OCT) state. Clusters are generated spontaneously near the on-ramp and after some time of backwards movements, they decay out since $f_{\mathrm{up}} < f_b$. Here, one important property is that "younger" clusters, which

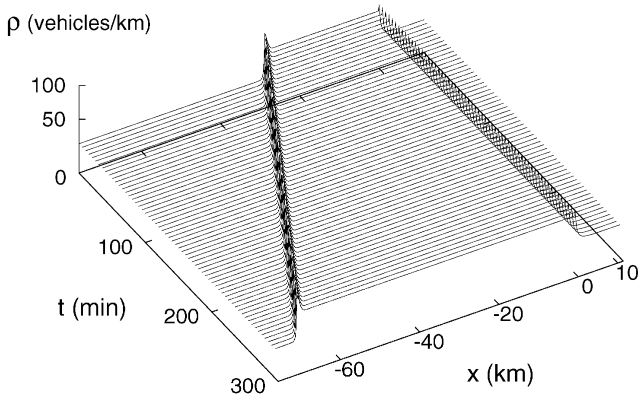

Fig. 5. The evolution of the HCT state for $f_{\rm up}$=1497 vehicles/h and $f_{\rm rmp}$=794 vehicles/h. The congested region is homogeneous and it expands with time. After Ref. [13].

are generated later, propagate farther up before the decay. As a result, the congested region, containing many moving clusters, expands monotonically with time, which is in clear contrast to the non-expanding congested regions in the PLC and the RH states. Although it is not marked in Fig. 1, the OCT-like state is also possible for the large (2119 veh/h) $f_{\rm up}$. In this case, however, each cluster does not decay at all during its backwards movement since $f_{\rm up} > f_b$, which provides a noticeable difference from the OCT state mentioned above.

In the OCT state, spacing between the clusters varies considerably with $f_{\rm rmp}$. When $f_{\rm rmp}$ becomes smaller, the spacing becomes larger and for sufficiently small $f_{\rm rmp}$, even homogeneous regions can appear between neighboring clusters. In Ref. [12], this traffic state at relatively small $f_{\rm rmp}$ is given a separate name, triggered-stop-and-go (TSG) state, and a sharp phase boundary between the OCT and TSG states is found. In this model [Eqs. (1) and (2)], however, the OCT state makes a smooth crossover to the TSG state without any signature of a sharp phase transition. Thus unlike Ref. [12], the OCT and the TSG states in this model are just two different limiting forms of a single traffic phase.

At still larger values of $f_{\rm rmp}$, the homogeneous congested traffic (HCT) state can appear (Fig. 5) where the congested region expands backwards monotonically. Unlike the OCT state, however, the congested region is homogeneous and moving clusters do not develop spontaneously. An essentially identical traffic state is also found in Ref. [12]. Predictions regarding the transition between the OCT and HCT states are different however. While Ref. [12] predicts a direct transition, in Ref. [13], the transition is found to go through an intermediate traffic state (mixed congested traffic (MCT) state) where a part of the congested region near the on-ramp is homogeneous and the rest of the region is filled with clusters.

In terms of the "strength" of the on-ramp effects, these traffic states can be classified into two groups. In the PLC and the RH states, the congested region remains finite and the vehicle flux measured at a downstream point from the on-ramp equals $f_{\rm up} + f_{\rm rmp}$.

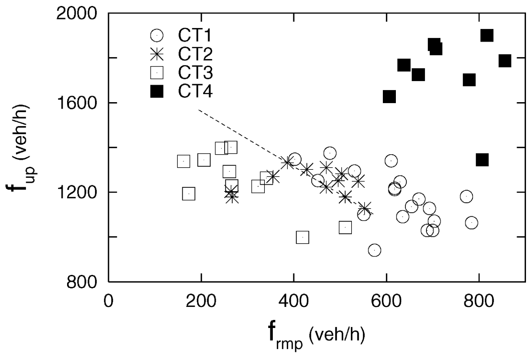

Fig. 6. Empirical phase diagram of the congested traffic flow. After Ref. [20].

