Classical E&M Quiz 02

May 8, 2006

1. Let us assume an infinite solenoid of radius R which has N turns of wire per unit length with current I. For simplicity, the axis of the solenoid is along the z-axis and let us use the cylindrical coordinate system. You are going to calculate magnetic field inside and outside of the solenoid. You may assume that $\mathbf{B} = 0$ at $\rho = \infty$. **You are not allowed to use Biot and Savart Law.** You may use the following steps, if necessary.

   (a) Obtain z and θ dependence of the magnetic field.

   (b) Obtain B_ρ inside and outside of the solenoid.

   (c) Obtain B_θ inside and outside of the solenoid.

   (d) Obtain B_z outside of the solenoid.

   (e) Obtain B_z inside of the solenoid.

   For magnetostatics, the magnetic field satisfies the following two Maxwell equations,

   $$\nabla \cdot \mathbf{B} = 0 \qquad \nabla \times \mathbf{B} = \mu_0 \mathbf{J}$$

   The second relation can be written in integral form,

   $$\oint_C \mathbf{B} \cdot d\mathbf{l} = \mu_0 I$$

2. Using the method of contour integration, let us evaluate the following integral.

   $$I = \int_0^\infty \frac{dx}{1 + x^2}$$

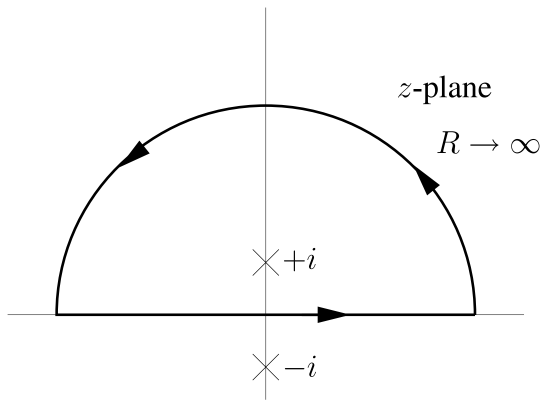
   It is easy to show that the function $1/(1+z^2)$ is analytic (differentiable) and single-valued on complex plane. For a regular (analytic and single-valued) complex function, we can use Cauchy's integral formula for a closed contour in z-plane.

   $$\int_C f(z) dz = 2\pi i \sum \text{residues}$$

Since our function $f(z) = 1/(1+z^2)$ has simple poles, the residue can be found as

$$a_{-1} = [(z-z_0)f(z)]_{z=z_0}$$

You can use the following contour in z-plane. Make sure to show that the contribution from the upper semi-circle is zero.



3. What happens if you use lower semicircle as return contour in the previous problem?