Coulomb Sum Rule

presented by

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HEP
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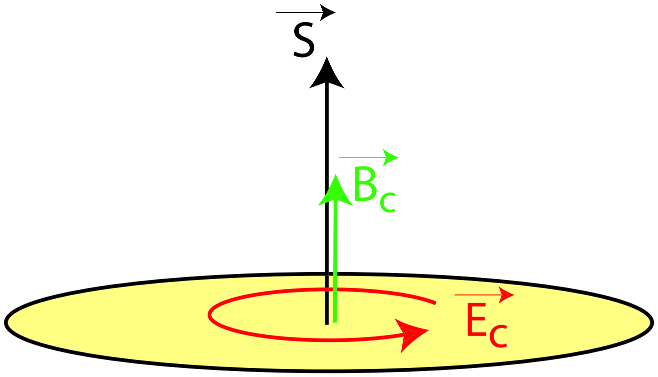
Gluon Field Polarizabilities

Polarized Nucleon



Induced Color
Magnetic and Electric Fields

$$\mathbf{B}_C \sim \chi_B \mathbf{S}$$
$$\mathbf{E}_C \sim \chi_E \mathbf{S}$$

Twists Expansion of Γ_1

- According to OPE, at $Q^2 \gg \Lambda^2_{\mathrm{QCD}}$,

$$\Gamma_1(Q^2) = \sum_{\tau=2,4,\cdots} \frac{\mu_\tau(Q^2)}{Q^{\tau-2}}$$

- Leading twist μ_2

$$\begin{aligned}
\mu_2(Q^2) &= C_{NS}(Q^2)\left(-\frac{1}{12}a_3 + \frac{1}{36}a_8\right) + C_S(Q^2)\frac{1}{9}\Delta\Sigma \\
a_3 &\equiv g_A = \Delta u - \Delta d, \quad a_8 = \Delta u + \Delta d - 2\Delta s \\
\Delta\Sigma &= \Delta u + \Delta d + \Delta s
\end{aligned}$$

- Wilson coefficients from pQCD

$$\begin{aligned}
C_{NS}(Q^2) &= 1 - \left(\frac{\alpha_S}{\pi}\right) - 3.58\left(\frac{\alpha_S}{\pi}\right)^2 - 20.22\left(\frac{\alpha_S}{\pi}\right)^2 \\
C_S(Q^2) &= 1 - 0.33\left(\frac{\alpha_S}{\pi}\right) - 0.55\left(\frac{\alpha_S}{\pi}\right)^2 - 4.45\left(\frac{\alpha_S}{\pi}\right)^2
\end{aligned}$$

Higher Twist Component

- Subtracting $\mu_2^n(Q^2)$,

$$\Delta\Gamma_1(Q^2) \equiv \Gamma_1(Q^2) - \mu_2(Q^2) = \frac{\mu_4(Q^2)}{Q^2} + \mathcal{O}\left(\frac{1}{Q^2}\right)$$

- $\mathcal{O}(1/Q^2)$ term

$$\mu_4(Q^2) = \frac{1}{9}M^2\left(a_2(Q^2) + 4d_2(Q^2) + 4f_2(Q^2)\right)$$

- a_2 and d_2 from spin structure functions

$$\begin{aligned} a_2(Q^2) &= 2\int_0^1 x^2 g_1(x,Q^2)\,dx \\ d_2(Q^2) &= 3\int_0^1 x^2 \bar{g}_2(x,Q^2)\,dx \\ &= \int_0^1 x^2\left(2g_1(x,Q^2) + 3g_2(x,Q^2)\right)\,dx \end{aligned}$$

Higher Twist Effect of $g_2(x, Q^2)$

$$d_2(Q^2) = \int_0^1 x^2 \left[2g_1(x, Q^2) + 3g_2(x, Q^2)\right] dx$$

- ➜ At high Q^2, d_2 measures induced color field by target spin

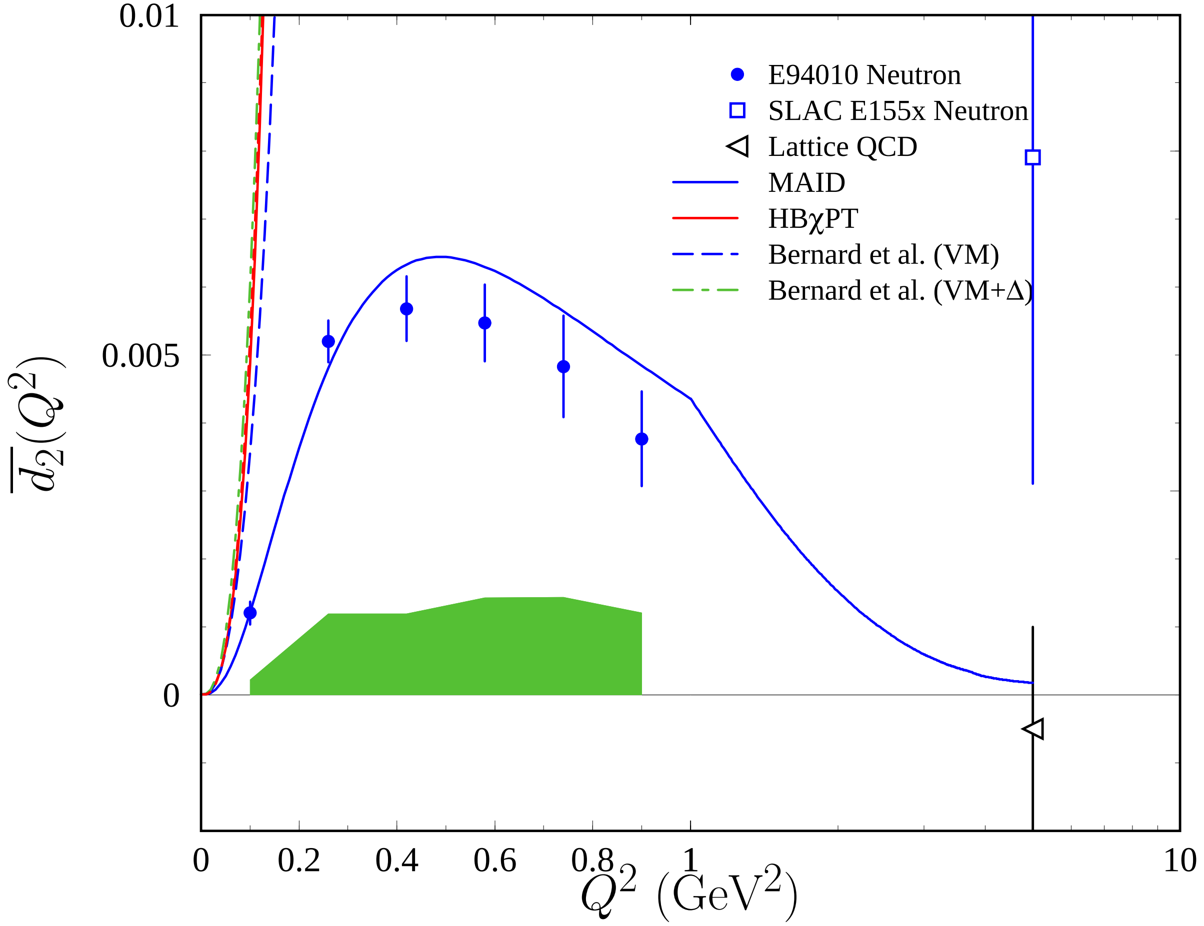
$$d_2 = \frac{1}{8}(\chi_E + 2\chi_B)$$

- ➜ At low Q^2, d_2 is related to spin polarizabilities

$$d_2(Q^2) = \frac{Q^6}{16M^2\alpha_{\rm em}} \left[3\delta_{LT}(Q^2) - \gamma_0(Q^2)\right]$$

- ➜ Intermediate Q^2: transition between partonic and hadronic scales
- ➜ Necessary to study higher twist effect from g_1 structure function
- ➜ At $Q^2 = 5$ GeV^2, SLAC E155x shows large, positive d_2 but with big error bar.

