QCD and Gluons

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Topics in High Energy Physics

October 5, 2004

Bjorken Scaling

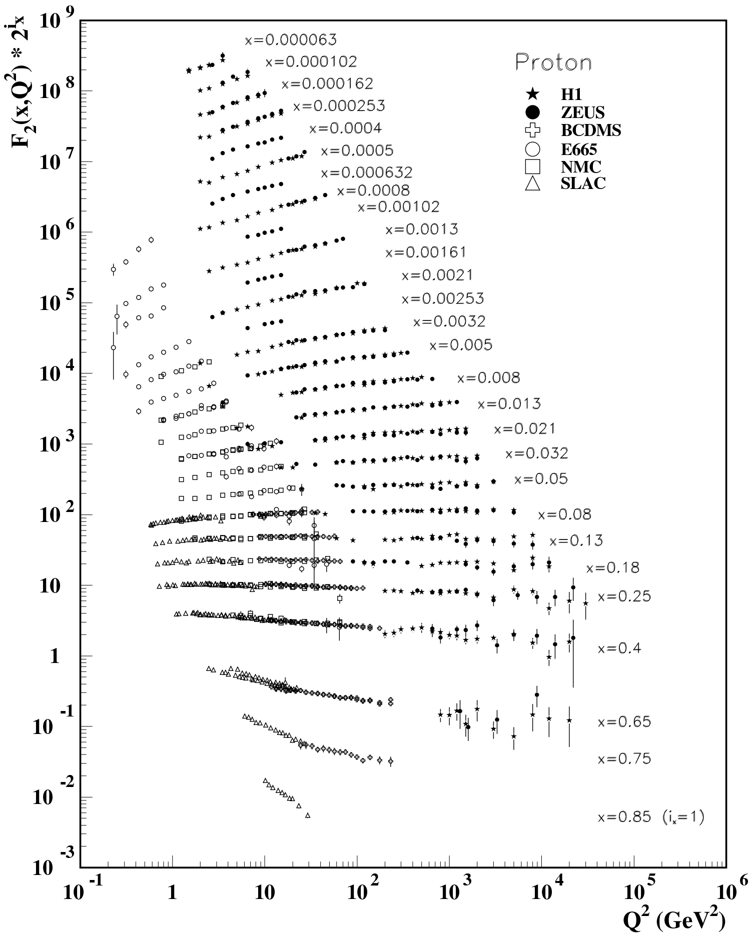
At large Q^2 and ν (or W),

- ➜ interaction with *point* constituents inside the nucleon
- ➜ scaling behavior with new variable x

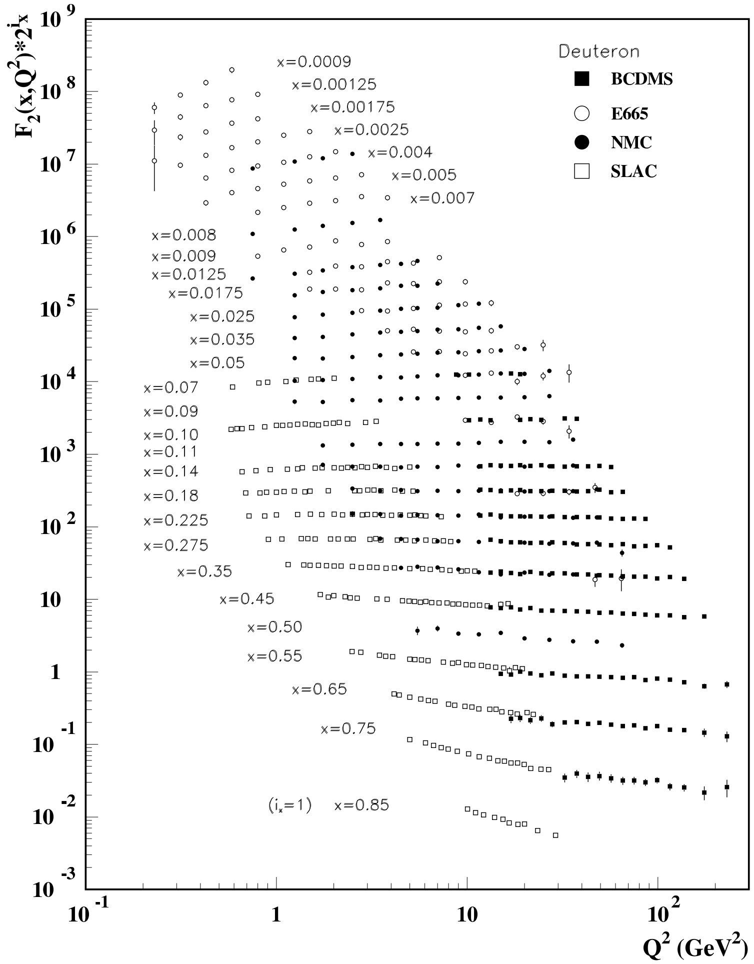
$$\begin{aligned} MW_1(\nu, Q^2) &\rightarrow F_1(x) \\ \nu W_2(\nu, Q^2) &\rightarrow F_2(x) \end{aligned}$$

