Probing with Neutrinos

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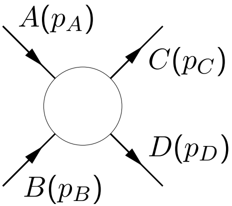
Cross Section

- For the process of $AB \to CD$ in the center-of-mass frame

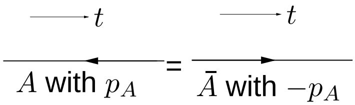
$$\left.\frac{d\sigma}{d\Omega}\right|_{\mathrm{cm}} = \frac{1}{64\pi^2 s}\frac{p_f}{p_i}\left|\mathcal{M}\right|^2$$

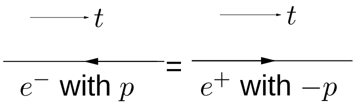
- $\mathcal{M}$ is the *invariant amplitude* calculated from the corresponding Feynman diagram



Particle/*Anti*-particle

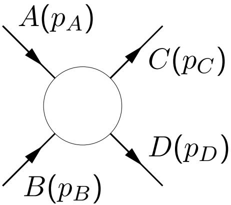
- A particle moving in the *opposite* direction in time $=$ *anti*-particle





Crossing Symmetry

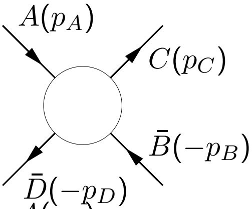
From the invariant amplitude $\mathcal{M}(p_A, p_B, p_C, p_D)$ for the following process



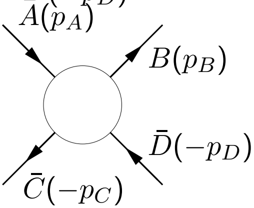
we can easily get $\mathcal{M}$ for other processes such as

$$A\bar{D} \quad \to \quad \bar{B}C \quad \text{exchange of B and D}$$
$$A\bar{C} \quad \to \quad \bar{B}D \quad \text{exchange of B and C}$$

In Detail

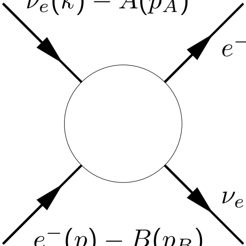


$$\mathcal{M}(p_A, -p_D, p_C, -p_B)$$



$$\mathcal{M}(p_A, -p_C, p_B, -p_D)$$

- $\nu_e + e^- \rightarrow \nu_e + e^-$

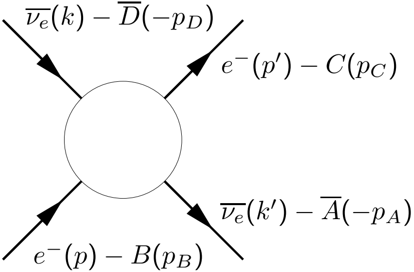


- $|\mathcal{M}|^2 = 16G^2 s^2$ with $s = (p+k)^2$
- Cross section is given by

$$\frac{d\sigma}{d\Omega}(\nu_e e^-) = \frac{1}{64\pi^2 s}\overline{|\mathcal{M}|^2} = \frac{G^2 s}{4\pi^2}$$

Crossing Both Neutrinos

- $\nu_e + e^- \to \nu_e + e^-$



- $|\mathcal{M}|^2 = 16G^2t^2$ with $t = (p - k')^2$
- $|\mathcal{M}|^2 = 4G^2s^2(1 + \cos\theta)^2$
- Cross section

$$\frac{d\sigma}{d\Omega}(\nu_e e^-) = \frac{1}{64\pi^2 s}\overline{|\mathcal{M}|^2} = \frac{G^2 s}{16\pi^2}(1 + \cos\theta)^2$$

- Changing target to quarks

$$\frac{d\sigma}{d\Omega}(\nu_e d \to e^- u) = \frac{G^2 s}{4\pi^2}$$
$$\frac{d\sigma}{d\Omega}(\overline{\nu_e} u \to e^+ d) = \frac{G^2 s}{16\pi^2}(1+\cos\theta)^2$$

- Using Chage Conjugation and Parity

$$\frac{d\sigma}{d\Omega}(\overline{\nu_e}\overline{d} \to e^+ \overline{u}) = \frac{G^2 s}{4\pi^2}$$
$$\frac{d\sigma}{d\Omega}(\nu_e \overline{u} \to e^- \overline{d}) = \frac{G^2 s}{16\pi^2}(1+\cos\theta)^2$$

- Using dimensionless variable y (also Lorentz invariant)

$$1 - y \equiv \frac{p \cdot k'}{p \cdot k} \simeq \frac{1}{2}(1 + \cos\theta)$$

$$\begin{aligned}
\frac{d\sigma}{dy}(\nu_e d \to e^- u) &= \frac{d\sigma}{dy}(\overline{\nu_e}\overline{d} \to e^+ \overline{u}) = \frac{G^2 x s}{\pi} \\
\frac{d\sigma}{dy}(\overline{\nu_e} u \to e^+ d) &= \frac{d\sigma}{dy}(\nu_e \overline{u} \to e^- \overline{d}) = \frac{G^2 x s}{\pi}(1 - y)^2
\end{aligned}$$

- Using isoscalar target (equal number of p and n)

$$\begin{aligned} d^p(x) + d^n(x) &= d(x) + u(x) \equiv Q(x) \\ \bar{u}^p(x) + \bar{u}^n(x) &= \bar{u}(x) + \bar{d}(x) \equiv \overline{Q}(x) \end{aligned}$$

$$\begin{aligned} \frac{d\sigma}{dxdy}(\nu_\mu N \to \mu^- X) &= \frac{G^2 xs}{2\pi}(Q(x) + (1-y)^2 \overline{Q}(x)) \\ \frac{d\sigma}{dxdy}(\bar{\nu}_\mu N \to \mu^+ X) &= \frac{G^2 xs}{2\pi}(\overline{Q}(x) + (1-y)^2 Q(x)) \end{aligned}$$

- For the same isoscalar target

$$\frac{d\sigma}{dxdy}(eN \to eX) = \frac{2\pi\alpha^2}{Q^4} xs \left[1 + (1-y)^2\right] \frac{5}{18} \left[Q(x) + \bar{Q}(x)\right]$$

- If $\overline{Q}(x) = 0$

$$\frac{d\sigma}{dy}(\nu) \sim c, \quad \frac{d\sigma}{dy}(\overline{\nu}) \sim c(1-y)^2$$

- After integration,

$$\frac{\sigma(\bar{\nu})}{\sigma(\nu)} = \frac{1}{3}$$