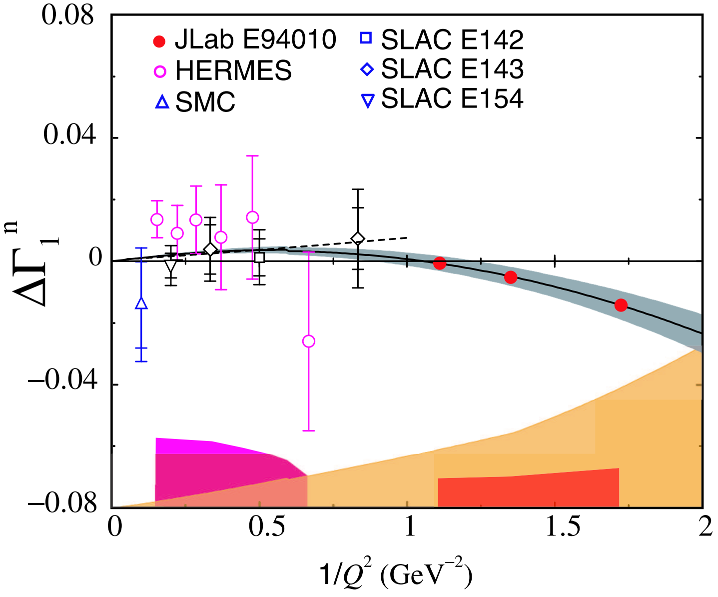
$d_2(Q^2)$


0.01
0.005
0
$\overline{d}_2(Q^2)$
0
0.2
0.4
0.6
0.8
1
10
Q^2 (GeV^2)
● E94010 Neutron
□ SLAC E155x Neutron
◁ Lattice QCD
MAID
HBχPT
Bernard et al. (VM)
Bernard et al. (VM+Δ)

Color Polarizabilities

Obtaining $f_2(Q^2)$ from Q^2 variation of $\Gamma_1(Q^2)$

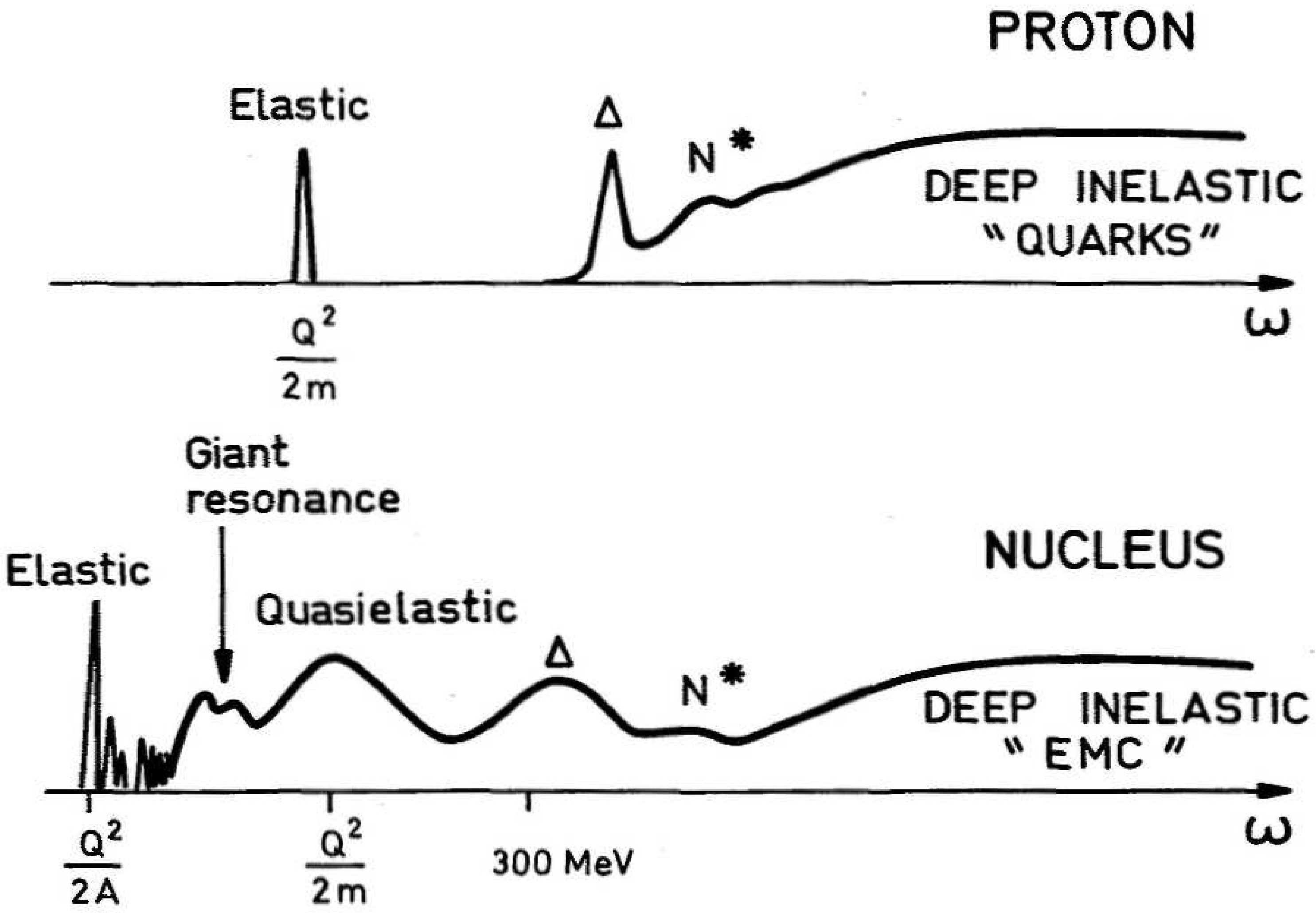
$$\begin{aligned} \chi_E^n &= \frac{2}{3}(2d_2^n + f_2^n) \\ \chi_B^n &= \frac{1}{3}(4d_2^n - f_2^n) \end{aligned}$$



$$\begin{aligned} f_2^n &= 0.034 \pm 0.043 \\ \mu_6^n &= (-0.019 \pm 0.017)M^4 \end{aligned}$$

$$\begin{aligned} \chi_E^n &= 0.033 \pm 0.029 \\ \chi_B^n &= -0.001 \pm 0.016 \end{aligned}$$

Electron Scattering On Nucleus



Probing Inside the Proton

- For a target with *non-zero* spin - form factors for *charge* and *magnetization*

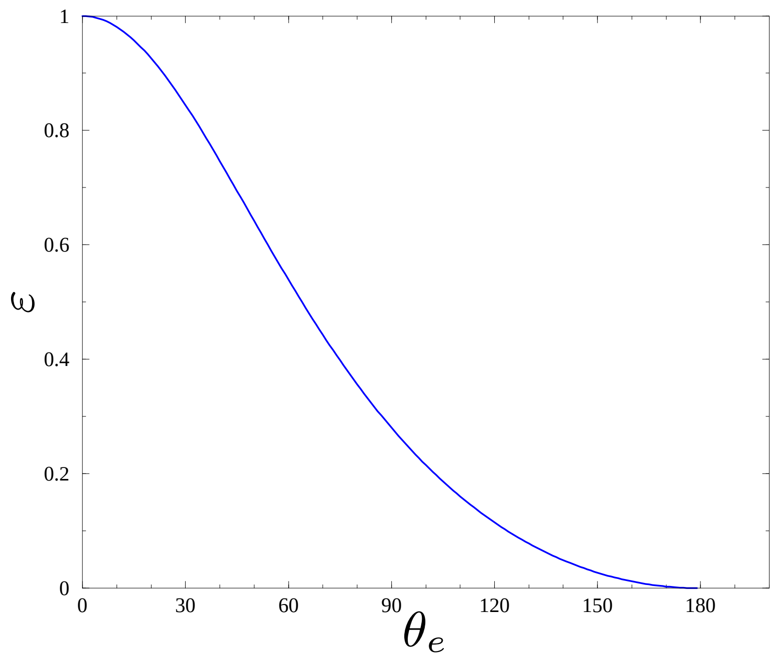
$$\frac{d\sigma}{d\Omega}_{\rm lab} = \left(\frac{\alpha^2\cos^2\frac{\theta}{2}}{4E^2\sin^4\frac{\theta}{2}}\right)\left(\frac{E'}{E}\right)\left[(G_E^p)^2+\frac{\tau}{\varepsilon}(G_M^p)^2\right]\left(\frac{1}{1+\tau}\right)$$

