Classical E&M Homework 01

March 15, 2006
Due date: March 29, 2006

Each problem has 10 points except Problem 7, which has 40 points.

1. Imagine a spherical shell of charge, of radius R and surface charge σ. If we remove a small circular piece of radius $a \ll R$, what is the direction and magnitude of the field at the midpoint of the aperture?

2. Use Gauss's theorem to prove that at the surface of a curved charged conductor, the normal derivative of the electric field is given by

$$\frac{1}{E}\frac{\partial E}{\partial n} = -\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

   where R_1 and R_2 are the principal radii of curvature of the surface.

3. Prove that no *stable* equilibrium is possible in electrostatics.

4. Prove Green's reciprocation theorem: If Φ is the potential due to a volume-charge density ρ within a volume V and a surface-charge density σ on the conducting surface S bounding the volume V, while Φ' is the potential due to another charge distribution ρ' and σ', then

$$\int_V \rho\Phi'\, d^3x + \int_S \sigma\Phi'\, da = \int_V \rho'\Phi\, d^3x + \int_S \sigma'\Phi\, da$$

5. Prove *Thomson's theorem*: If a number of surfaces are fixed in position and a given total charge is placed on each surface, then the electrostatic energy in the region bounded by the surfaces is an absolute minimum when the charges are placed so that every surface is an equipotential, as happens when they are conductors.

6. Prove the following theorem: If a number of conducting surfaces are fixed in position with a given total charge on each, the introduction of an uncharged, insulated conductor into the region bounded by the surfaces lowers the electrostatic energy.

7. Let's imagine that we are living in a world where the electric field from a point charge q at $\mathbf{r}'$ is given by

$$\mathbf{E}(\mathbf{r}) = \frac{q}{4\pi\epsilon_0}\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^{3+\varepsilon}}.$$

   (a) What would be the electric field inside a conductor in this new world? (Justify your answer.)

   (b) Show that in this new world, it is still possible to find a potential Φ which satisfies $\mathbf{E} = -\nabla\Phi$. Write down the expression for $\Phi(\mathbf{x})$ from an arbitrary charge distribution $\rho(\mathbf{x}')$.

   (c) What would be the divergence $\nabla\cdot\mathbf{E}$ for an arbitrary charge distribution $\rho(\mathbf{x}')$ in this new world?

   (d) In our world where $\varepsilon = 0$, if you have a hollow area inside a conductor, the field inside the this hollow area is always **zero**. Would it be true in this new world? (Justify your answer.)

   (e) Suppose that we give a uniform surface charge density σ_0 on the surface of a spherical shell of radius a. Calculate the potential $\psi(r)$ inside the shell in this new world. Show that in the limit of $\varepsilon \to 0$, $\psi(r)$ inside the conducting spherical shell becomes constant, making $\mathbf{E} = 0$.

   (f) Now explain how Cavendish has tested inverse square nature of the electrostatic force using two concentric spheres. Discuss the way to improve the precision on ε.

   (g) In modern version of Cavendish's experiment by Williams, Faller and Hill, they applied 4MHz voltage of 10kV peak between two concentric icosahedrons. What's the advantage of applying oscillating voltage?