- ➜ $x = \dfrac{Q^2}{2M\nu}$
- ➜ x is the fraction of the nucleon momentum carried by this *point* constituents

Measured F_2



Proton
H1
ZEUS
BCDMS
E665
NMC
SLAC
F2(x,Q2) * 2^ix
Q2 (GeV2)
x=0.000063
x=0.000102
x=0.000162
x=0.000253
x=0.0004
x=0.0005
x=0.000632
x=0.0008
x=0.00102
x=0.0013
x=0.00161
x=0.0021
x=0.00253
x=0.0032
x=0.005
x=0.008
x=0.013
x=0.021
x=0.032
x=0.05
x=0.08
x=0.13
x=0.18
x=0.25
x=0.4
x=0.65
x=0.75
x=0.85 (ix=1)



Deuteron
BCDMS
E665
NMC
SLAC
F2(x,Q2)*2^ix
Q2 (GeV2)
x=0.0009
x=0.00125
x=0.00175
x=0.0025
x=0.004
x=0.005
x=0.007
x=0.008
x=0.009
x=0.0125
x=0.0175
x=0.025
x=0.035
x=0.05
x=0.07
x=0.09
x=0.10
x=0.11
x=0.14
x=0.18
x=0.225
x=0.275
x=0.35
x=0.45
x=0.50
x=0.55
x=0.65
x=0.75
(ix=1) x=0.85

Parton Model Interpretation

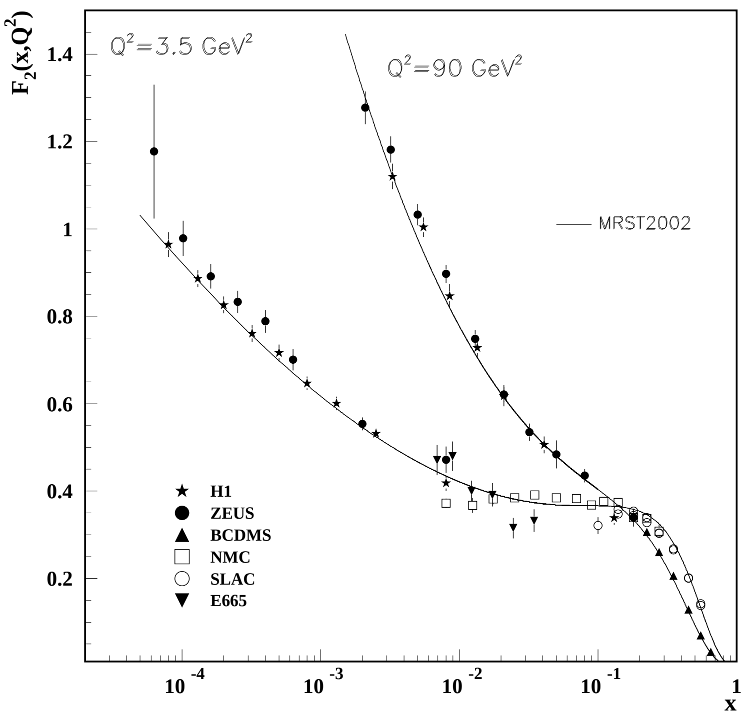
$$F_2(x) = \sum_i e_i^2 \, x \, f_i(x)$$

$$F_1(x) = \frac{1}{2} \sum_i e_i^2 \, f_i(x)$$

where $f_i(x)$ is the probability to find a parton i with momentum fraction x inside the nucleon Callan-Gross relation