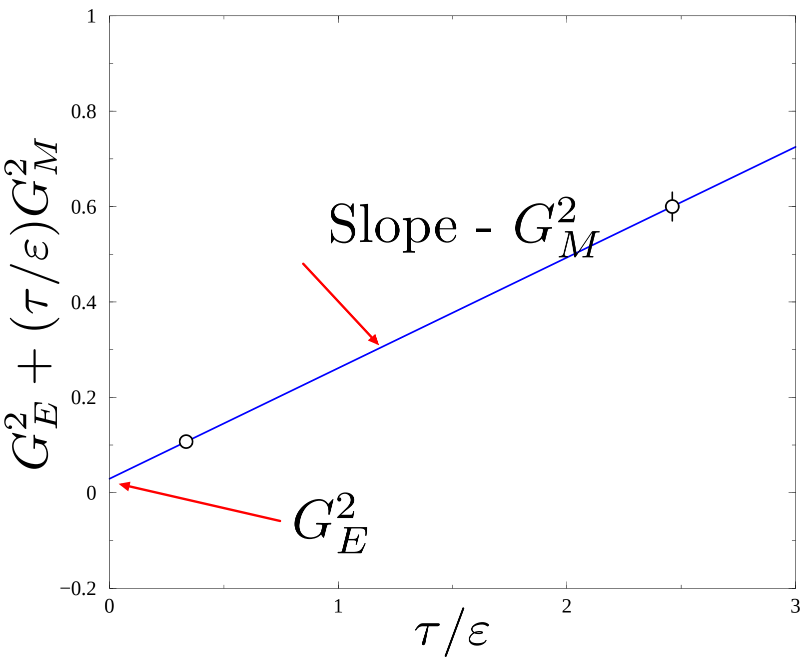
$$\tau \equiv \frac{Q^2}{4M^2}, \quad \frac{1}{\varepsilon} = 1+2(1+\tau)\tan^2\frac{\theta}{2}$$

G_E^p distribution of charge inside the proton

G_M^p distribution of magnetization inside the proton

Rosenbluth Separation





Cross Section for Quasi-Elastic Scattering

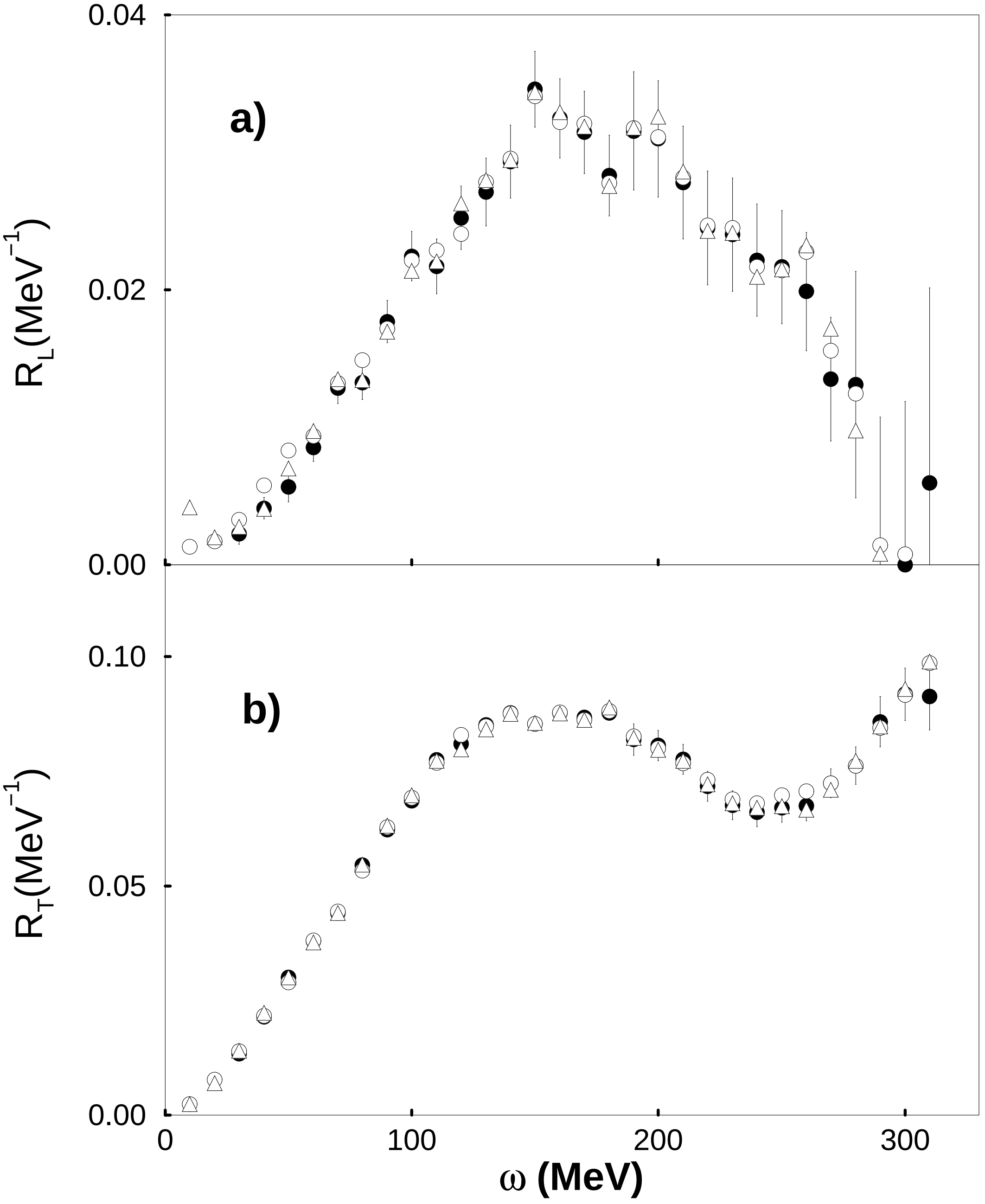
$$\frac{d^2\sigma}{d\Omega d\omega} = \sigma_{\mathrm{Mott}} \left[\frac{Q^4}{q^4} R_L(q,\omega) + \frac{Q^2}{2q^2} \frac{1}{\varepsilon} R_T(q,\omega) \right]$$

- $R_L(q,\omega)$, $R_T(q,\omega)$: Response functions
- Analogy of G_E^p and G_M^p of the free proton
- $R_L(q,\omega)$ characterizes charge interaction in the nucleus.
- Coulomb Sum

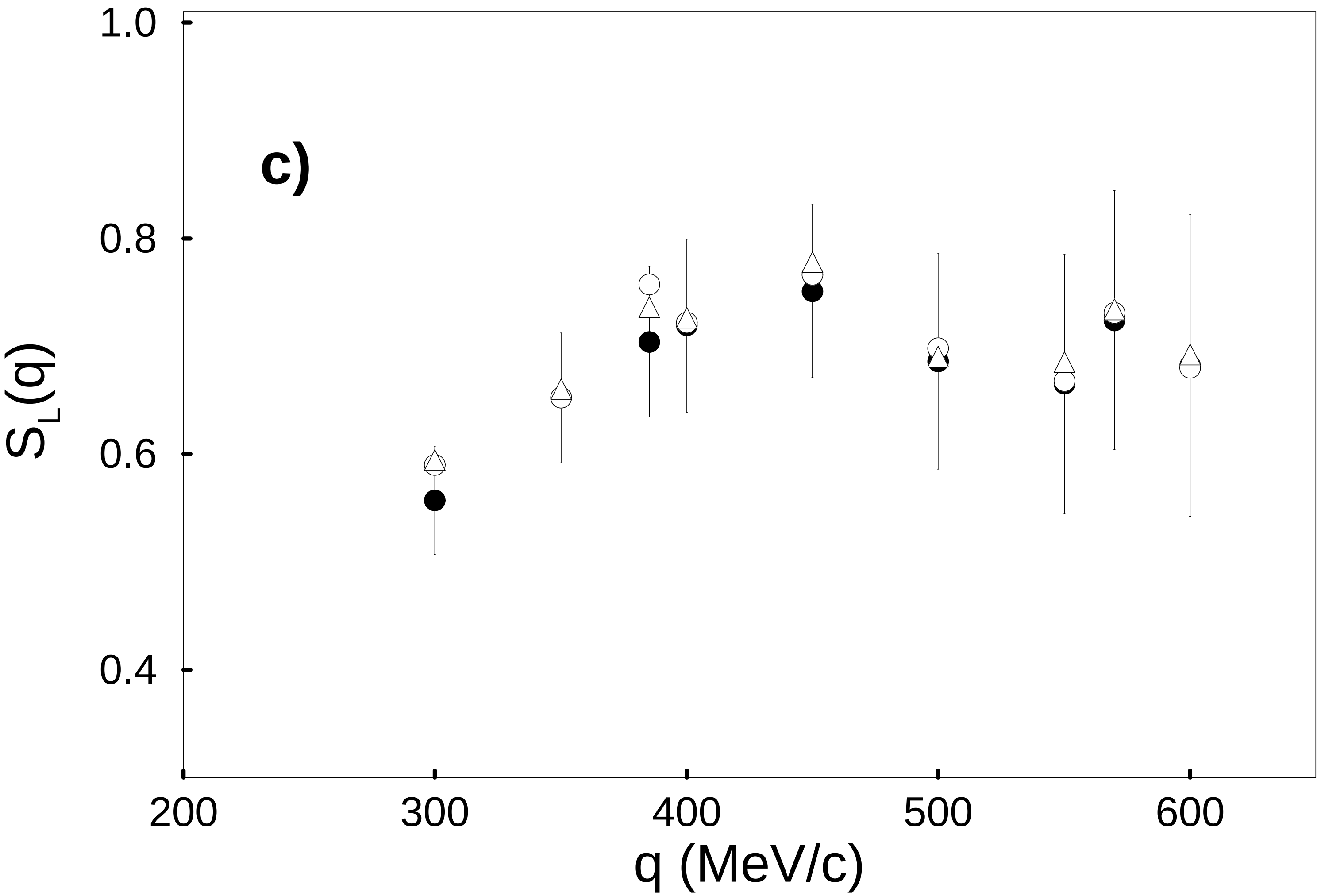
$$S_L(q) = \int_{\omega_{\mathrm{el}}^+}^{\infty} d\omega \frac{R_L(q,\omega)}{Z\tilde{G_E}^2(Q^2)}$$

