Mid Term

October 28, 2004

Name:

Notes

- Try to describe in detail as much as possible. Any reasonable argument will be given partial credits.
- General argument is more important than details, such as proportional factors.
- At the same time, try to be as concise as possible.
- Formally, the test ends at 5:15pm, but if there is a request, it can be extended **up to** 6:00pm.
- It is highly recommended to use English, but writing in other languages is also fine.
- No other book or lecture notes are allowed.
- You can use the calculator.
- Although the problem might look horrible, in close inspection, it can be just a simple algebra problem. Please try as much as you can.

1. The LEP at CERN used 50 GeV e^+ and 50 GeV e^- beam and produces collisions between them.

   (a) What's the value of the Mandelstam variable s? (1 point)

   (b) What's the total cross section of $\mu^+\mu^-$ pair creation at LEP? (2 points)

   (c) The luminosity of LEP was $10^{32}\mathrm{cm}^{-2}\mathrm{s}^{-1}$. What would be the reaction rate? (2 points)

2. When we describe Deep Inelastic Scattering (DIS) process with parton model, what's the meaning of the Bjorken variable x? (5 points)

3. What's the physical interpretation of the Parton Distribution Functions (PDF), $u(x)$, $d(x)$ etc? (5 points)

4. From naïve parton model, structure functions $F_1(x)$ and $F_2(x)$ are expected to show scaling behavior at sufficiently high Q^2 region. Experimental observation shows that the scaling is actually violated. Give a reasonably simple argument for this scaling violation. (5 points)

5. The general form of the Lagrangian for fermions (without interaction) can be written in a compact form as

$$\begin{aligned} \mathcal{L} &= \overline{\psi}(i\gamma^\mu\partial_\mu - m)\psi \\ &= \psi^\dagger\gamma^0(i\gamma^\mu\partial_\mu - m)\psi \end{aligned}$$

   where γ^μ's are Dirac matrices. In high energy physics (especially in dealing with weak interactions), chiral (or helicity) decomposition is very convenient tool. In this representation, we use helicity projection operator P_R and P_L and divide ψ into two components, L and R.

$$\begin{aligned} P_R &= \frac{1+\gamma^5}{2}, \quad P_L = \frac{1-\gamma^5}{2} \\ \psi_R &= P_R\psi, \quad \psi_L = P_L\psi \end{aligned}$$

   (a) Show that $P_R + P_L = 1$ to make $\psi = \psi_R + \psi_L$ (2 points)

   (b) Show that $P_R P_R = P_R$, $P_L P_L = P_L$, $P_R P_L = P_L P_R = 0$ (3 points)

(c) Show the following (5 points)

$$\begin{aligned}
\overline{\psi}_L\psi_L &= \psi_L^\dagger\gamma^0\psi_L = 0 \\
\overline{\psi}_R\psi_R &= \psi_R^\dagger\gamma^0\psi_R = 0 \\
\overline{\psi}_L\gamma^\mu\psi_R &= \psi_L^\dagger\gamma^0\gamma^\mu\psi_R = 0 \\
\overline{\psi}_R\gamma^\mu\psi_L &= \psi_R^\dagger\gamma^0\gamma^\mu\psi_L = 0
\end{aligned}$$

(d) Rewrite $\mathcal{L}$ in terms of ψ_R and ψ_L (5 points)

(e) Try to decompose $\mathcal{L}$ into $\mathcal{L}_L$ and $\mathcal{L}_R$ components, each containing only ψ_L or ψ_R. Discuss the role of mass m in this decomposition. (5 points)

(f) Discuss the situation when "chiral symmetry" is complete, that is ψ_L and ψ_R do not mix each other. (5 points)

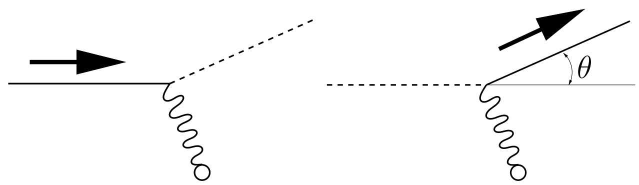
6. One direct application of the previous concept of "chirality" is helicity conservation in high energy lepton scattering process. As an example, let's consider the electron scattering over the spinless target. The differential cross section for this process is given by

$$\frac{d\sigma}{d\Omega_{\rm lab}} = \left(\frac{\alpha^2}{4E^2\sin^4\frac{\theta}{2}}\right)\frac{E'}{E}\cos^2\frac{\theta}{2}.$$