- ➜ $2xF_1(x) = F_2(x)$
- ➜ Consequence of spin 1/2 nature of the *partons*

Scaling Violation



- ➜ $F_2(x)$ (or $2xF_1(x)$) is not *completely* constant over Q^2
- ➜ Due to the finite value of Q^2: finite spatial resolution
- ➜ Slow $\log Q^2$ dependence
- ➜ Originally considered *experimental* problem
- ➜ Turned out to be one of the success of QCD
- ➜ Another evidence of the role of the *gluons*

QCD - Basics

E&M	QCD
electric charge	*color* charge
$\pm$ charge	*three* colors: R, G, B
neutral photon	8 *bicolored* gluons
Similar Feynman diagram rules	
pbotons do not interact	gluons *do* interact each other
α_{em} increases	α_{S} decreases at short distance
linear	*non*-linear

Effect of Gluon Emission

- ➜ Gluon emission at $\gamma^* q$ vertex
- ➜ Scaling violation
- ➜ Jets with large p_T

Using σ_T and σ_L,

$$\begin{aligned} 2F_1 &= \frac{\sigma_T}{\sigma_0} \\ \frac{F_2}{x} &= \frac{\sigma_T + \sigma_L}{\sigma_0} \end{aligned}$$

DIS cross section with gluon emission

- $\gamma*$ interacts with parton of momentum fraction y
- emits a gluon of momentum fraction $y(1-z)$

$$\left(\frac{\sigma_T(x,Q^2)}{\sigma_0}\right)_{\gamma^* p} = \sum_i \int dz \int dy f_i(y) \delta(x - zy) \left(\frac{\hat{\sigma}_T(z,Q^2)}{\hat{\sigma}_0}\right)_{\gamma^* i}$$

After integration of δ function

$$\left(\frac{\sigma_T(x,Q^2)}{\sigma_0}\right)_{\gamma^* p} = \sum_i \int_x^1 \frac{dy}{y} f_i(y) \left(\frac{\hat{\sigma}_T(x/y,Q^2)}{\hat{\sigma}_0}\right)_{\gamma^* i}$$

Using cross section ratio for $\gamma^* q \to q$

$$\frac{\hat{\sigma}_T + \hat{\sigma}_L}{\hat{\sigma}_0} = e_i^2 \delta(1-z)$$

$$\frac{F_2(x,Q^2)}{x} = \sum_i e_i^2 \int_x^1 \frac{dy}{y} f_i(y) \delta\left(1 - \frac{x}{y}\right) = \sum_i e_i^2 f_i(x)$$

Gluon Emission Cross Section

Differential cross section for $\gamma^* q \to qg$

$$\frac{d\hat{\sigma}}{dp_T^2} \simeq e_i^2 \hat{\sigma}_0 \frac{1}{p_T^2} \frac{\alpha_S}{2\pi} P_{qq}(z)$$

with

$$P_{qq}(z) = \frac{4}{3} \left(\frac{1 + z^2}{1 - z} \right)$$

is the probability of a quark emitting a gluon and becoming a quark with reduced momentum z.

Total cross section for $\gamma^* q \to qg$

$$\begin{aligned} \hat{\sigma}(\gamma^* q \to qg) &= \int_{\mu^2}^{\hat{s}/4} dp_T^2 \frac{d\hat{\sigma}}{dp_T^2} \\ &\simeq e_i^2 \hat{\sigma}_0 \int_{\mu^2}^{\hat{s}/4} \frac{dp_T^2}{p_T^2} \frac{\alpha_S}{2\pi} P_{qq}(z) \\ &\simeq e_i^2 \hat{\sigma}_0 \left(\frac{\alpha_S}{2\pi} P_{qq}(z) \log \frac{Q^2}{\mu^2} \right) \end{aligned}$$

Finally, new F_2

$$\frac{F_2(x,Q^2)}{x} = \sum_q e_q^2 \int_x^1 \frac{dy}{y} q(y) \left(\delta(1-\frac{x}{y}) + \frac{\alpha_S}{2\pi} P_{qq}\left(\frac{x}{y}\right) \log\frac{Q^2}{\mu^2} \right)$$