$$\tilde{G_E}^2(Q^2)=\left([G_E^p(Q^2)]^2+(N/Z)[G_E^n(Q^2)]^2\right)\frac{1+Q^2/4M^2}{1+Q^2/2M^2}$$

R_L and R_T



Coulomb Sum



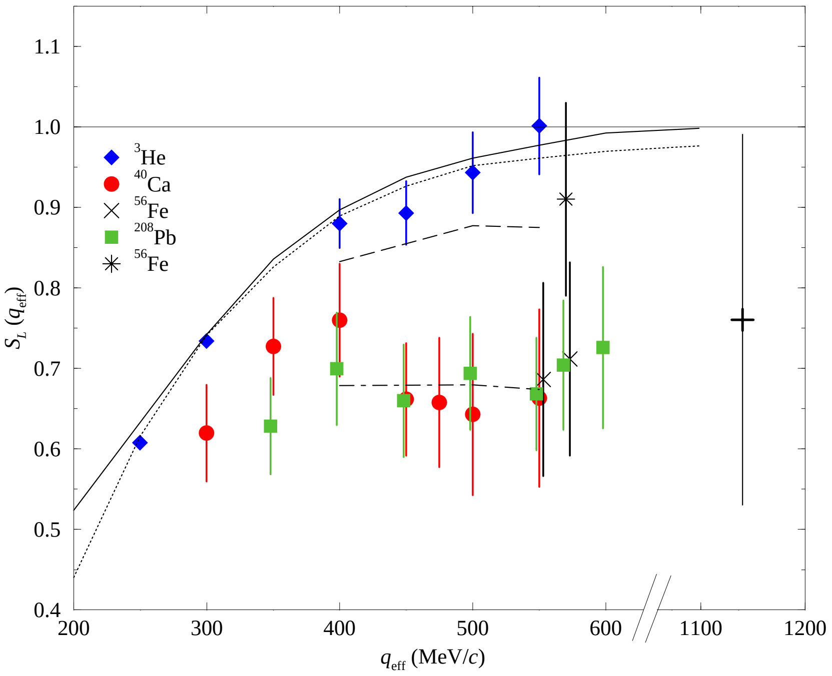
Coulomb Sum Rule

- By definition, $S_L(q) = 1$ for the free proton
- Saturation of the Coulomb Sum

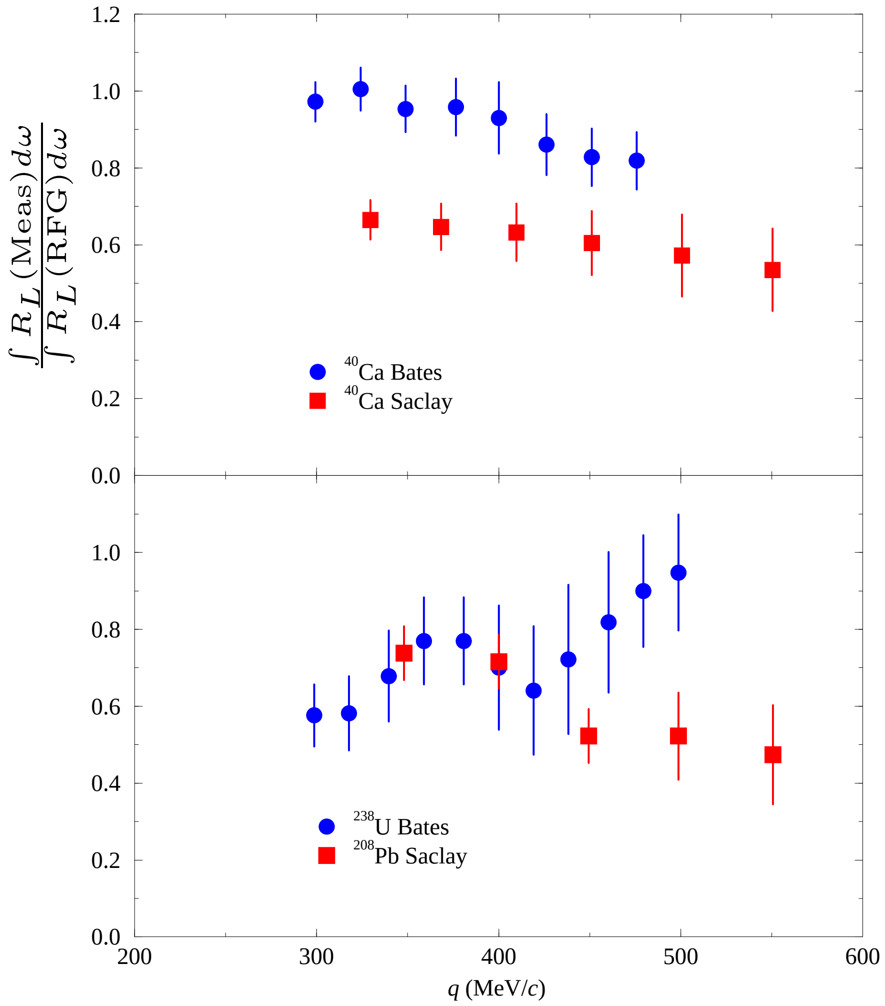
  $S_L(q) \to 1$ at sufficiently large q
- Deviation of the Coulomb Sum Rule
  - at small q – Nucleon-nucleon long-range correlations and Pauli blocking
  - at large q – Short range correlations and modification of the free nucleon electromagnetic properties inside the nuclear medium
- Nuclear density dependence (^{4}He to ^{208}Pb)
- Related to chiral symmetry restoration in dense nuclear medium

Measurements

- For the past twenty years, a large experimental program at Bates, Saclay and SLAC
- Limited kinematic coverage in q and ω due to machine limitations

