The Other Structure Function, *etc*

presented by

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Spin Structure Function $g_2(x, Q^2)$

- In Quark-Parton Model, $g_2(x, Q^2) = 0$
- Using Operator Product Expansion (OPE) from perturbative QCD

$$\begin{aligned}
\int_0^1 x^n g_1(x, Q^2)\, dx &= \frac{1}{2} a_n, \quad n = 0, 2, 4, ..., \\
\int_0^1 x^n g_2(x, Q^2)\, dx &= \frac{1}{2} \frac{n}{n+1} (d_n - a_n), \quad n = 2, 4, ...,
\end{aligned}$$

- Decomposition of $g_2(x, Q^2)$ into two parts

$$g_2(x, Q^2) = g_2^{WW}(x, Q^2) + \bar{g}_2(x, Q^2)$$

Interesting Part of $g_2(x, Q^2)$

- Wandzura and Wilczek(Nobel prize, 2004) has shown

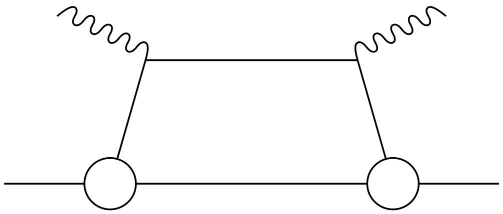
$$g_2^{WW}(x, Q^2) = -g_1(x, Q^2) + \int_x^1 \frac{g_1(y, Q^2)}{y}\, dy$$

- g_2^{WW} is the *trivial* part of $g_2(x, Q^2)$.
- Wandzura and Wilczek suggested that $\bar{g}_2(x, Q^2) = 0$.
- Moments of $\bar{g}_2(x, Q^2)$

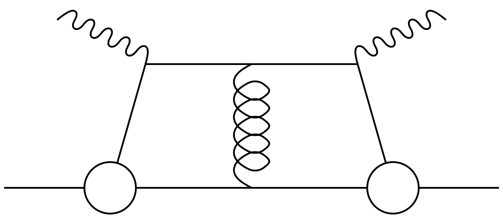
$$\int_0^1 x^n \bar{g}_2(x, Q^2)\, dx = \frac{n}{2(n+1)} d_n(Q^2)$$

Higher Twist Effect

- $\bar{g}_2(x, Q^2)$ is interesting since it is from *twist*-3 and higher.
- Twist-3 and higher effect is coming from quark-gluon correlations.
- In very simple picture, twist-2 (lowest possible twist) contribution.

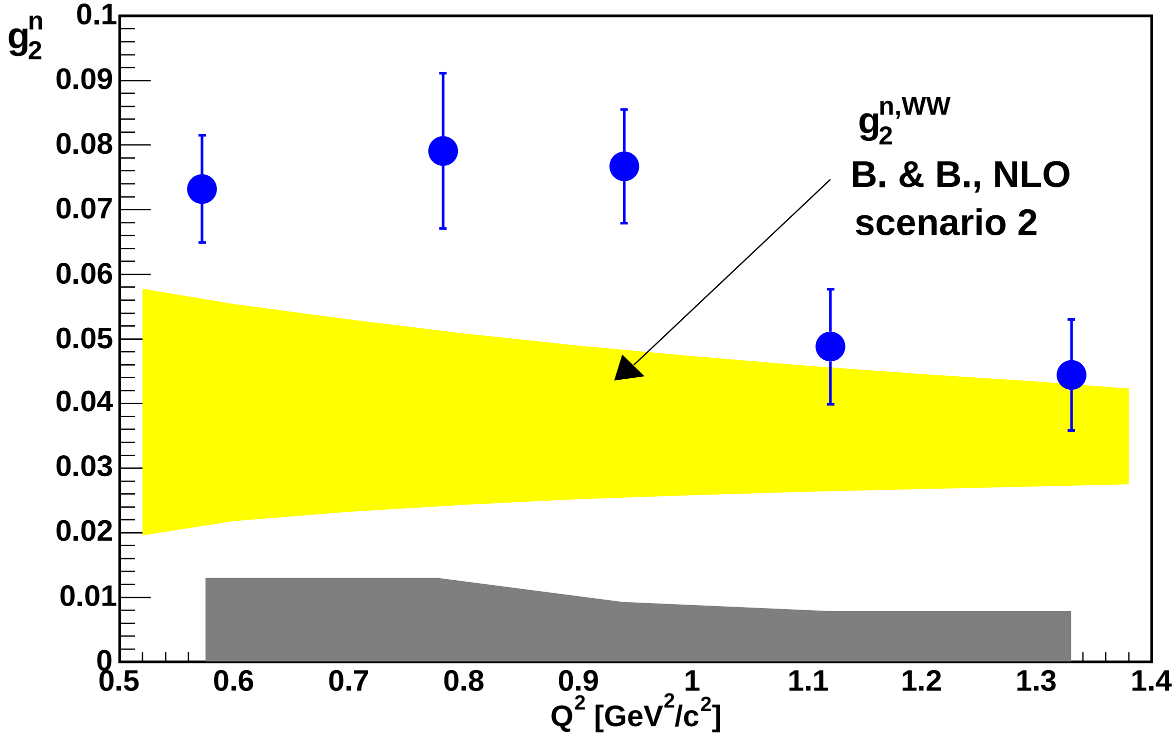


- Again in very simple picture, twist-3 contribution.