In this sense, this first group of traffic states represents *local* disturbances caused by the on-ramp. In the OCT and the HCT states, on the other hand, the ranges of the congested region expand indefinitely with time and the flux at the downstream can be significantly lower than $f_{\rm up} + f_{\rm rmp}$. In this sense, this second group of traffic states can be classified as traffic jams [19]. [3]

Lastly, we mention that for all three values of $f_{\rm up}$, there exist considerable ranges of $f_{\rm rmp}$ where the free flow can coexist with certain congested traffic states. For the intermediate $f_{\rm up}$, in particular, three traffic states can coexist [see Fig. 1(b)]. The possible coexistence of the free flow with other traffic states is briefly discussed in Ref. [12] but the triple coexistence is found only in Ref. [13].

4. Comparison with empirical results and discussions

An evidence for the predicted traffic phases was provided recently by Lee et al. [20]. Traffic data collected from a highway in Korea are analyzed. From the combined information on local density-flow relations and global structure of the congested traffic region, four congested traffic states are identified, each of which has qualitatively distinct properties. Particular attention is paid to possible correlations between the values of $f_{\rm up}, f_{\rm rmp}$ and the occurrence of traffic states, and it is found that traffic states appear at different regions of the $(f_{\rm rmp}, f_{\rm up})$ plane. Fig. 6 shows the empirical phase diagram obtained in this way where four congested traffic states, CT1, CT2, CT3, and CT4, are marked with different symbols and the phase boundary of the free flow is marked with a dashed line.

In the CT1, CT2, and CT3 states, the congested flow region does not expand while in the CT4 state, it expands monotonically with time. The clusters also introduce differences. While moving clusters develop spontaneously in the CT1 state, such a feature is not observed in the other states. From these properties, the CT1 state can be

[3] Simulations of the Bando model [2] with hindrances also found the formation of the OCT- and HCT-like states [19].

related to the RH state, the CT2 and CT3 states to the PLC state, and the CT4 state to the HCT state.

An additional distinction comes from the cross-correlation between the density ρ and the flux q. While strong cross-correlation between the temporal fluctuations of ρ and those of q is found in the CT3 state, no such correlation is found in the other states. As a result, the temporal evolution of the local density-flow relation $(\rho(x,t), q(x,t))$ measured at a fixed point x generates a straight line in the ρ–q plane for the CT3 state while it generates a two-dimensional area filled with data points in the ρ–q plane for the other states. To our knowledge, existing traffic models fail to reproduce the phase transition in the behavior of the cross-correlations.

We now compare the empirical phase diagram (Fig. 6) with the theoretical prediction (Fig. 1). Note that the relative positions of the traffic states in Fig. 6 are in qualitative agreement with those in Fig. 1: The RH-like state (CT1) occurs to the right of the PLC-like state (CT2,CT3), and the HCT-like state appears at still higher values of $f_{\rm rmp}$. Also, the significant overlap between the free flow and other congested traffic states is in agreement with the prediction (Fig. 1).

A comparison shows that the present traffic models reproduce many properties of highway traffic flow. There are some deficiencies as well. One is the failure to describe the nontrivial behaviors in the density-flow cross-correlation. Another is that the agreement between two phase diagrams, Figs. 1 and 6, is unsatisfactory quantitatively. Still another is the behavior of moving clusters. It is found [8,20] that the spontaneous generation and the growth of the moving clusters exhibit some nontrivial properties, which are not reproducible with this model [Eqs. (1) and (2)]. Thus, in order to take care of these deficiencies, more improved models are required.

Lastly, we mention that the empirical study [20] covers only a limited area in the $(f_{\rm rmp}, f_{\rm up})$ plane. It is possible that additional traffic states may exist in the unexplored area. In fact, an OCT-like state is not found in Ref. [20] whereas theoretical predictions [13,12] suggest that the OCT state appears to the left of the HCT state. More empirical studies are necessary.

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