Re-arranging

$$\begin{aligned} \frac{F_2(x,Q^2)}{x} &\equiv \sum_q e_q^2 \int_x^1 \frac{dy}{y} \left(q(y) + \Delta q(y,Q^2))\right) \delta\left(1-\frac{x}{y}\right) \\ &= \sum_q e_q^2 \left(q(x) + \Delta q(x,Q^2)\right) \end{aligned}$$

where

$$\Delta q(x,Q^2) \equiv \frac{\alpha_S}{2\pi} \log\left(\frac{Q^2}{\mu^2}\right) \int_x^1 \frac{dy}{y} q(y) P_{qq}\left(\frac{x}{y}\right)$$

Or

$$\frac{d}{d\log Q^2} q(x,Q^2) = \frac{\alpha_S}{2\pi} \int_x^1 \frac{dy}{y} q(y,Q^2) P_{qq}\left(\frac{x}{y}\right)$$

Production of $q\bar{q}$

Including $\gamma^* g \to q\bar{q}$ cross section

$$\left|\frac{F_2(x,Q^2)}{x}\right|_{\gamma^* g\to q\bar{q}} = \sum_q e_q^2 \int_x^1 \frac{dy}{y} g(y) \frac{\alpha_S}{2\pi} P_{qg}\left(\frac{x}{y}\right) \log\frac{Q^2}{\mu^2}$$

with

$$P_{qg}(z) = \frac{1}{2}\left(z^2 + (1-z)^2\right)$$

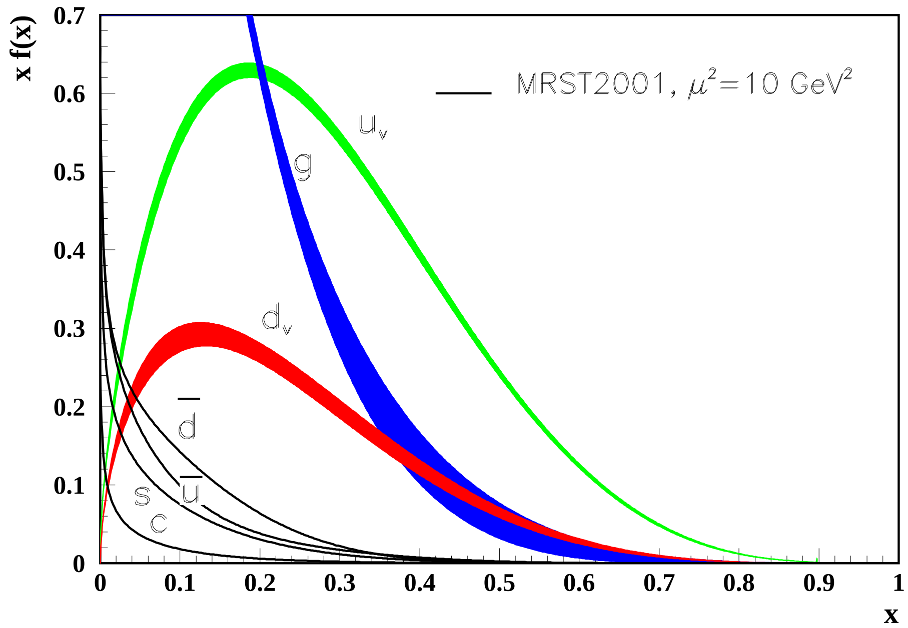
is the probability that a gluon produces $q\bar{q}$ pair with quark momentum z

Complete Evolution Equations

$$\frac{dq_i(x,Q^2)}{d\log Q^2} = \frac{\alpha_S}{2\pi}\int_x^1 \frac{dy}{y}\left(q_i(y,Q^2)P_{qq}\left(\frac{x}{y}\right)\right) + g(y,Q^2)P_{qg}\left(\frac{x}{y}\right)$$

$$\frac{dg(x,Q^2)}{d\log Q^2} = \frac{\alpha_S}{2\pi}\int_x^1 \frac{dy}{y}\left(q_i(y,Q^2)P_{gq}\left(\frac{x}{y}\right)\right) + g(y,Q^2)P_{gg}\left(\frac{x}{y}\right)$$

Parton Distribution Functions



- Determined from various experiments
- Fitting with evolution equation