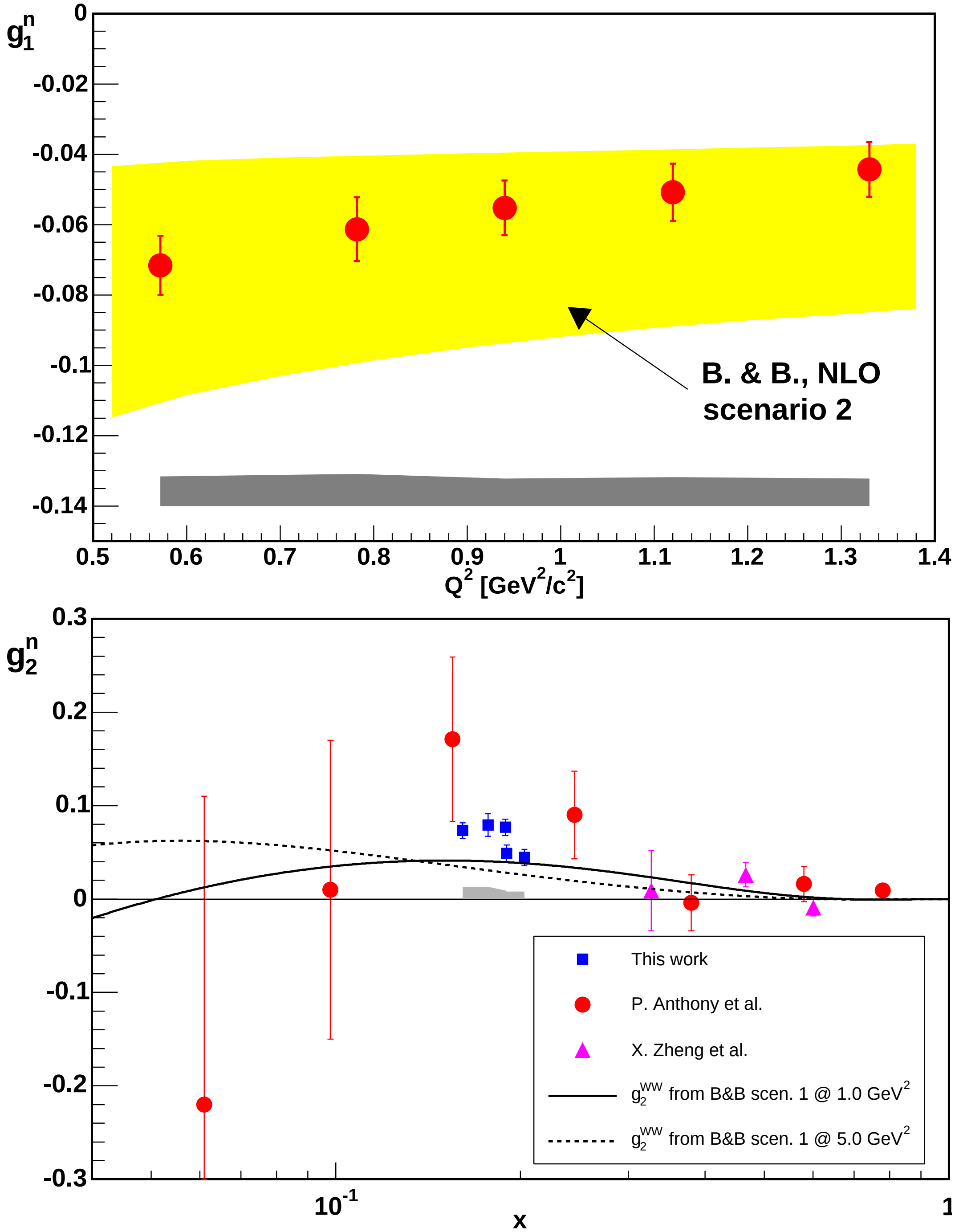


Measurement of g_2^n

- Precision g_2^n data covering $0.57 < Q^2 < 1.34$ GeV^2 at $x \sim 0.2$
- Direct comparison with g_2^{WW} prediction using world g_1^n data
- Measured g_2^n consistently higher than g_2^{WW} at low Q^2



g_2^n (Cont.)



g_1^n
B. & B., NLO
scenario 2
Q^2 [GeV^2/c^2]
g_2^n
x
This work
P. Anthony et al.
X. Zheng et al.
g_2^{WW} from B&B scen. 1 @ 1.0 GeV^2
g_2^{WW} from B&B scen. 1 @ 5.0 GeV^2

