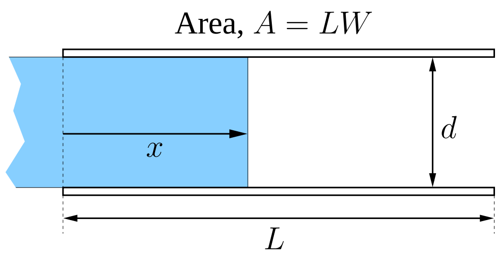
Classical E&M Homework 07

May 11, 2005

Due date: 11:59 PM, May 23, 2005

1. (60 points) Let's do some sanity check as regard to the work involved with dielectric material. Let's consider a parallel capacitor (two conducting plates separated by distance d, as in the figure).



Now, as in the figure, let's try to insert a dielectric slab with ε_1 into this capacitor. Let's assume that the whole set up is in the vacuum so that we have ε_0 for the empty space ($\varepsilon_1 > \varepsilon_0$).

   (a) (12 points) Charges Q and $-Q$ ($Q > 0$) are given to each plate and the system is isolated.

      i. What's the appropriate boundary condition between the dielectric material and the vacuum? Justify.
      ii. Get $\mathbf{D}$ and $\mathbf{E}$ in dielectric material and vacuum.
      iii. Surface charge density on the plates.
      iv. Total energy (W) of the system
      v. Force on the dielectric slab.

   (b) (8 points) Now Connect a battery to the capacitor so that the potential difference between the two plates is V.

      i. Write down the relation between V and Q, total charge on the upper plate.
      ii. Write down the expression for W in terms of V.

   (c) (30 points) Now we will move the slab by δx to see the change of the total energy. Let's try to follow the steps given in the textbook.

      i. Before moving the slab, disconnect the battery so that there is no change of charge. Calculate the change of total energy δW_1. For this purpose, we can use the expression from 1(a)iv. Be careful of the sign, $\delta W_1 = W(x + \delta x) - W(x)$.

ii. Now using the relation from 1(b)i, express δW_1 using V.

iii. What's the change in potential difference (δV) between the two plates after this move of the dielectric material? In our case, is the potential difference increasing or decreasing?

iv. What's the additional charge (δQ) to the upper plate to restore the original potential difference V?

v. Now let's reconnect the battery and move charge δQ to restore the original potential difference V.

A. What's the difference in total energy (δW_2) between the starting configuration $(V + \delta V, Q)$ and $(V, Q + \delta Q)$? What's the relation between δW_2 and δW_1 obtained in step 1(c)i?

B. In the process, the battery should move the charge δQ from lower plate to upper plate. What's the work done by the battery (δW_B)? Compare δW_B with δW_2.

vi. Calculate the force on the slab from the relation

$$F = -\frac{\delta W}{\delta x} = -\frac{\delta W_1 + \delta W_2}{\delta x},$$

and compare with the result from the step 1(a)v.

(d) (10 points) While solving this problem, you probably have ignored the edge effect (or finite size of the capacitor). Discuss the effect of this assumption for the case of (a) the total energy (W) and (b) the force on the slab (F).

2. (20 points) Jackson problem 5.3

3. (10 points) In the class, we discussed hyperfine structure of the atoms. For spherically symmetric s states, derive Eq. (5.74) from (5.73). (Is this quantum mechanics class or classical E&M class?)

4. (20 points) While we are studying magnetic field, we used a lot of *exotic* vector algebra equations. From the cover of the textbook, there are two non-trivial formulae involving ∇. Derive

$$\begin{aligned}
\nabla(\mathbf{a} \cdot \mathbf{b}) &= (\mathbf{a} \cdot \nabla)\mathbf{b} + (\mathbf{b} \cdot \nabla)\mathbf{a} + \mathbf{a} \times (\nabla \times \mathbf{b}) + \mathbf{b} \times (\nabla \times \mathbf{a}) \\
\nabla \times (\mathbf{a} \times \mathbf{b}) &= \mathbf{a}(\nabla \cdot \mathbf{b}) - \mathbf{b}(\nabla \cdot \mathbf{a}) + (\mathbf{b} \cdot \nabla)\mathbf{a} - (\mathbf{a} \cdot \nabla)\mathbf{b}
\end{aligned}$$