Homepage : http://apctp.org
E-mail : sec@apctp.org

$$\Phi_{out}^{(a)} = \frac{q}{4\pi\varepsilon_0}\sum_{l=0}^{\infty}\frac{a^{2l+1}}{d^{l+1}r^{l+1}}\frac{l(\epsilon-\epsilon)}{l\epsilon+(l+1)\epsilon}P_l(\cos\theta)$$

Note that $l=0$ term is ZERO

$$= \frac{q}{4\pi\varepsilon_0}\sum_{l=1}^{\infty}\frac{a^{2l+1}}{d^{l+1}r^{l+1}}\frac{l(\epsilon-\epsilon)}{l\epsilon+(l+1)\epsilon}P_l(\cos\theta)$$

Taking limit $\epsilon \to \infty$

$$\Phi_{out}^{(a)} = \frac{q}{4\pi\varepsilon_0}\sum_{l=1}^{\infty}\frac{a^{2l+1}}{d^{l+1}r^{l+1}}(-1)P_l(\cos\theta)$$

we need to add $l=0$ term and subtract it

$$= \frac{-q}{4\pi\varepsilon_0}\left\{\sum_{l=1}^{\infty}\left(\frac{a}{d}\right)\frac{(a^2/d)^l}{r^{l+1}}P_l(\cos\theta) + \left(\frac{a}{d}\right)\frac{1}{r} - \frac{a}{d}\frac{1}{r}\right\}$$

$$= -\frac{q}{4\pi\varepsilon_0}\sum_{l=0}^{\infty}\left(\frac{a}{d}\right)\frac{(a^2/d)^l}{r^{l+1}}P_l(\cos\theta) + \frac{1}{4\pi\varepsilon_0}\frac{\left(\frac{a}{d}q\right)}{r}$$

$$= \frac{\left(-\frac{a}{d}q\right)}{4\pi\varepsilon_0}\sum_{l=0}^{\infty}\left(\frac{a}{d}\right)\frac{(a^2/d)^l}{r^{l+1}}P_l(\cos\theta) + \frac{1}{4\pi\varepsilon_0}\frac{\left(\frac{a}{d}q\right)}{r}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\left(-\frac{a}{d}q\right)}{|\vec{x}-\vec{x}_1|} + \underbrace{\frac{1}{4\pi\varepsilon_0}\frac{\left(\frac{a}{d}q\right)}{r}}$$

negative image charge at the center to make total charge of the sphere ZERO

Headquarters
San 31, Hyoja-dong, Nam-gu, Pohang, Gyeongbuk, 790-784, Korea
Tel : 82-54-279-8661~4 Fax : 82-54-279-8679

Branch Office
The Korean Federation of Science and Technology Societies Building 11th floor,
Yoksam-dong 635-4, Kangnam-gu, Seoul, 135-703, Korea
Tel : +82-2-561-7641~2 Fax : +82-2-561-7140