Sum Rule for $g_2(x, Q^2)$

- Burkhardt and Cottingham has shown that

$$\int_0^1 g_2(x, Q^2)\, dx = 0$$

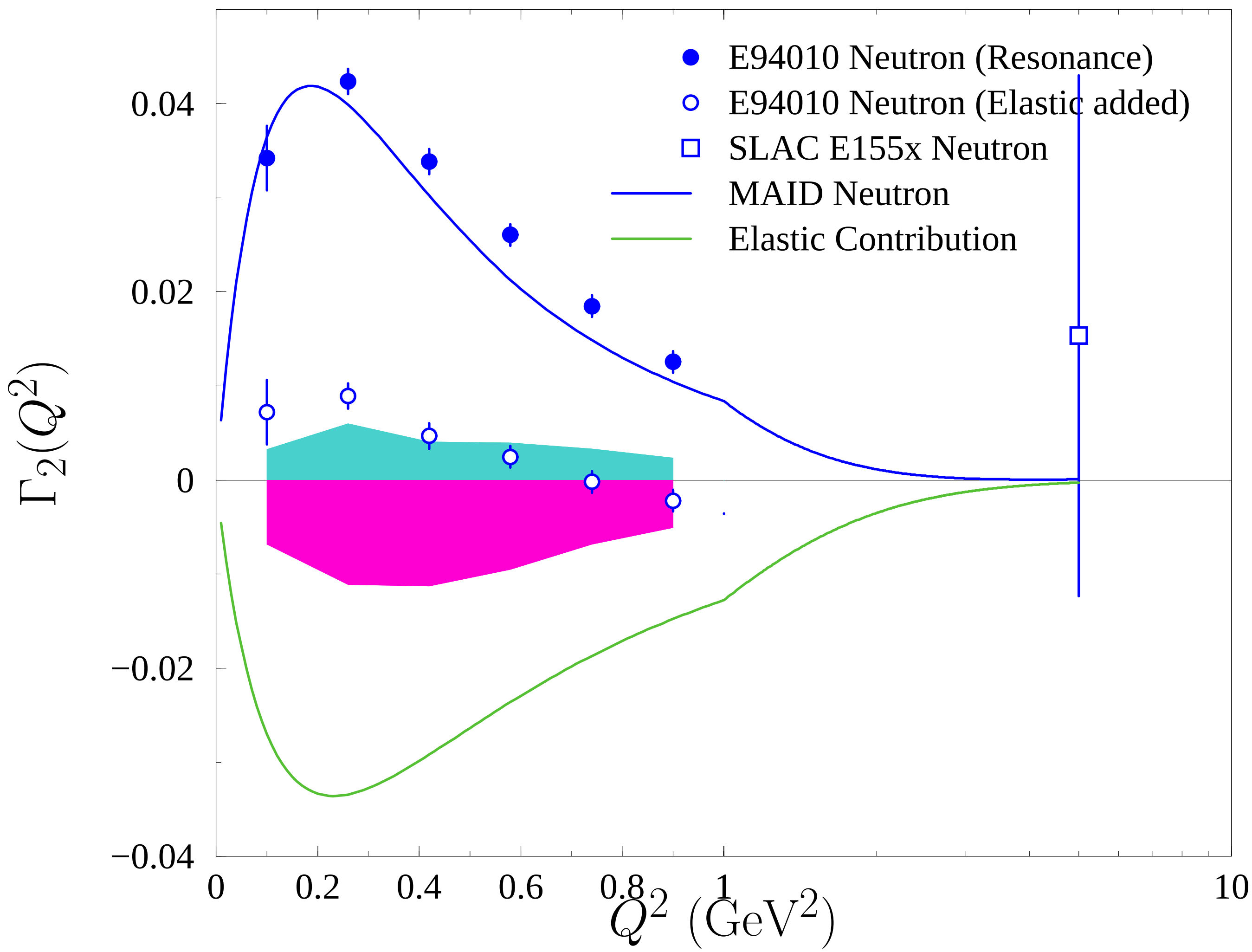
- Similar assumptions as Gerasimov-Drell-Hearn sum rule
- Vanishing amplitude as $|R| \to \infty$ is *questionable.*
  - Possibility of divergence of the integral - quite slim
  - Possibility of δ function at $x = 0$

$$g_2(x, Q^2) = g_2^{\text{observable}}(x, Q^2) + c\delta(x)$$

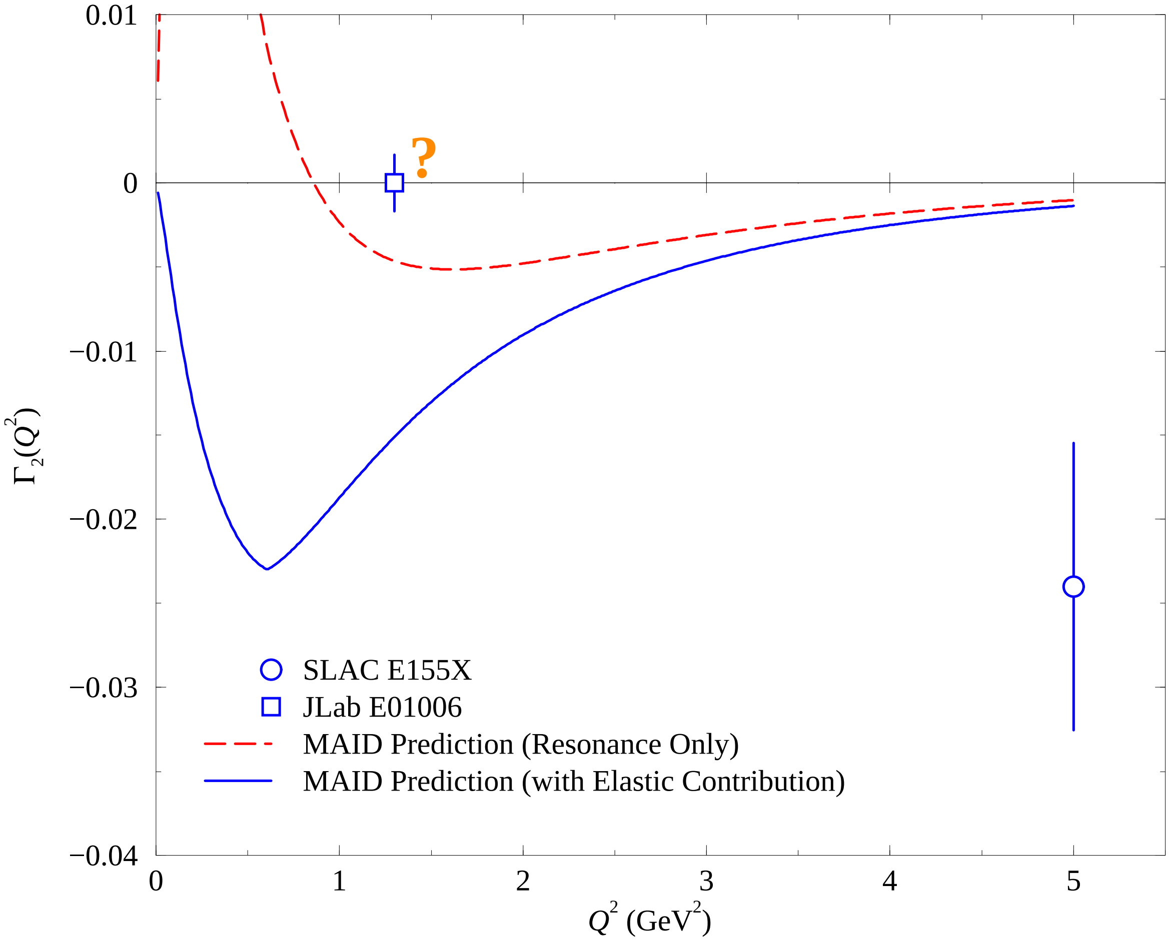
  - Sum rule holds, but can not be observed.

$$\int_0^1 g_2^{\text{observable}}(x, Q^2)\, dx = -\frac{1}{2}c$$

Experiments on the Neutron



What about the Proton?