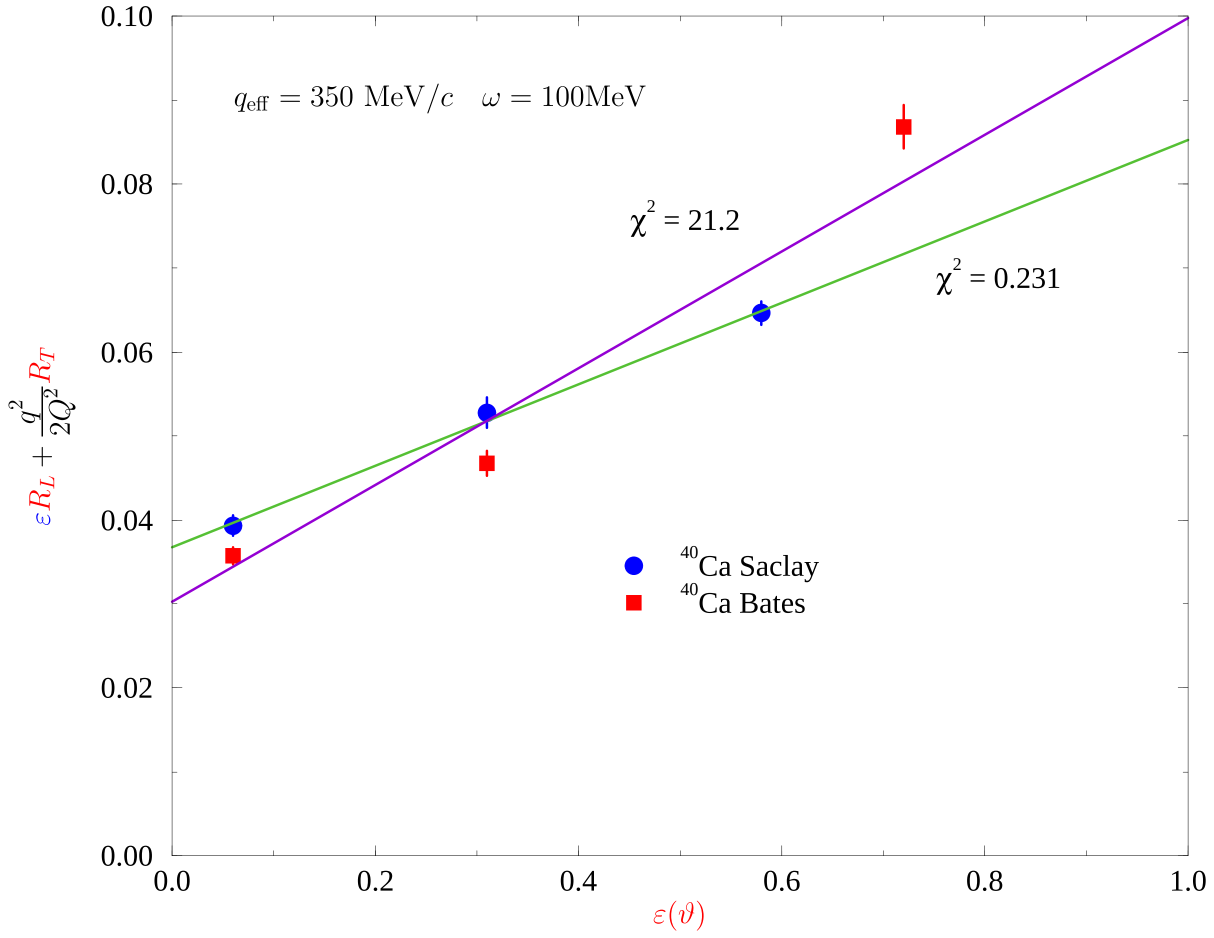


Controversy

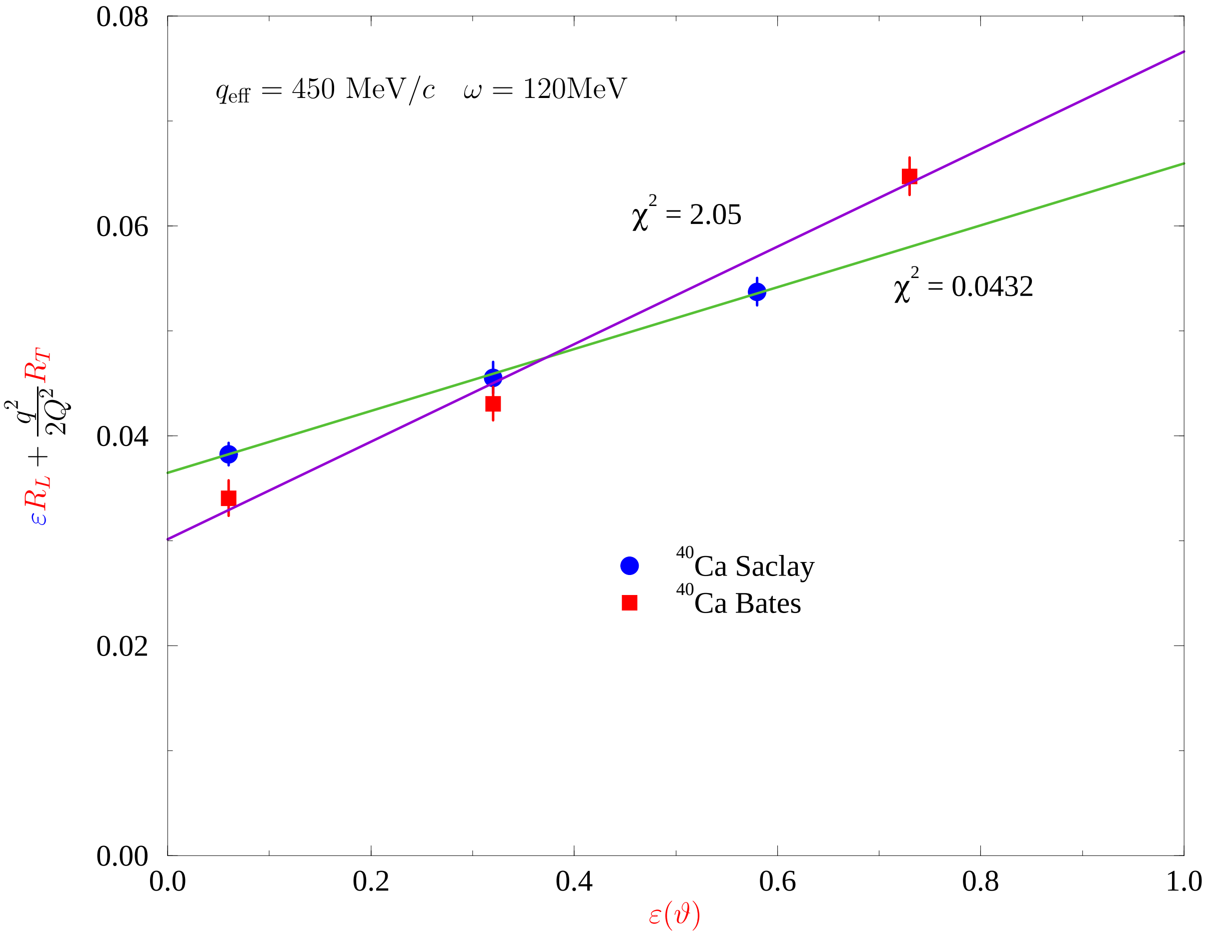


- Early data shows significant quenching of the CSR.
- With the addition of forward angle data, Bates claims no significant quenching.
- Saclay new analysis claims that quenching persists.

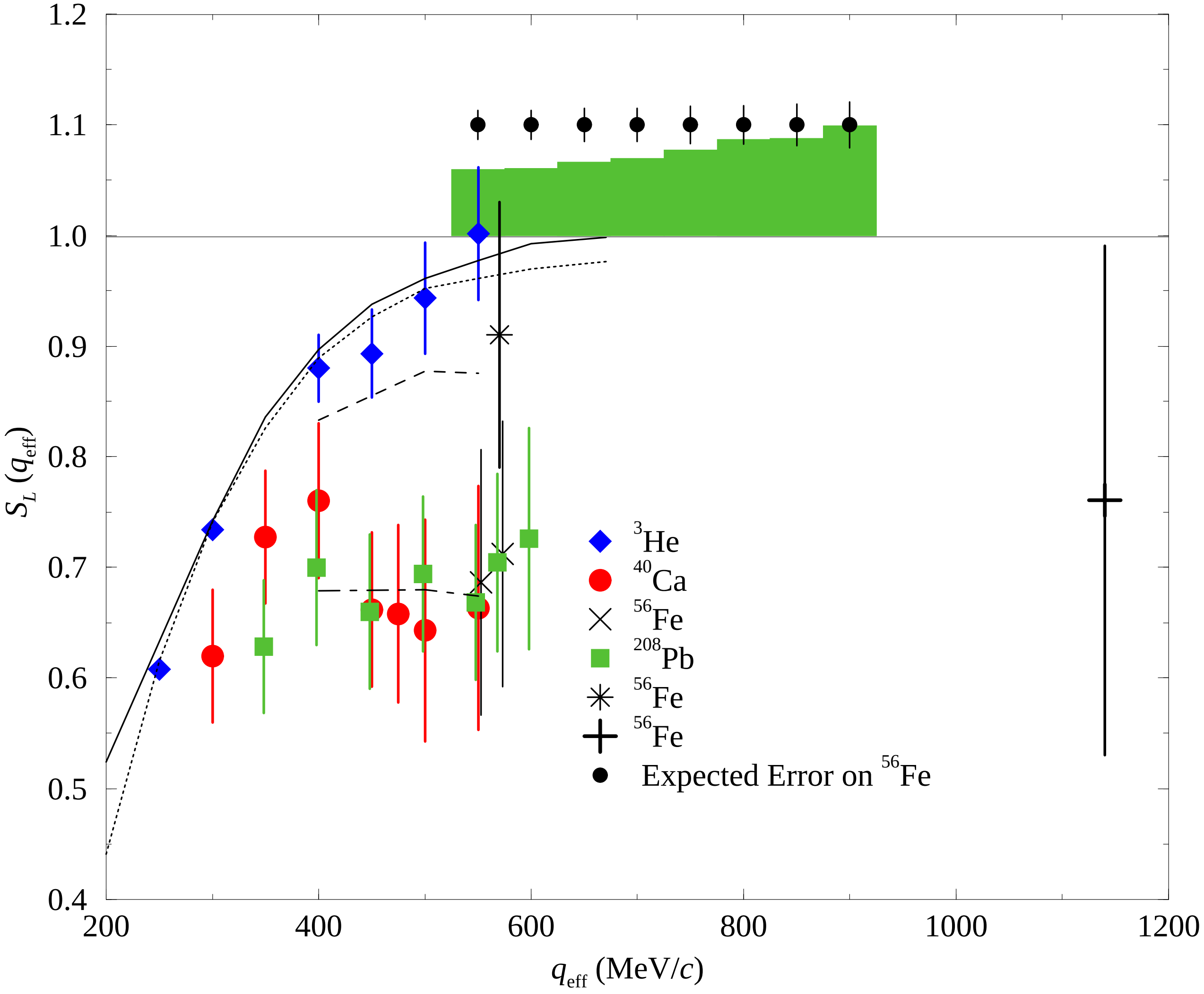
Saclay(France) vs. MIT(US)



