Answers for the Final Term

December 17, 2004

1. (a) From $(N_+ - N_-)/(N_+ + N_-) = -0.8$, we have $N_+ = 0.1N$ and $N_- = 0.9N$ with $N = N_+ + N_-$. That is 80% of the electrons are in $-$ spin state while the remaining 20% is *un*polarized (equal number of $+$ and $-$ states).

   (b) Assuming ρ, Δl, Q and $\Delta\Omega$ is the same for both experiments with electron beam polarization $+$ and $-$, the result is straightforward.

   (c) Using the propagation of error with careful differentiation, we get

$$\delta A = \sqrt{\frac{1 - A^2}{N}}.$$

   Some of you have made the following error,

$$\frac{\partial}{\partial N_+}\left(\frac{N_+ - N_-}{N}\right) = \frac{1}{N},$$

   which is not true since N contains N_+.

   (d) Since f is just a number without an error,

$$\begin{aligned}
\delta A_{\text{physics}} &= \frac{1}{f}\delta A_{\text{measured}} \\
&= \frac{1}{f}\sqrt{\frac{1 - A^2_{\text{measured}}}{N}} \\
&= \frac{1}{f}\sqrt{\frac{1 - f^2 A^2_{\text{physics}}}{N}}.
\end{aligned}$$

   Most of you has forgot the factor f in front of A_{physics}, which is not that important for this case since we are going to assume $A^2 \ll 1$. With this assumption,

$$\delta A \simeq \frac{1}{f\sqrt{N}}$$

   Note that the final precision is inversely proportional to f and $\sqrt{N}$.

(e) For both cases, we want to get $\delta A = 0.01$. The number of events required to achieve this precision is

$$N \geq \frac{1}{f^2(\delta A)^2}$$

For the experiment A,

$$N_A \geq \frac{1}{0.8^2 0.01^2} \simeq 16000,$$

while for the experiment B,

$$N_B \geq \frac{1}{0.6^2 0.01^2} \simeq 28000,$$

From the experiment A to B, the beam polarization has reduced by 25%, but the required number of events has almost doubled (1.8 times more). This shows the power of polarization. In general, it's more economical to invest time and money to improve the polarization of the beam (or target).

(f) From step (b), we assumed ρ, Δl, Q and $\Delta\Omega$ is the same between $+$ and $-$ beam polarization. But in the real world, everything changes. For the case of Alice, there is much more chance that they will be different. For Bob's case, unless these things changes more than 60 times per second, chances are we will be using the same ρ, Δl, Q and $\Delta\Omega$ for bothe polarization states, thus reducing *systematic* errors of the experiment.

2. From the reflection symmetry, we only need to consider positive J_z, $1/2$ and $3/2$. From the angular momentum conservation, there can not be any spin flip either for the real photon nor for the nucleon. As a result, we have only *two* independent amplitudes, $\mathcal{M}_{1,1/2,1,1/2}$ and $\mathcal{M}_{1,-1/2,1,-1/2}$.

3. Using the definition of g_2^{WW},

$$\int_0^1 g_2^{WW}(x)\,dx = -\int_0^1 g_1(x)\,dx + \int_0^1 \int_x^1 \frac{g_1(y)}{y}\,dy\,dx.$$

Using the integration by parts, the second term simplifies into

$$\int_0^1 \int_x^1 \frac{g_1(y)}{y}\,dy\,dx \quad = \quad x \int_x^1 \frac{g_1(y)}{y}\,dy\bigg|_0^1 - \int_0^1 x \frac{d}{dx}\left(\int_x^1 \frac{g_1(y)}{y}\,dy\right)dx$$

$$\begin{aligned} &= 0 - \int_0^1 x \cdot \left(-\frac{g_1(x)}{x} \right) dx \\ &= +\int_0^1 g_1(x)\, dx, \end{aligned}$$

which cancels the first term.

Some of you have used change of integration order in 2 dimensional xy plane. It seems that that method is also correct.

4. (a) Using Lorentz Boost formula, in the new frame,

$$\begin{aligned} E' &= \gamma E - \gamma\beta p_z \\ p'_z &= \gamma p_z - \gamma\beta E. \end{aligned}$$

Simple algebra shows the result.

(b) Since y' and y is linear, the number density per dy does not change via Lorentz Boost.

(c) In the limit $E \gg m$, $E \simeq p$ and $p_z/p = \cos\theta$. So

$$y = \frac{1}{2} \ln \left(\frac{1 + \cos\theta}{1 - \cos\theta} \right)$$

Using trigonometric relation for $\cos^2\theta/2$ and $\sin^2\theta/2$ gives the result.