A_1 at Large x

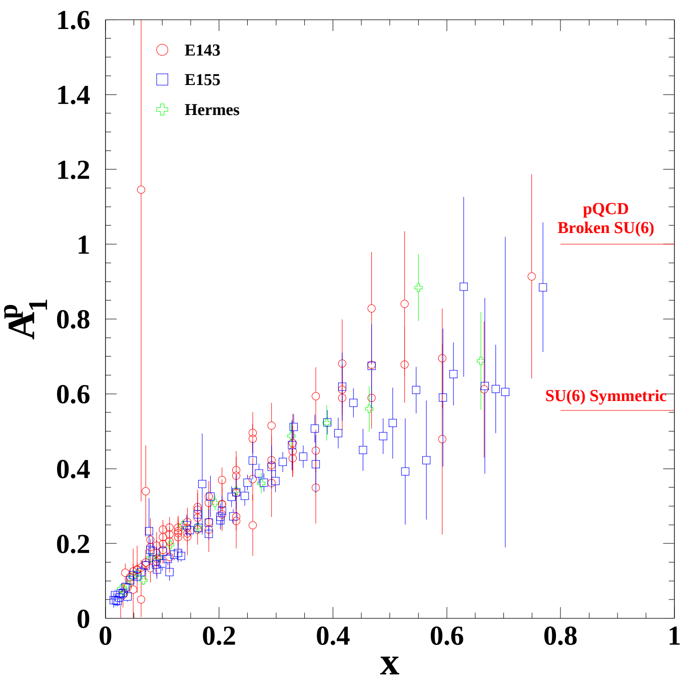
$$A_1(Q^2) = \frac{\sigma_{1/2} - \sigma_{3/2}}{\sigma_{1/2} + \sigma_{3/2}} = \frac{g_1 - \gamma^2 g_2}{F_1}$$

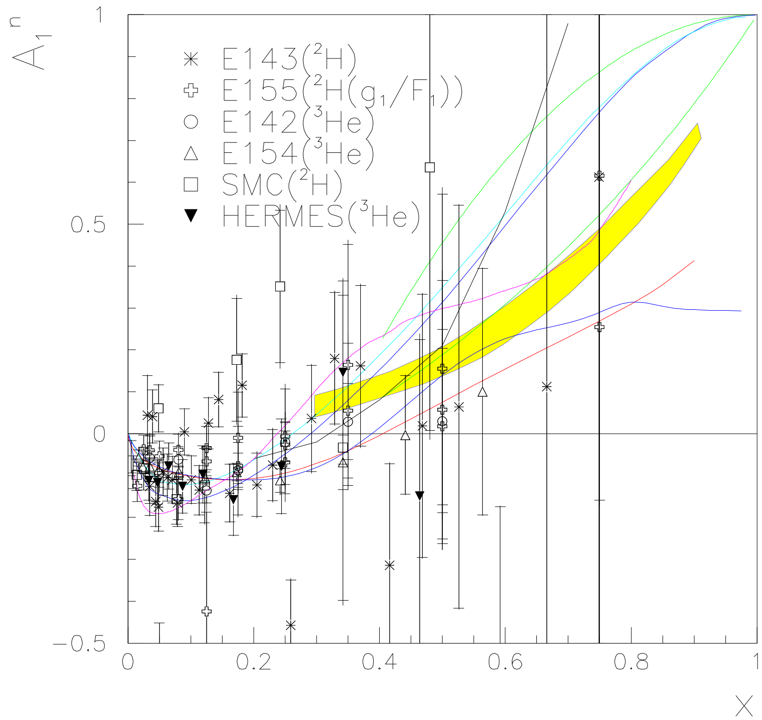
$$\text{with} \quad \gamma^2 = \frac{4M^2x^2}{Q^2}$$

- Large x region dominated by valence quarks
- Relatively clean region to study the nucleon structure
- Predictions from various models

	SU(6)	CQM	pQCD
A_1^p	5/9	1	1
A_1^n	**0**	**1**	**1**

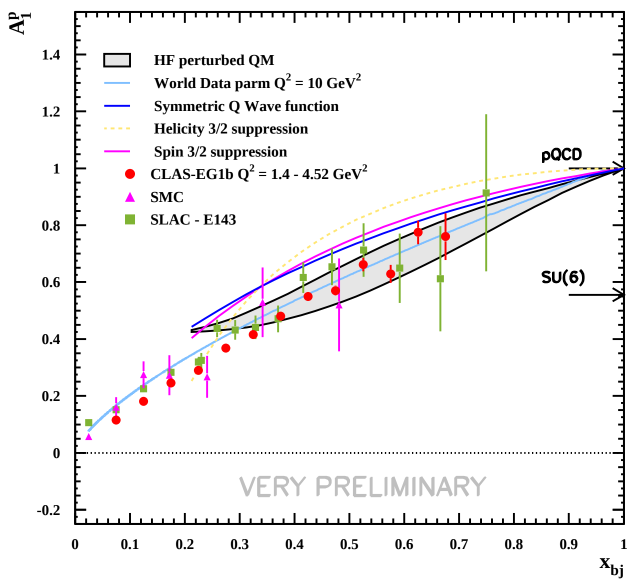
$A_1^{p,n}$ World Data

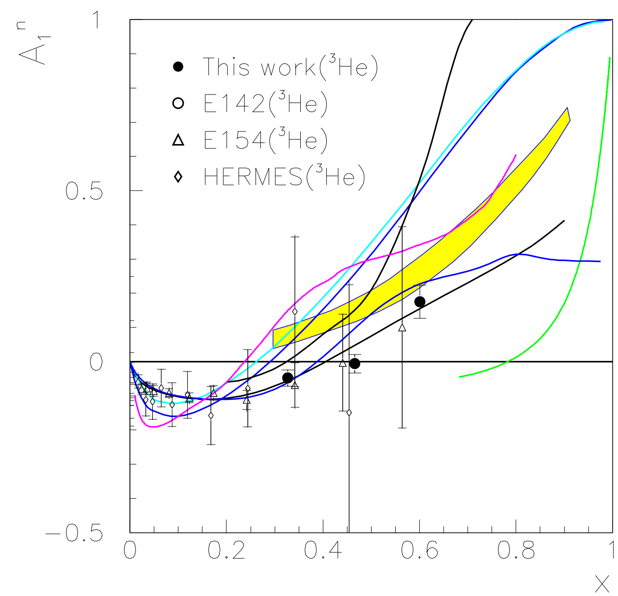




- Poor precision as x approaches to 1
- Not yet close enough to $x = 1$
- Hard to distinguish predictions from various models