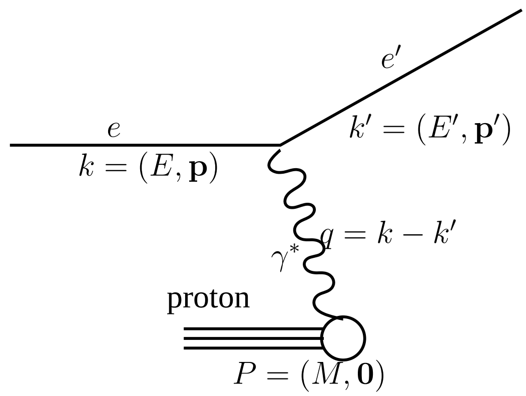
During the scattering process at high energy, the helicity (spin component along the direction of the particle) is a good quantum number. If the incident electron had helicity +1 (or -1), the scattered electron will have the same helicity (helicity conservation).

(a) Give an argument why helicity (or chirality) is a good quantum number in high energy electron scattering process. (Refer the results of problem set 5f.) (5 points)

(b) Try to convince yourself the extra term $\cos^2\theta/2$ in terms of helicity conservation. In the following two diagrams showing initial and final state, the thick arrow represents electron spin direction from helicity conservation. Consider the projection of the initial spin to the final electron direction.) (10 points)



7. Consider the elastic scattering process $e + p \to e + p$. For this reaction, show that $x = 1$. Refer the following diagram. (10 points)



8. Let's consider the process $p(\nu, e^-\pi^+)X$. We will try to calculate cross section of this reaction. Let's assign four momenta for each particle as

$$\nu(k) + p(p) \to e^-(k') + \pi^+(p_h) + X$$

   (a) Draw a leading order diagram for this process. (10 points)

   (b) To simplify the situation, let's assume that the proton is composed of only valence quarks. Identify the quark flavor which enters into the reaction. Give an argument why the other quark flavor can not be involved in the process. (5 points)

   (c) Write down the cross section $d\sigma/dxdy$ for the process $\nu p \to e^- X$. (10 points)

   (d) Write down the cross section $d\sigma/dxdydz$ for the whole process $\nu p \to e^-\pi^+ X$. (10 points)

   (e) Discuss advantages and/or disadvantages of neutrino scattering experiments compared to electron scattering experiments. (5 points)

Useful Formulae

1. A few numerical constants

$$
\begin{aligned}
1\,\text{barn} &= 10^{-24}\,\text{cm}^2 \\
\alpha_{\text{em}} &= \frac{1}{137} \\
(\hbar c)^2 &= 0.389\text{GeV}^2\,\text{mb}
\end{aligned}
$$

2. Kinematic variables in the lepton scattering (refer diagram in problem set 7)

$$
\begin{aligned}
x &\equiv \frac{-q^2}{2P\cdot q} \equiv \left.\frac{Q^2}{2M\nu}\right|_{\text{Lab}} \\
\nu &= E - E'|_{\text{Lab}} \\
Q^2 &= -q^2 = 4EE'\sin^2\left.\frac{\theta}{2}\right|_{\text{Lab}}
\end{aligned}
$$

3. $e^+e^- \to \mu^+\mu^-$ cross sections

$$
\begin{aligned}
\left.\frac{d\sigma}{d\Omega}\right|_{cm}(e^+e^- \to \mu^+\mu^-) &= \frac{\alpha^2}{4s}(1+\cos^2\theta) \\
\sigma(e^+e^- \to \mu^+\mu^-) &= \frac{4\pi\alpha^2}{3s}
\end{aligned}
$$

4. neutrino scattering cross section

$$
\frac{d\sigma}{dy}(\nu d \to e^- u) = \frac{G^2 s}{\pi}, \qquad s = (p_\nu + p_d)^2
$$

5. γ matrices

$$
\begin{aligned}
\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu &= 2g^{\mu\nu} \\
\gamma^\mu\gamma^5 + \gamma^5\gamma^\mu &= 0 \\
\gamma^{5\dagger} &= \gamma^5 \\
(\gamma^5)^2 = 1
\end{aligned}
$$

6. Spinors

$$\overline{\psi} = \psi^\dagger \gamma^0$$

7. d functions for spin 1/2

$$\begin{aligned}
d^{1/2}_{1/2,1/2} &= \cos\frac{\theta}{2} \\
d^{1/2}_{1/2,-1/2} &= -\sin\frac{\theta}{2} \\
\left|\frac{1}{2}\right\rangle_z &= d^{1/2}_{1/2,1/2}\left|\frac{1}{2}\right\rangle_{z'} + d^{1/2}_{1/2,-1/2}\left|-\frac{1}{2}\right\rangle_{z'}
\end{aligned}$$

when the angle between z-axis and z'-axis is θ.