Classical E&M Quiz 01

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1. According to Coulomb's law, the electric field from a unit charge at origin is given by (SI system)
$$\mathbf{E} = \frac{q}{4\pi\epsilon_0}\frac{\mathbf{x}}{|\mathbf{x}|^3}$$
The SI unit of charge is the *coulomb* (C), and the electric field is measured in *volts per meter* (V/m). If we put one coulomb (1C) of point charge at the origin, what is the electric field at a distance of 1 meter? (*Hint*: $4\pi\epsilon_0 c^2 = 10^7$ with the speed of light c gives $\epsilon_0 \simeq 8.854 \times 10^{-12}$ *farad per meter*, or F/m.)

2. I am sure that you have learned about the vector.

   (a) What is a vector?

   (b) Suppose that we have a vector field $\mathbf{F} = (F_x, F_y, F_z)$. Let's try to form a new vector in the following way.
   $$\mathbf{G} = \hat{\mathbf{x}}\frac{\partial F_x}{\partial x} + \hat{\mathbf{y}}\frac{\partial F_y}{\partial y} + \hat{\mathbf{z}}\frac{\partial F_z}{\partial z}$$
   Prove or disprove that $\mathbf{G}$ is a vector.

3. You should have learned some vector algebra. Let's review a few of them.

   (a) δ_{ij} and ϵ_{ijk} are quite convenient symbols. Show the following.
   $$\sum_{i=1}^{3} \epsilon_{ijk}\epsilon_{ilm} = \delta_{jl}\delta_{km} - \delta_{jm}\delta_{kl}$$

   (b) Using the previous result, show that
   $$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}$$

   (c) What about
   $$\nabla \times (\nabla \times \mathbf{a}) = \nabla(\nabla \cdot \mathbf{a}) - \nabla^2\mathbf{a}$$

4. Let us consider an *ideal* conductor inside a uniform electric field.

   (a) What is the electric field inside the conductor?

   (b) Give a physical argument for your answer.

   (c) Now with some magic, the conductor has been converted into an insulator with all the charge distribution fixed. Then turn off the uniform electric field. What is the electric field inside this *insulator*?

   (d) If this *insulator* has surface charge density σ, what is the electric field just outside of the surface? You should give a derivation to your answer even though you know the answer by heart.