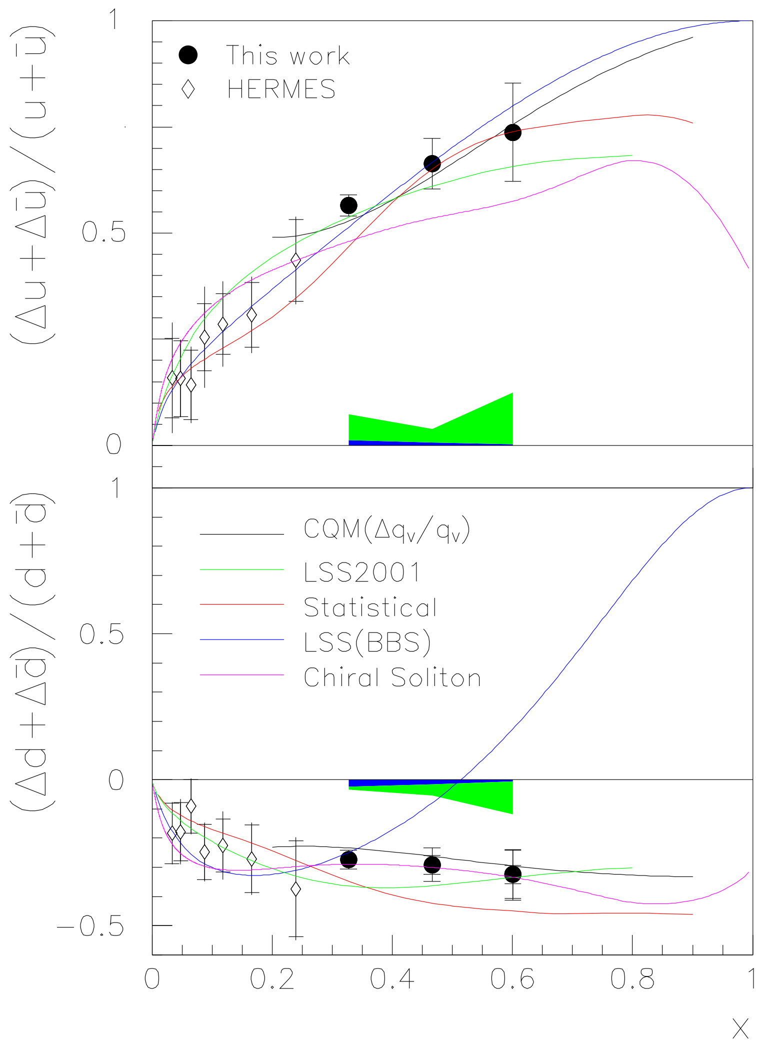
$A_1^{p,n}$ at Large x





- First precision $A_1^{p,n}$ data at high x
- Test of fundamental understanding of valence quark picture
- Extracting valence quark spin distributions
- Crucial input for pQCD fit to PDF

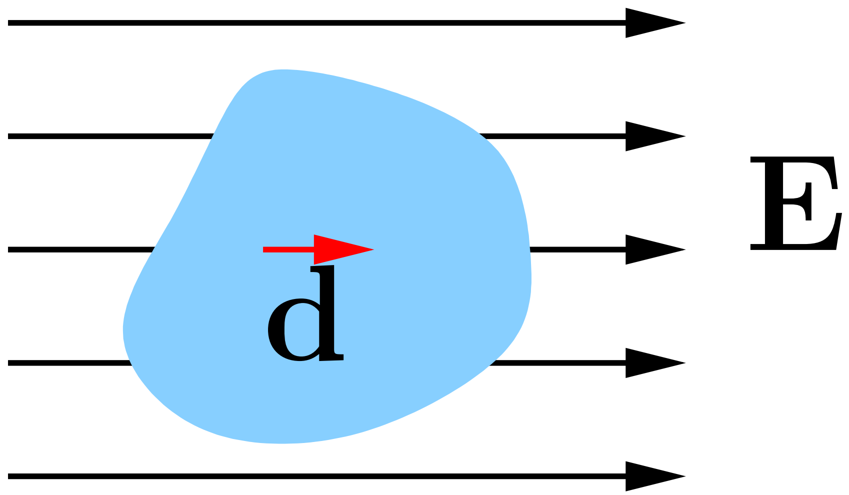
Polarized Quark Distributions



- ➜ Combining A_1^n and A_1^p results
- ➜ u quark spin as expected
- ➜ d quark spin stays negative
  - Disagrees with pQCD model calculation assuming HHC
  - non-negligible quark orbital angular momentum
- ➜ Consistent with valence quark models or pQCD PDF fits

Polarizability

Classical Electromagnetism



$\mathbf{d} = \alpha \mathbf{E}$ electric dipole polarizability

Spin Polarizabilites

Virtual Compton scattering

$$T(\nu,Q^2) = \varepsilon'^* \cdot \varepsilon f_T(\nu,Q^2) + f_L(\nu,Q^2)$$
$$+i\sigma\cdot(\varepsilon'^* \times \varepsilon) g_{TT}(\nu,Q^2) - i\sigma\cdot[(\varepsilon'^* - \varepsilon)\times\hat{q}] g_{LT}(\nu,Q^2)$$

