Extension to Spins

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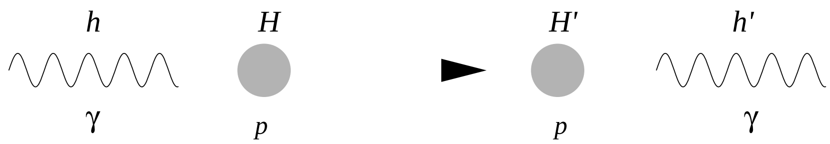
Spin Structure Functions

- Two more structure functions $g_1(x, Q^2)$ and $g_2(x, Q^2)$
- in addition to $F_1(x, Q^2)$ and $F_2(x, Q^2)$
- Why total *four* structure functions?
  - Invariance under reflection (parity)
  - Invariance under time reversal

Virtual Compton Scattering



$$\gamma^*(h_\gamma) + N(h_N) \rightarrow \gamma^*(h'_\gamma) + N(h'_N)$$

$$J_z = h_\gamma - h_N = h'_\gamma - h'_N$$

$$\begin{aligned} h_\gamma &= -1, 0, 1 \\ h_N &= -\frac{1}{2}, \frac{1}{2} \\ h'_\gamma &= -1, 0, 1 \\ h'_N &= -\frac{1}{2}, \frac{1}{2} \end{aligned}$$

36 combinations are possible, but angular momentum conservation gives only 10 possibilities.

Possible cases

	h_γ	h_N	J_z	h'_γ	h'_N	$\mathcal{M}$
A	1	$\frac{1}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$\mathcal{M}_{1,\frac{1}{2},0,-\frac{1}{2}}$
B	1	$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{1}{2}$	$\mathcal{M}_{1,\frac{1}{2},1,\frac{1}{2}}$
C	1	$-\frac{1}{2}$	$\frac{3}{2}$	1	$-\frac{1}{2}$	$\mathcal{M}_{1,-\frac{1}{2},1,-\frac{1}{2}}$
D	0	$\frac{1}{2}$	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	$\mathcal{M}_{0,\frac{1}{2},-1,-\frac{1}{2}}$
E	0	$\frac{1}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\mathcal{M}_{0,\frac{1}{2},0,\frac{1}{2}}$
F	-1	$-\frac{1}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\mathcal{M}_{-1,-\frac{1}{2},0,\frac{1}{2}}$
G	-1	$-\frac{1}{2}$	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	$\mathcal{M}_{-1,-\frac{1}{2},-1,-\frac{1}{2}}$
H	-1	$\frac{1}{2}$	$-\frac{3}{2}$	-1	$\frac{1}{2}$	$\mathcal{M}_{-1,\frac{1}{2},-1,\frac{1}{2}}$
I	0	$-\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{1}{2}$	$\mathcal{M}_{0,-\frac{1}{2},1,\frac{1}{2}}$
J	0	$-\frac{1}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$\mathcal{M}_{0,-\frac{1}{2},0,-\frac{1}{2}}$

Applying Symmetries

Reflection symmetry

$$\mathcal{M}_{\alpha,\beta,\gamma,\delta} = \mathcal{M}_{-\alpha,-\beta,-\gamma,-\delta}$$

$$A = F \quad B = G \quad C = H \quad D = I \quad E = J$$

Only 5 amplitudes are independent

$$\mathcal{M}_{1,\frac{1}{2},0,-\frac{1}{2}}, \mathcal{M}_{1,\frac{1}{2},1,\frac{1}{2}}, \mathcal{M}_{1,-\frac{1}{2},1,-\frac{1}{2}}, \mathcal{M}_{0,-\frac{1}{2},1,\frac{1}{2}}, \mathcal{M}_{0,\frac{1}{2},0,\frac{1}{2}}$$

Time reversal symmetry (exchange of initial and final state)

$$\mathcal{M}_{0,-\frac{1}{2},1,\frac{1}{2}} = \mathcal{M}_{1,\frac{1}{2},0,-\frac{1}{2}}$$

leaving only 4 independent amplitudes

$$\mathcal{M}_{1,\frac{1}{2},0,-\frac{1}{2}}, \mathcal{M}_{1,\frac{1}{2},1,\frac{1}{2}}, \mathcal{M}_{1,-\frac{1}{2},1,-\frac{1}{2}}, \mathcal{M}_{0,\frac{1}{2},0,\frac{1}{2}}$$

How to determine $g_1(x, Q^2)$ and $g_2(x, Q^2)$?

$$\frac{d^2\sigma}{d\Omega dE'} = \frac{4\alpha^2 E'^2 \cos^2\frac{\theta}{2}}{Q^4}\left[\frac{F_2}{\nu} + 2\frac{F_1}{M}\tan^2\frac{\theta}{2}\right]$$

$$\frac{d^2\sigma}{dE' d\Omega}(\downarrow\Uparrow - \uparrow\Uparrow) = \frac{4\alpha^2}{MQ^2}\frac{E'}{\nu E}\left[(E + E'\cos\theta)g_1 - \frac{Q^2}{\nu}g_2\right]$$

$$\frac{d^2\sigma}{dE' d\Omega}(\downarrow\Rightarrow - \uparrow\Rightarrow) = \frac{4\alpha^2 \sin\theta}{MQ^2}\frac{E'^2}{E}\frac{1}{\nu^2}(\nu g_1 + 2E g_2)$$

- First arrow ($\downarrow$,$\uparrow$) is for the electron
- Second arrow ($\Uparrow$,$\Rightarrow$) is for the nucleon

Structure Functions in Parton Model

$$\left.\begin{aligned} F_1(x) &= \tfrac{1}{2}\textstyle\sum_i e_i^2[q_i(x)+\bar{q}_i(x)] \\ F_2(x) &= \textstyle\sum_i e_i^2 x[q_i(x)+\bar{q}_i(x)] \end{aligned}\right\} F_2(x) = 2xF_1(x)(\text{Callan-Gross})$$

$$\begin{aligned} g_1(x) &= \frac{1}{2}\sum_i e_i^2 \Delta q_i(x) \\ \Delta q_i(x) &= q_i^+(x) + \overline{q}_i^+(x) - q_i^-(x) - \overline{q}_i^-(x) \\ g_2(x) &= 0 \end{aligned}$$

Sum Rules - Introduction

Consider a complex function

$$f(z) = u(z) + iv(z)$$

Dispersion relation gives

$$u(x_0) = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{v(x)}{x - x_0}\, dx$$

$$v(x_0) = -\frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{u(x)}{x - x_0}\, dx$$

Application to optics: Kronig-Kramers optical dispersion relations

$$\Re[n^2(\omega_0) - 1] = \frac{2}{\pi} P \int_{0}^{\infty} \frac{\omega \Im[n^2(\omega) - 1]}{\omega^2 - \omega_0^2}\, d\omega$$

$$\Im[n^2(\omega_0) - 1] = -\frac{2}{\pi} P \int_{0}^{\infty} \frac{\omega_0 \Re[n^2(\omega) - 1]}{\omega^2 - \omega_0^2}\, d\omega$$

QCD Sum Rules

- Currently, no way to calculate structure functions from QCD
- Easier to calculate various integrals of the structure functions - QCD sum rules
- For example
  - Gottfried sum rule: unpolarized structure functions
  - Gerasimov-Drell-Hearn sum rule: polarized structure functions
  - Bjorken sum rule, Ellis-Jaffe sum rule *etc*