QCD Sum Rules

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Integral of g_1

- In Quark-Parton Model

$$g_1(x) = \frac{1}{2}\sum_i e_i^2 \left(f_i^{\uparrow}(x) - f_i^{\downarrow}(x) \right)$$

- For the proton

$$g_1^p(x) = \frac{1}{2}\left(\frac{4}{9}\Delta u(x) + \frac{1}{9}\Delta d(x) + \frac{1}{9}\Delta s(x) \right)$$

- Using

$$\Delta u = \int_0^1 \Delta u(x)\, dx \quad etc.$$

- Integral of $g_1(x)$ can be written as

$$\Gamma_1^p = \int_0^1 g_1^p(x)\, dx = \frac{1}{2}\left(\frac{4}{9}\Delta u + \frac{1}{9}\Delta d + \frac{1}{9}\Delta s \right)$$

Proton Spin

- Naïve Quark Model

$$\frac{1}{2}(\text{proton}) = \frac{1}{2}(\Delta u + \Delta d + \Delta s)$$

- Experimental measurement of Γ_1^p gives *one* linear combination of Δu, Δd and Δs.
- We need two more relations. $\rightarrow$ SU(3) flavor symmetry
- In QPM, linear combinations of these moments $\sim$ weal axial-vector couplings

$$\begin{aligned} a_0 &= \Delta u + \Delta d + \Delta s \equiv \Delta\Sigma \\ a_3 &= \Delta u - \Delta d \\ a_8 &= \Delta u + \Delta d - 2\Delta s \end{aligned}$$

Baryon Decays

- Semi-leptonic decays of baryon octet under SU(3) symmetry involves the transition $d \to u$ or $s \to u$
- Decay lifetime can be parametrized by two numbers F and D

$$
\begin{array}{ccc}
n \to p e^- \bar{\nu} & d \to u & F + D \\
\Xi^- \to \Xi^0 e^- \bar{\nu} & d \to s & F - D \\
\Xi^- \to \Lambda e^- \bar{\nu} & d \to s & F - D/3 \\
\Lambda \to p e^- \bar{\nu} & d \to u & F + D/3
\end{array}
$$

etc

- Current algebra under SU(3) gives

$$
\begin{aligned}
a_3 &= F + D \\
a_8 &= 3F - D
\end{aligned}
$$

Summary

- Three unknowns with three equations

$$\begin{aligned}\Gamma_1^p &= \frac{1}{2}\left(\frac{4}{9}\Delta u + \frac{1}{9}\Delta d + \frac{1}{9}\Delta s\right)\\ a_3 &= \Delta u - \Delta d = F + D\\ a_8 &= \Delta u + \Delta d - 2\Delta s = 3F - D\end{aligned}$$

- a_3 is related to the neutron β-decay and $a_3 = g_A$, axial coupling constant.

$$\begin{aligned}\Gamma_1^p &= \int_0^1 g_1^p(x)\,dx = \frac{g_A}{12}\left(\frac{1}{3}\frac{3F/D-1}{F/D+1} + 1\right) + \frac{1}{9}\Sigma\\ \Gamma_1^n &= \int_0^1 g_1^n(x)\,dx = \frac{g_A}{12}\left(\frac{1}{3}\frac{3F/D-1}{F/D+1} - 1\right) + \frac{1}{9}\Sigma\end{aligned}$$

Spin Crisis

- Measurements from EMC (European Muon Collaboration)

$$\Gamma_1^p = 0.126 \pm 0.010 \pm 0.015$$

- Hyperon semileptonic decay measurements

$$F = 0.47 \pm 0.04, \quad D = 0.81 \pm 0.03$$

- $\Delta u = 0.74 \pm 0.10$, $\Delta d = -0.54 \pm 0.10$, $\Delta s = -0.20 \pm 0.11$
- Nucleon spin from the *quarks*

$$\Delta\Sigma = \Delta u + \Delta d + \Delta s = 0.01 \pm 0.29$$

- **Spin Crisis**: quarks contribute almost *nothing* to the nucleon spin

Corrections

- Problems with previous estimate
  - SU(3) is not exact symmetry
  - At finite Q^2, QCD effect
- Considering QCD effects in $g_1(x)$

$$\Delta\Sigma = 0.2 \pm 0.1$$

- Recent results from HERMES

$$\begin{aligned}
\Delta\Sigma &= 0.347 \pm 0.024 \pm 0.066 \\
a_3 &= 0.880 \pm 0.045 \pm 0.107 \\
a_8 &= 0.262 \pm 0.078 \pm 0.045
\end{aligned}$$

Proton Spin Puzzle

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L$$

- ➜ $\Delta\Sigma = 0.347 \pm 0.024 \pm 0.066$
- ➜ $\Delta G = 0.41 \pm 0.23$
  - SMC, HERMES, COMPASS, RHIC-spin
  - From extended database of $g_1(x, Q^2)$
  - or from $\vec{p}\vec{p}$ collision (RHIC)
- ➜ L?

Bjorken Sum Rule

- Start from

$$\Gamma_1^p = \int_0^1 g_1^p(x)\,dx = \frac{g_A}{12}\left(\frac{1}{3}\frac{3F/D-1}{F/D+1}+1\right)+\frac{1}{9}\Sigma$$

$$\Gamma_1^n = \int_0^1 g_1^n(x)\,dx = \frac{g_A}{12}\left(\frac{1}{3}\frac{3F/D-1}{F/D+1}-1\right)+\frac{1}{9}\Sigma$$

- Taking the difference between p and n

$$\Gamma_1^p - \Gamma_1^n = \frac{g_A}{6}$$

- Experimentally

$$\Gamma_1^p - \Gamma_1^n\,(\text{at 5 GeV}^2) = 0.181 \quad \text{SMC}$$

$$\Gamma_1^p - \Gamma_1^n\,(\text{at 5 GeV}^2) = 0.171 \quad \text{SLAC}$$

- To be compared with 0.181 ± 0.003 from Bjorken sum rule with QCD corrections.
- Note that Bjorken Sum Rule requires only SU(2) symmetry, which is a good symmetry.

Ellis-Jaffe Sum Rule

- Assume exact SU(3) symmetry
- Assume $\Delta s = 0$

$$\Gamma_1^p = \int_0^1 g_1^p(x)\,dx = \frac{g_A}{12}\left(\frac{5}{3}\frac{3F/D-1}{F/D+1}+1\right)$$
$$\Gamma_1^n = \int_0^1 g_1^n(x)\,dx = \frac{g_A}{12}\left(\frac{5}{3}\frac{3F/D-1}{F/D+1}-1\right)$$

- Numerically

$$\Gamma_1^p = 0.199 \pm 0.005 \quad \Gamma_1^n = -0.009 \pm 0.005$$

- Experimentally (SLAC)

$$\Gamma_1^p = 0.114 \quad \Gamma_1^n = -0.051$$