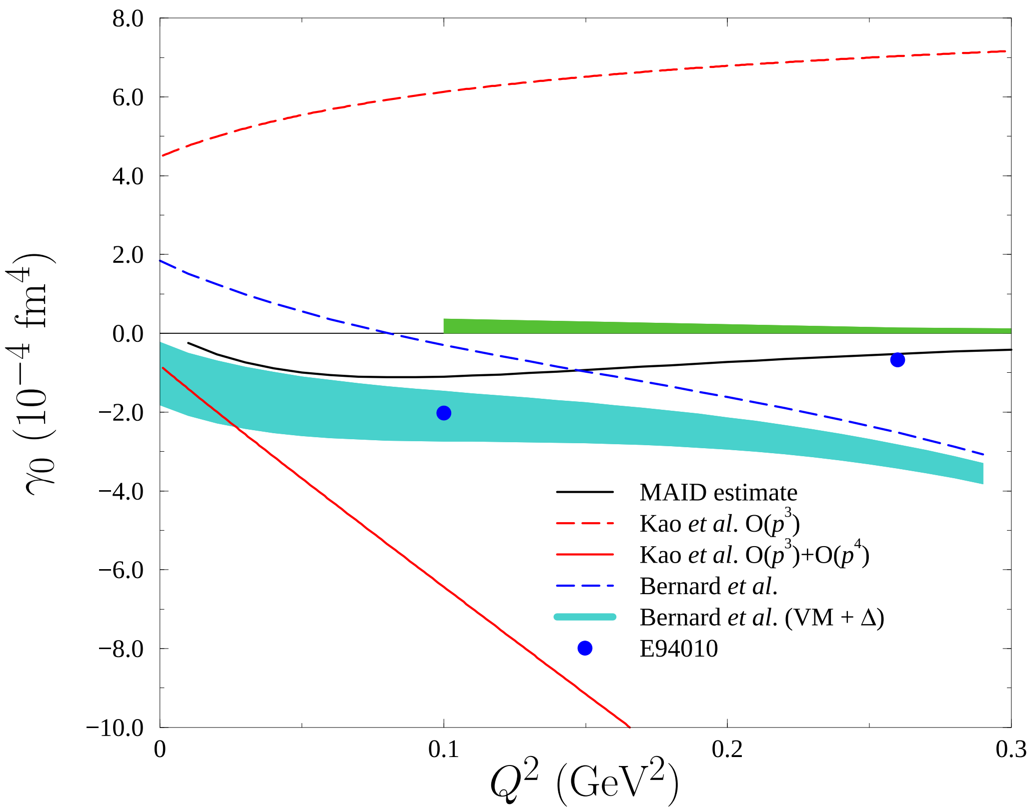
f_T	$\rightarrow \alpha + \beta$	electric and magnetic polarizabilities
g_{TT}	$\rightarrow \gamma_0$	forward spin polarizability
f_L	$\rightarrow \alpha_L$	longitudinal polarizability
g_{LT}	$\rightarrow \delta_{LT}$	longitudinal-transverse polarizability

$$\gamma_0(Q^2) = \frac{16M^2\alpha_{\rm em}}{Q^6}\int_0^{x_0} x^2\left\{g_1(x,Q^2) - \frac{Q^2}{\nu^2}g_2(x,Q^2)\right\}dx$$

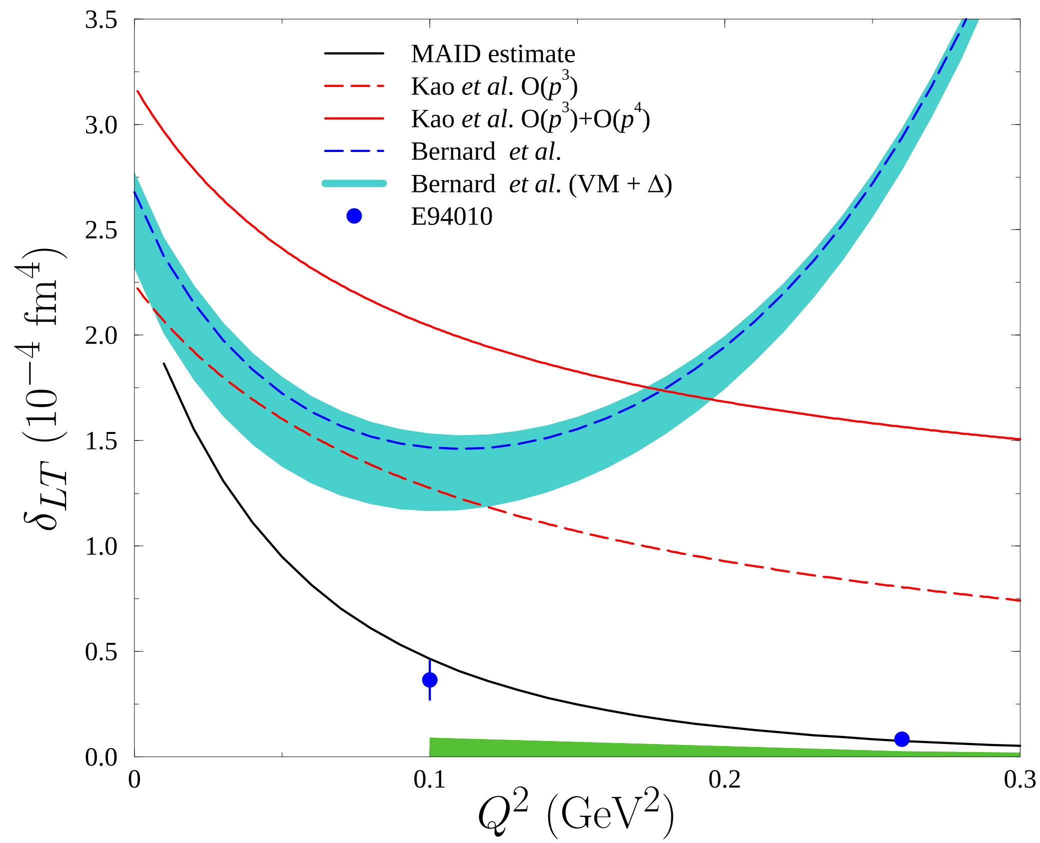
$$\delta_{LT}(Q^2) = \frac{16M^2\alpha_{\rm em}}{Q^6}\int_0^{x_0} x^2\left\{g_1(x,Q^2) + g_2(x,Q^2)\right\}dx$$

$$\delta_{LT}(Q^2) \rightarrow \frac{1}{3}\gamma_0(Q^2), \quad Q^2\rightarrow\infty$$