Saclay(France) vs. MIT(US)



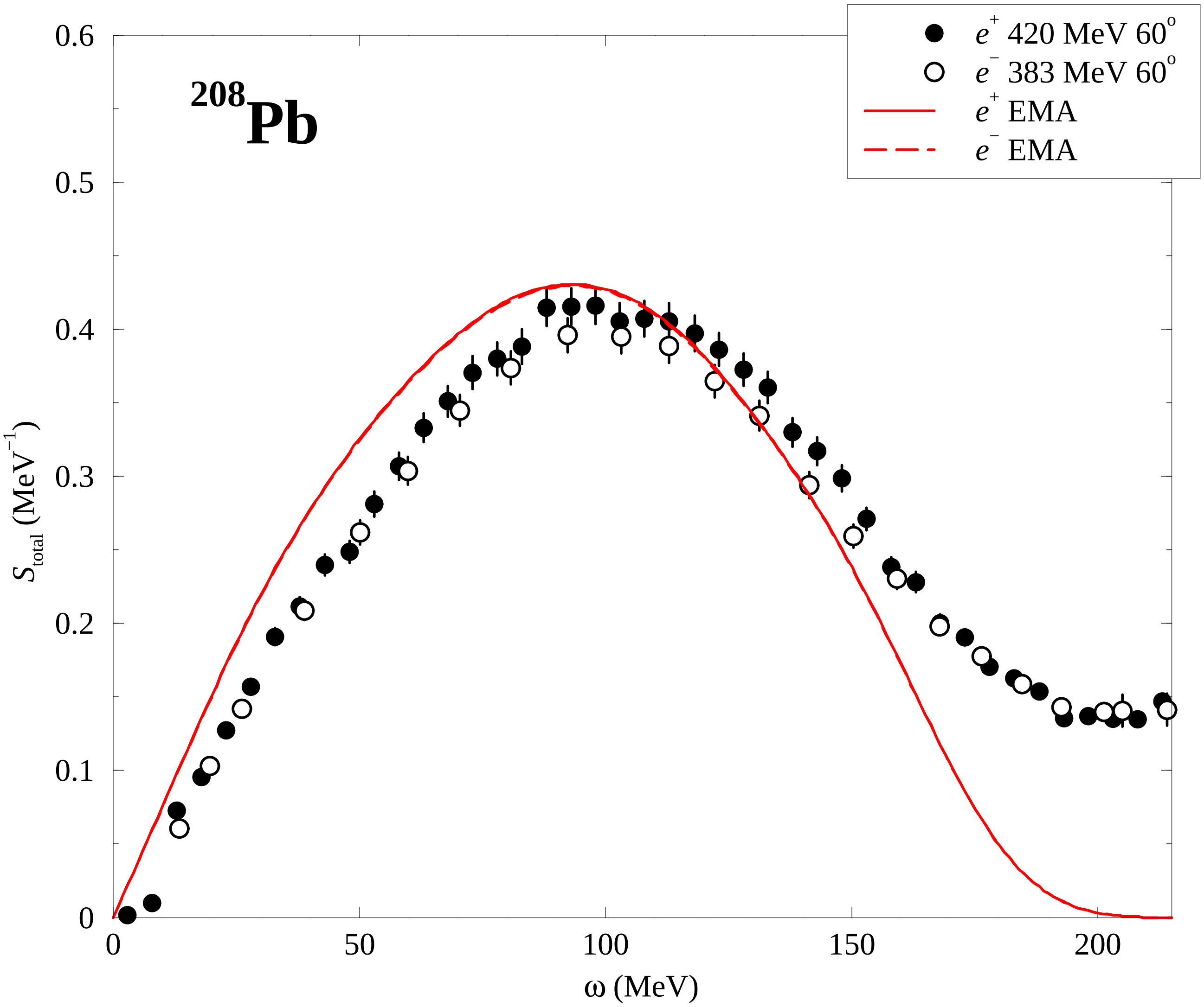
New Proposed Experiment at JLab



Coulomb Corrections

- Due to high Z of the nucleus
  - electrons are *accelerated* while entering into the nucleus
  - electrons are *decelerated* while leaving the nucleus
- As a result, when we send electron beam of energy E and detect outgoing electron of energy E'
  - Inside the nucleus, electron beam will have energy $E + \Delta E$
  - Just after the scattering, outgoing electron *had* an energy $E + \Delta E'$
- Several ways to account for this effect
  - Full calculation using *Distorted Wave Born Approximation*
  - Approximate way with *Effective Momentum Approximation*
- Experimental cross check with positron (e^+) beam

