Structure Functions - Miscellany

prepared by

Seonho Choi

Seoul National University

Topics in High Energy Physics

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Drell-Yan Process

- Production of lepton pairs from pp collision

$$p + p \rightarrow l^+ l^- X$$

- Annihilation of q and $\bar{q}$ inside the proton
- Again, using *factorization*
  - core process: $q\bar{q} \rightarrow \mu^- \mu^+$
  - q, $\bar{q}$ from the proton: $f_q(x)$ and $f_{\bar{q}}(y)$

Drell-Yan (Cont.)

Core process

$$\begin{aligned} \hat{\sigma}(q\bar{q} \to \mu^- \mu^+) &= \frac{4\pi\alpha^2}{3\hat{s}} e_q^2 \\ \hat{s} &= (p_q + p_{\bar{q}})^2 = Q^2 \end{aligned}$$

In differential form,

$$\frac{d\hat{\sigma}}{dQ^2} = \frac{4\pi\alpha^2}{3Q^2} e_q^2 \delta(Q^2 - \hat{s})$$

Using structure functions,

$$\begin{aligned} &\frac{d\sigma}{dQ^2}(pp \to \mu^- \mu^+ X) \\ &= \left(\frac{1}{3}\right)\left(\frac{1}{3}\right) 3 \sum_q \int dx \int dy\, f_q(x) f_{\bar{q}}(y) \frac{d\hat{\sigma}}{dQ^2} \end{aligned}$$

Using

$$\hat{s} = (xp_1 + yp_2)^2 \simeq xys,$$

$$\frac{d\sigma}{dQ^2}(pp \to \mu^- \mu^+ X)$$

$$= \frac{4\pi\alpha^2}{9Q^4} \sum_q e_q^2 \int dx \int dy\, f_q(x) f_{\bar{q}}(y) \delta\left(1 - xy\frac{s}{Q^2}\right)$$

Scaling

$$Q^4 \frac{d\sigma}{dQ^2} = \mathcal{F}\left(\frac{s}{Q^2}\right)$$

Weak Interactions - Basics

- Decays of π or μ: long lifetime $\rightarrow$ weak interaction

$$\pi^- \rightarrow \mu^- \bar{\nu}_\mu \qquad \tau = 2.6 \times 10^{-8}\,\text{sec}$$

$$\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu \qquad \tau = 2.2 \times 10^{-6}\,\text{sec}$$

- Mediated by gauge bosons Z^0, $W^\pm$
- Parity violation - *chirality*

$$^{60}\text{Co} \rightarrow ^{60}\text{Ni}^* + e^- + \bar{\nu}_e$$

- Only *right*-handed $\bar{\nu}$ or *left*-handed ν observed in nature
- CP invariance

$$\Gamma(\pi^+ \rightarrow \mu^+ \nu_L) \neq \Gamma(\pi^+ \rightarrow \mu^+ \nu_R) = 0 \quad \text{P violation}$$

$$\Gamma(\pi^+ \rightarrow \mu^+ \nu_L) \neq \Gamma(\pi^- \rightarrow \mu^- \bar{\nu}_L) = 0 \quad \text{C violation}$$

$$\Gamma(\pi^+ \rightarrow \mu^+ \nu_L) \neq \Gamma(\pi^- \rightarrow \mu^- \bar{\nu}_R) = 0 \quad \text{CP invariance}$$

Charge Current $\nu - q$ scattering

- $W^{\pm}$ connects $\nu_\mu - \mu^-$ or $d - u$ etc
- Only *left*-handed particles or *right*-handed *anti*-particles
- Basic core processes

$$\frac{d\sigma}{d\Omega}(\nu_\mu d(\bar{u}) \to \mu^- u(\bar{d})) = \frac{G^2 s}{4\pi^2}$$

$$\frac{d\sigma}{d\Omega}(\bar{\nu}_\mu u(\bar{d}) \to \mu^+ d(\bar{u})) = \frac{G^2 s}{16\pi^2}(1 + \cos\theta)^2$$

- For $\nu N \to \mu N$, again *factorization*

$$\frac{d\sigma}{dxdy}(\nu N \to \mu X) = \sum_i f_i(x) \left(\frac{d\sigma_i}{dy}\right)_{\hat{s}=xs}$$

$\nu - N$ scattering

$$1 - y = \frac{p \cdot k'}{p \cdot k} \simeq \frac{1}{2}(1 + \cos\theta)$$

$$\frac{d\hat{\sigma}}{dy}(\nu_\mu d \to \mu^- u) = \frac{G^2 x s}{\pi}$$

$$\frac{d\hat{\sigma}}{dy}(\bar{\nu}_\mu d \to \mu + -d) = \frac{G^2 x s}{\pi}(1 - y)^2$$

Using isoscalar target (equal number of p and n)

$$d^p(x) + d^n(x) = d(x) + u(x) \equiv Q(x)$$

$$\bar{u}^p(x) + \bar{u}^n(x) = \bar{u}(x) + \bar{d}(x) \equiv \bar{Q}(x)$$

$\nu - N$ scattering

$$\frac{d\sigma}{dxdy}(\nu_\mu N \to \mu^- X) = \frac{G^2 xs}{2\pi}(Q(x) + (1-y)^2 \bar{Q}(x))$$
$$\frac{d\sigma}{dxdy}(\bar{\nu}_\mu N \to \mu^+ X) = \frac{G^2 xs}{2\pi}(\bar{Q}(x) + (1-y)^2 Q(x))$$

Compare with

$$\frac{d\sigma}{dxdy}(eN \to eX) = \frac{2\pi\alpha^2}{Q^4} xs \left[1 + (1-y)^2\right] \frac{5}{18} \left[Q(x) + \bar{Q}(x)\right]$$