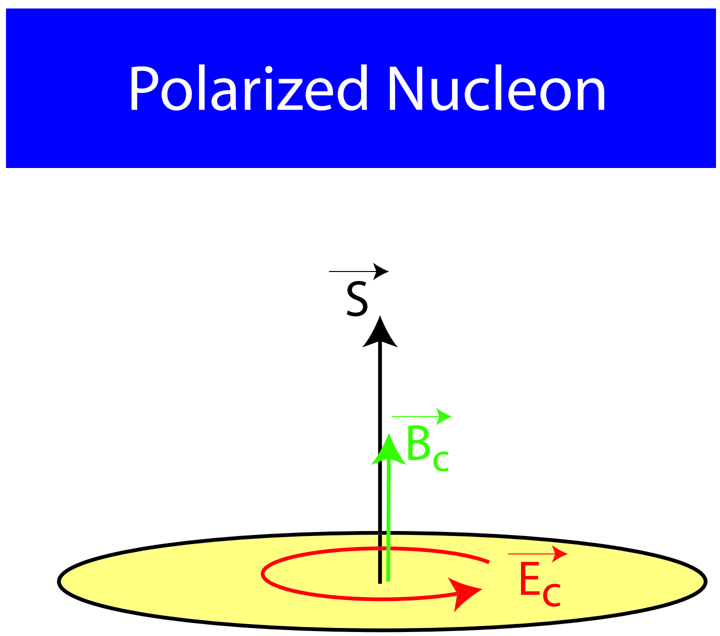
Spin Polarizability γ_0



Spin Polarizability δ_{LT}



Gluon Field Polarizabilities



Induced Color
Magnetic and Electric Fields

$$\mathbf{B}_C \sim \chi_B \mathbf{S}$$
$$\mathbf{E}_C \sim \chi_E \mathbf{S}$$

Twists Expansion of Γ_1

- According to OPE, at $Q^2 \gg \Lambda^2_{\rm QCD}$,

$$\Gamma_1(Q^2) = \sum_{\tau=2,4,\cdots} \frac{\mu_\tau(Q^2)}{Q^{\tau-2}}$$

- Leading twist μ_2

$$\begin{aligned}
\mu_2(Q^2) &= C_{NS}(Q^2)\left(-\frac{1}{12}a_3 + \frac{1}{36}a_8\right) + C_S(Q^2)\frac{1}{9}\Delta\Sigma \\
a_3 &\equiv g_A = \Delta u - \Delta d, \quad a_8 = \Delta u + \Delta d - 2\Delta s \\
\Delta\Sigma &= \Delta u + \Delta d + \Delta s
\end{aligned}$$

- Wilson coefficients from pQCD

$$\begin{aligned}
C_{NS}(Q^2) &= 1 - \left(\frac{\alpha_S}{\pi}\right) - 3.58\left(\frac{\alpha_S}{\pi}\right)^2 - 20.22\left(\frac{\alpha_S}{\pi}\right)^2 \\
C_S(Q^2) &= 1 - 0.33\left(\frac{\alpha_S}{\pi}\right) - 0.55\left(\frac{\alpha_S}{\pi}\right)^2 - 4.45\left(\frac{\alpha_S}{\pi}\right)^2
\end{aligned}$$

Higher Twist Component

- Subtracting $\mu_2^n(Q^2)$,

$$\Delta\Gamma_1(Q^2) \equiv \Gamma_1(Q^2) - \mu_2(Q^2) = \frac{\mu_4(Q^2)}{Q^2} + \mathcal{O}\left(\frac{1}{Q^2}\right)$$

- $\mathcal{O}(1/Q^2)$ term

$$\mu_4(Q^2) = \frac{1}{9}M^2\left(a_2(Q^2) + 4d_2(Q^2) + 4f_2(Q^2)\right)$$

- a_2 and d_2 from spin structure functions

$$\begin{aligned} a_2(Q^2) &= 2\int_0^1 x^2 g_1(x,Q^2)\,dx \\ d_2(Q^2) &= 3\int_0^1 x^2 \bar{g}_2(x,Q^2)\,dx \\ &= \int_0^1 x^2\left(2g_1(x,Q^2) + 3g_2(x,Q^2)\right)\,dx \end{aligned}$$

