Classical E&M Homework 05

April 11, 2005

Due date: 11:59 PM, April 20, 2005

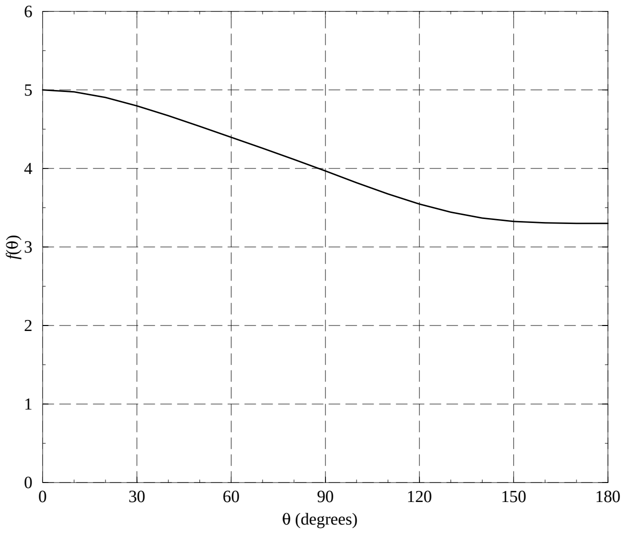
1. (20 points) Jackson 3.2
   (d) Find the magnitude and the direction of the electric field at the north pole ($\theta = 0$, $r = R$).
   (e) Discuss limiting forms of the electric field as the spherical cap becomes very small. Compare with the result of Problem 1 of HW 2.
2. (20 points) Jackson 3.4
3. (20 points) Jackson 3.6
4. (20 points) Jackson 3.7
5. (20 points) Solve *one* of the following two problems. You may solve both of them, of course, but you won't get extra credits. If you **DO** solve both of them, your final credit would be the greatest of the two, not the sum. Note that the problem choice #2 is composed of two sub questions. (Some computer programming knowledge would be very handy to solve this problem. But clever use of Micro$oft Excel or Mathematica would do the job. Otherwise, simple programmable calculator should be enough.)
   (a) Place 3 charges, each q at $(x, y) = (0, 0)$, $(0, 3)$ and $(4, 0)$. Can you find a point in space (probably in xy plane) where the electric field vanishes? If you think there is no such point in space, prove it. Otherwise, find the (x, y) coordinates for such a point. (Note that although the electric field is zero, but this would be still *unstable* equilibrium point.) You should provide a numerical answer up to 6 significant digits.
   (b) Problem choice #2
      i. (15 points) When a quantum system with angular momentum L decays, the decay output has angular distribution according to $P_l(\cos\theta)$. However, in nature no system is pure so the underlying system often has mixed angular momentum states (several

different values of L). In this case, the observed angular distribution will show incoherent sum of corresponding P_l's. The goal of physicists is to discern each individual L states from the measured angular distribution. As we discussed in the class, an arbitrary function of θ can be expanded in terms of Legendre polynomials:

$$\begin{aligned} f(\theta) &= \sum_{l=0}^{\infty} A_l P_l(\cos\theta) \quad \text{with} \\ A_l &= \frac{2l+1}{2}\int_{-1}^{1} f(\theta)P_l(\cos\theta)d(\cos\theta) \end{aligned}$$

In one experiment, the following angular distribution has been measured. Find the coefficients A_l up to $l = 5$.

Angle($^\circ$)	0	10	20	30	40	50	60	70	80	90
$f(\theta)$	5.00	4.98	4.90	4.80	4.67	4.54	4.40	4.26	4.12	3.97
Angle($^\circ$)	100	110	120	130	140	150	160	170	180	
$f(\theta)$	3.82	3.68	3.55	3.44	3.37	3.33	3.31	3.30	3.30	



ii. (5 points) Every measurement has finite error. In the previous measurements, if each measured $f(\theta)$ has constant error of 0.07 (for example, at $\theta = 0$, $f(\theta) = 5.0 \pm 0.07$), how are you going to evaluate the errors on your A_l's? A description of method would get full credit. If you actually evaluate the errors for each A_l, I will give you extra 5 points. (Bevington's *Data Reduction and Error Analysis for the Physical Sciences* will be a good reference.)