Data & EMA at Forward Angle



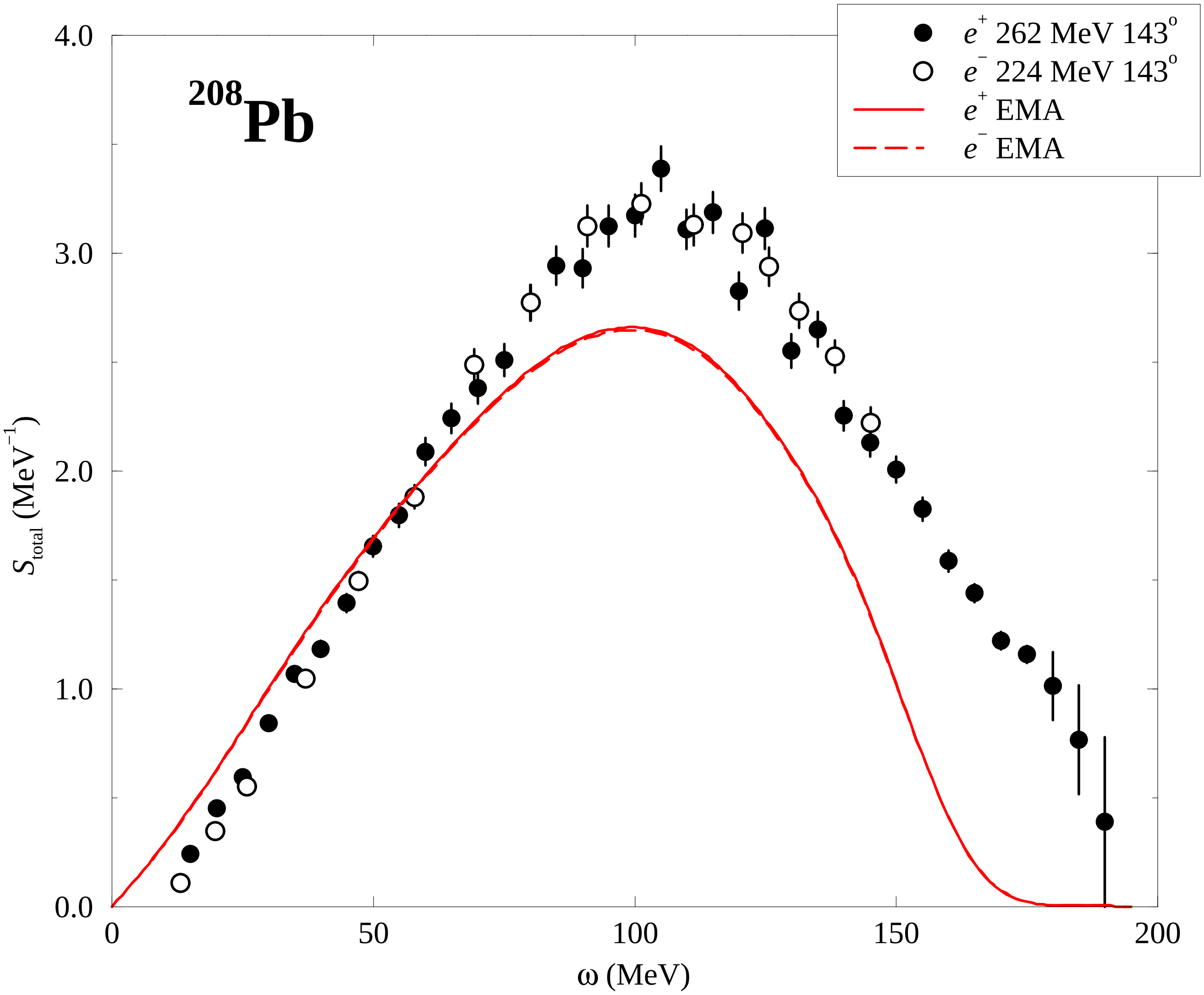
In EMA,

$$\left.\begin{array}{ccc} E & \rightarrow & E - \overline{V_C} \\ E' & \rightarrow & E' - \overline{V_C} \end{array}\right\} \longrightarrow \mathbf{q}_{\text{eff}}, Q^2_{\text{eff}}$$

with $|\overline{V_C}|$=19.0±1.5 MeV for ^{208}Pb

P. Guèye *et al.*, Physical Review C **60** 044308 (1999)

Data & EMA at Backward Angle



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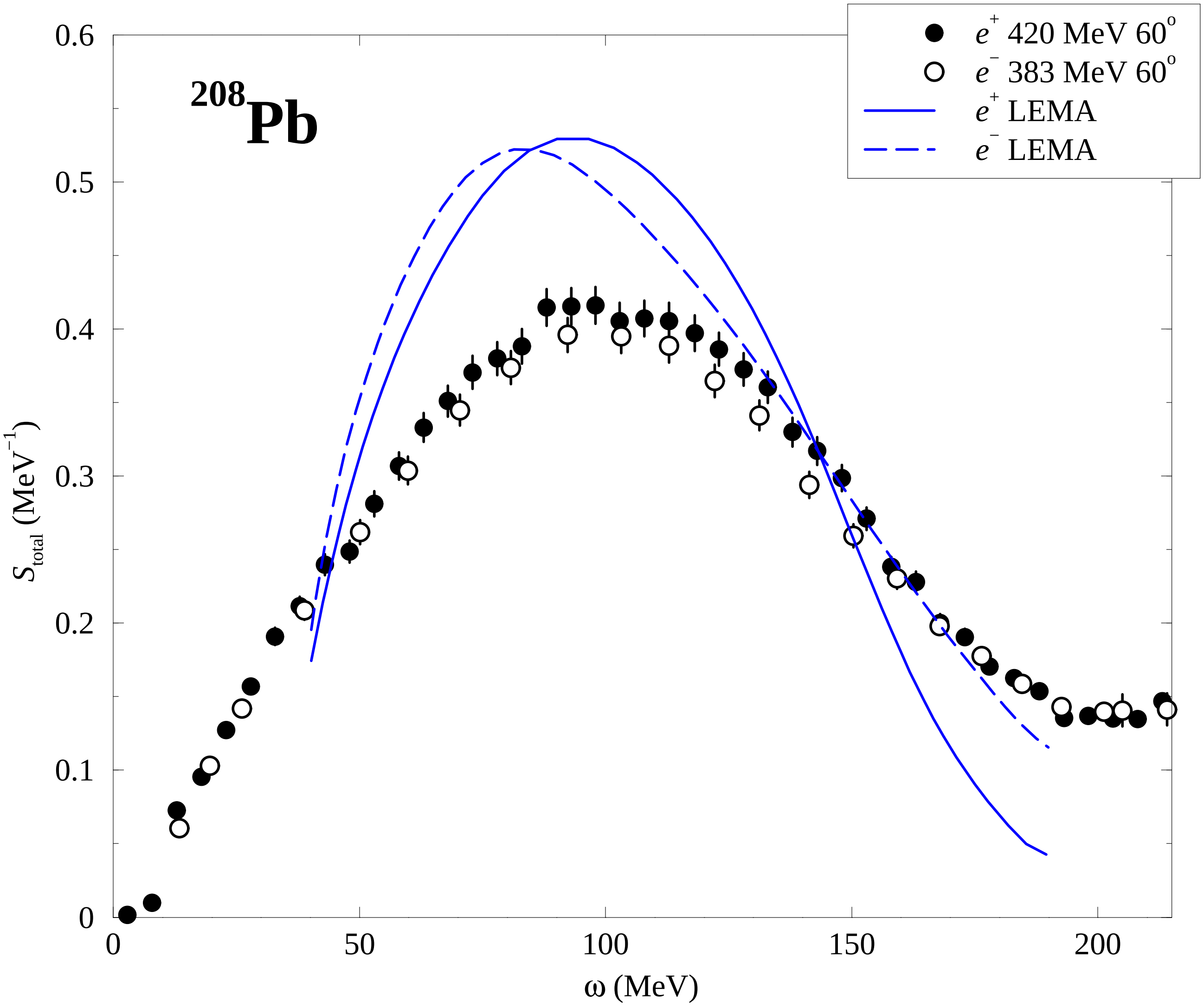
Data & LEMA at Forward Angle



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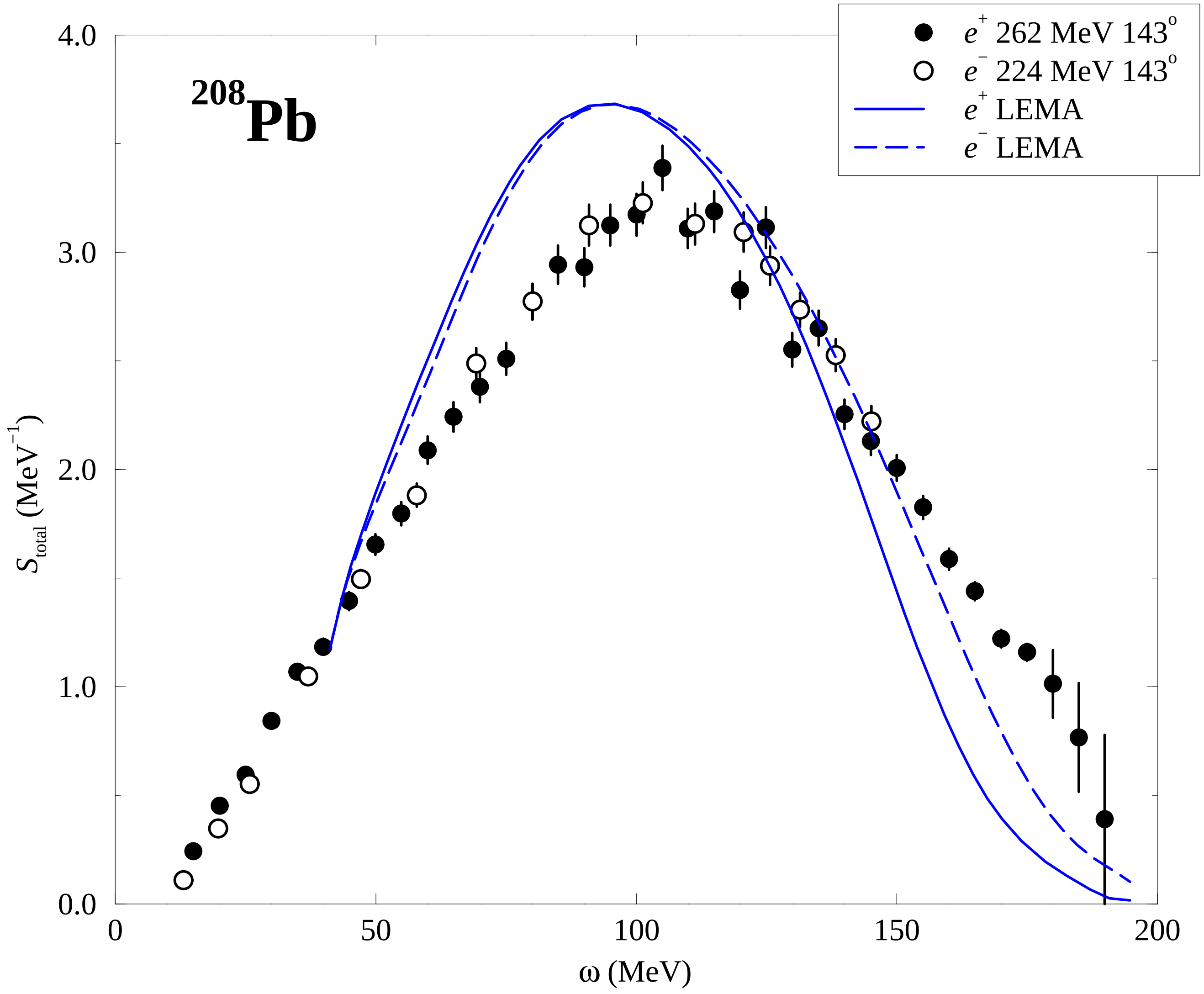
Data & LEMA at Backward Angle