Higher Twist Effect of $g_2(x, Q^2)$

$$d_2(Q^2) = \int_0^1 x^2 \left[2g_1(x, Q^2) + 3g_2(x, Q^2)\right] dx$$

- At high Q^2, d_2 measures induced color field by target spin

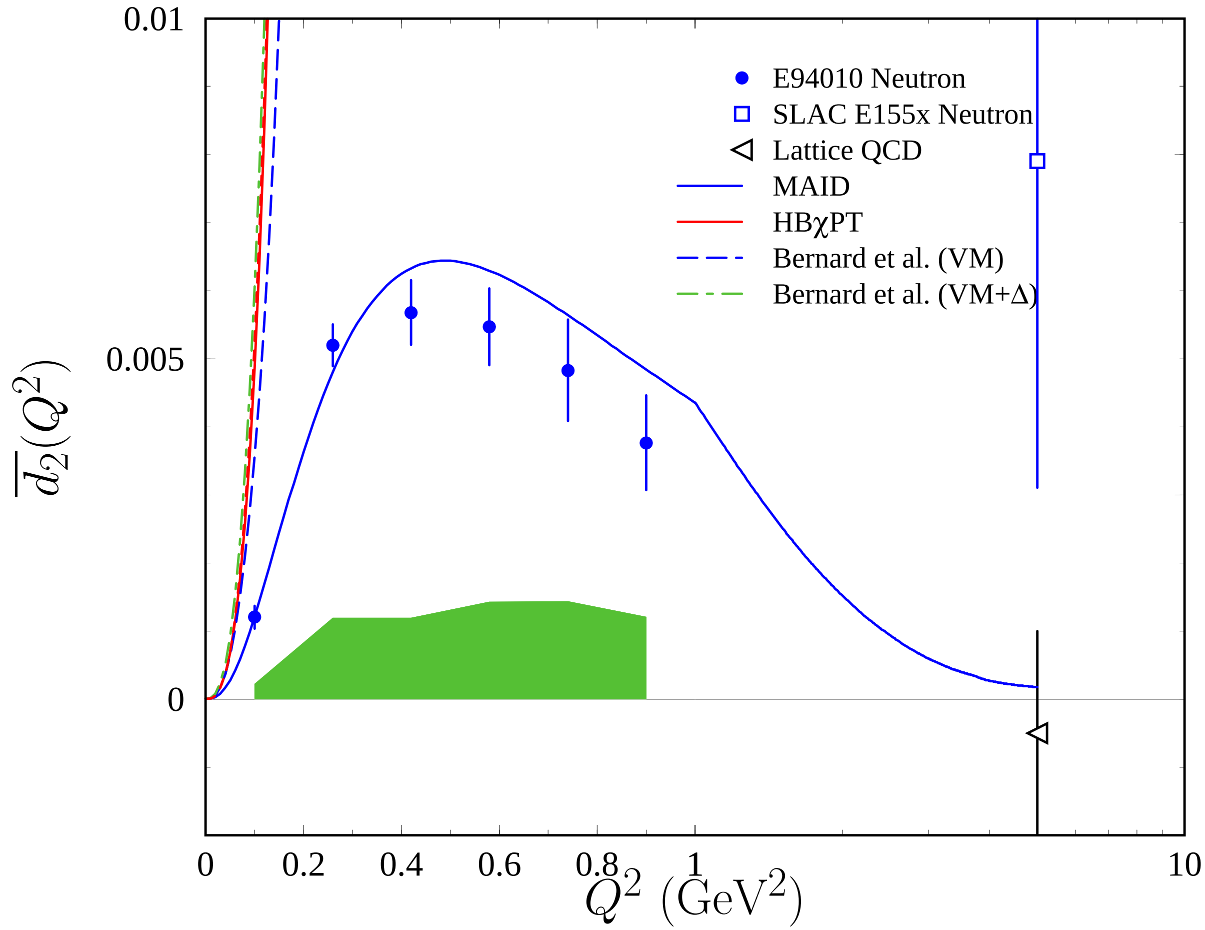
$$d_2 = \frac{1}{8}(\chi_E + 2\chi_B)$$

- At low Q^2, d_2 is related to spin polarizabilities

$$d_2(Q^2) = \frac{Q^6}{16M^2\alpha_{\rm em}}\left[3\delta_{LT}(Q^2) - \gamma_0(Q^2)\right]$$

- Intermediate Q^2: transition between partonic and hadronic scales
- Necessary to study higher twist effect from g_1 structure function
- At $Q^2 = 5$ GeV^2, SLAC E155x shows large, positive d_2 but with big error bar.

$d_2(Q^2)$



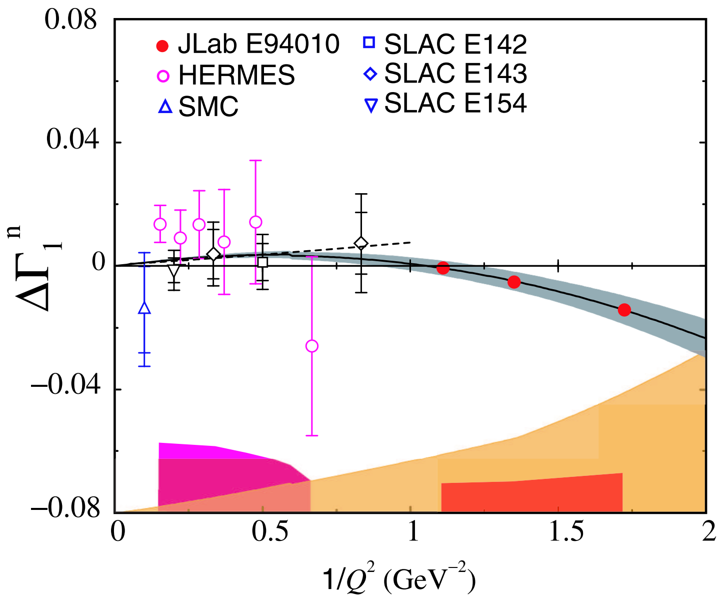
0.01
0.005
0
$\overline{d}_2(Q^2)$
0
0.2
0.4
0.6
0.8
1
10
Q^2 (GeV^2)
E94010 Neutron
SLAC E155x Neutron
Lattice QCD
MAID
HBχPT
Bernard et al. (VM)
Bernard et al. (VM+Δ)

Color Polarizabilities

Obtaining $f_2(Q^2)$ from Q^2 variation of $\Gamma_1(Q^2)$

$$\chi_E^n = \frac{2}{3}(2d_2^n + f_2^n)$$

$$\chi_B^n = \frac{1}{3}(4d_2^n - f_2^n)$$



$$f_2^n = 0.034 \pm 0.043$$

$$\mu_6^n = (-0.019 \pm 0.017)M^4$$

$$\chi_E^n = 0.033 \pm 0.029$$

$$\chi_B^n = -0.001 \pm 0.016$$