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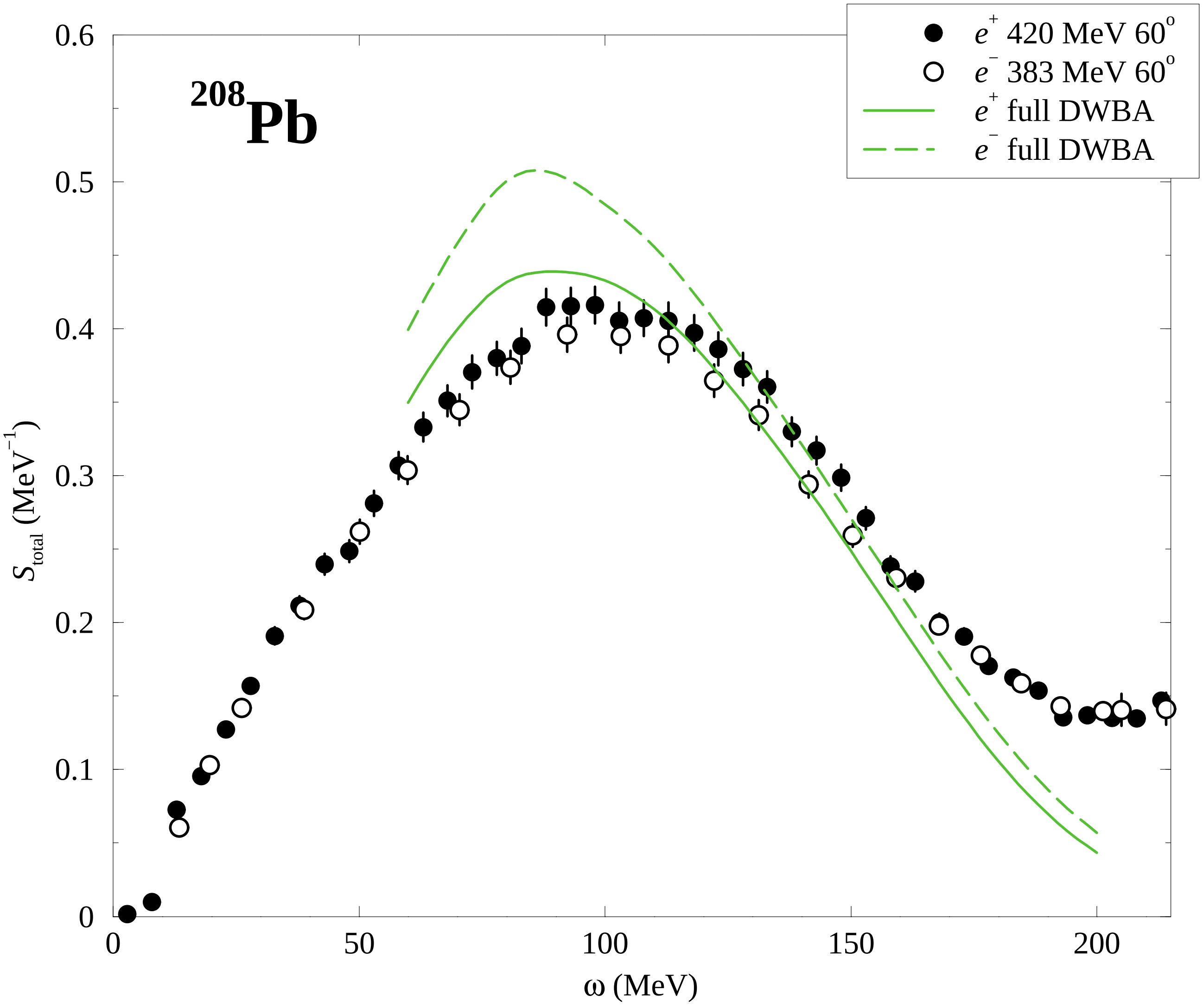
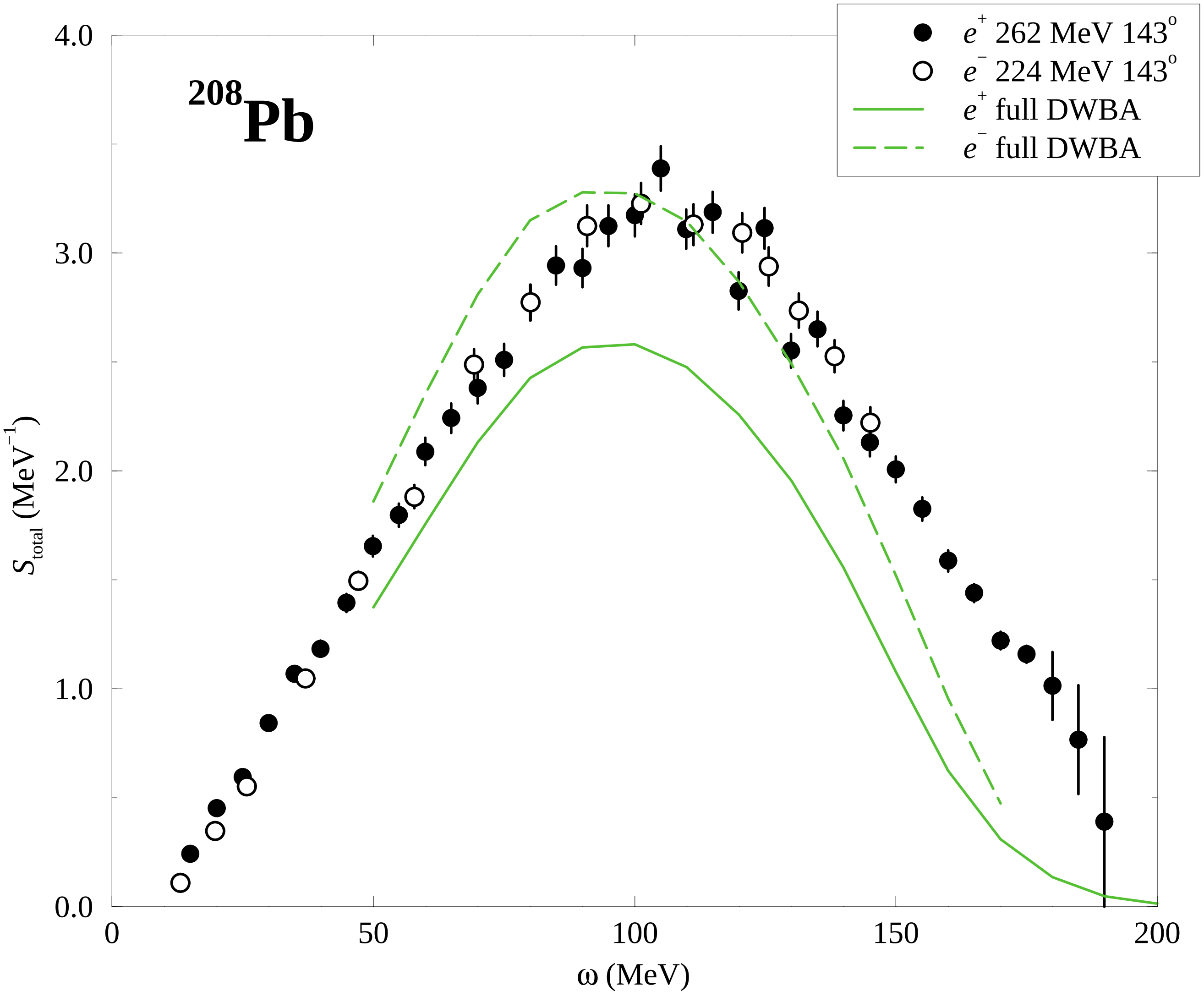
Data & DWBA at Forward Angle



K.S. Kim, L.E. Wright and D. A. Resler,
Physical Review C **64** 044607 (2001)

Data & DWBA at Backward Angle



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