Structure Functions

prepared by

Seonho Choi

Seoul National University

Topics in High Energy Physics

Sep 30, 2004

Summary of Formulae

$$\left.\frac{d\sigma}{dE' d\Omega}\right|_{e\mu\to e\mu} = \frac{\alpha^2}{4E^2 \sin^4 \frac{\theta}{2}} \times \left(\cos^2\frac{\theta}{2} + \frac{Q^2}{2m^2}\sin^2\frac{\theta}{2}\right)\delta\left(\nu - \frac{Q^2}{2m}\right)$$

$$\left.\frac{d\sigma}{dE' d\Omega}\right|_{ep\to ep} = \frac{\alpha^2}{4E^2 \sin^4 \frac{\theta}{2}} \times \left(\frac{G_E^2 + \tau G_M^2}{1+\tau}\cos^2\frac{\theta}{2} + \frac{Q^2}{2M^2}G_M^2\sin^2\frac{\theta}{2}\right)\delta\left(\nu - \frac{Q^2}{2M}\right)$$

$$\left.\frac{d\sigma}{dE' d\Omega}\right|_{ep\to eX} = \frac{\alpha^2}{4E^2 \sin^4 \frac{\theta}{2}} \times \left\{W_2(\nu, Q^2)\cos^2\frac{\theta}{2} + 2W_1(\nu, Q^2)\sin^2\frac{\theta}{2}\right\}$$

Bjorken Scaling

$$
\begin{aligned}
2W_1^{\text{point}} &= \frac{Q^2}{2m^2}\,\delta\left(\nu - \frac{Q^2}{2m}\right) \\
W_2^{\text{point}} &= \delta\left(\nu - \frac{Q^2}{2m}\right)
\end{aligned}
$$

Rearranged as

$$
\begin{aligned}
2MW_1^{\text{point}}(\nu, Q^2) &= \frac{Q^2}{2m\nu}\,\delta\left(1 - \frac{Q^2}{2m\nu}\right) \\
\nu W_2^{\text{point}}(\nu, Q^2) &= \delta\left(1 - \frac{Q^2}{2m\nu}\right)
\end{aligned}
$$

These point functions are only functions of $Q^2/2M\nu$, *not* of Q^2 and ν separately. — *Scaling*

Case of the Proton

$$2W_1^{\text{elastic}} = \frac{Q^2}{2M^2} G_M^2(Q^2)\, \delta\left(\nu - \frac{Q^2}{2M}\right)$$

$$W_2^{\text{elastic}} = \frac{G_E^2(Q^2) + \tau G_M^2(Q^2)}{1+\tau}\, \delta\left(\nu - \frac{Q^2}{2M}\right)$$

- W_1 and W_2 contain form factors $G_E(Q^2)$ and $G_M(Q^2)$
- Impossible to rearrange to be functions of single dimensionless variable
- A *mass* scale is explicitely present (in $G_E(Q^2)$ and $G_M(Q^2)$)
- Its value 0.71 GeV is set by experiment – size of the proton

Scaling Behavior

At large Q^2, virtual photon interacts with *point* constituents inside the proton, and

$$\begin{aligned} MW_1(\nu, Q^2) &\rightarrow F_1(x) \\ \nu W_2(\nu, Q^2) &\rightarrow F_2(x) \end{aligned}$$

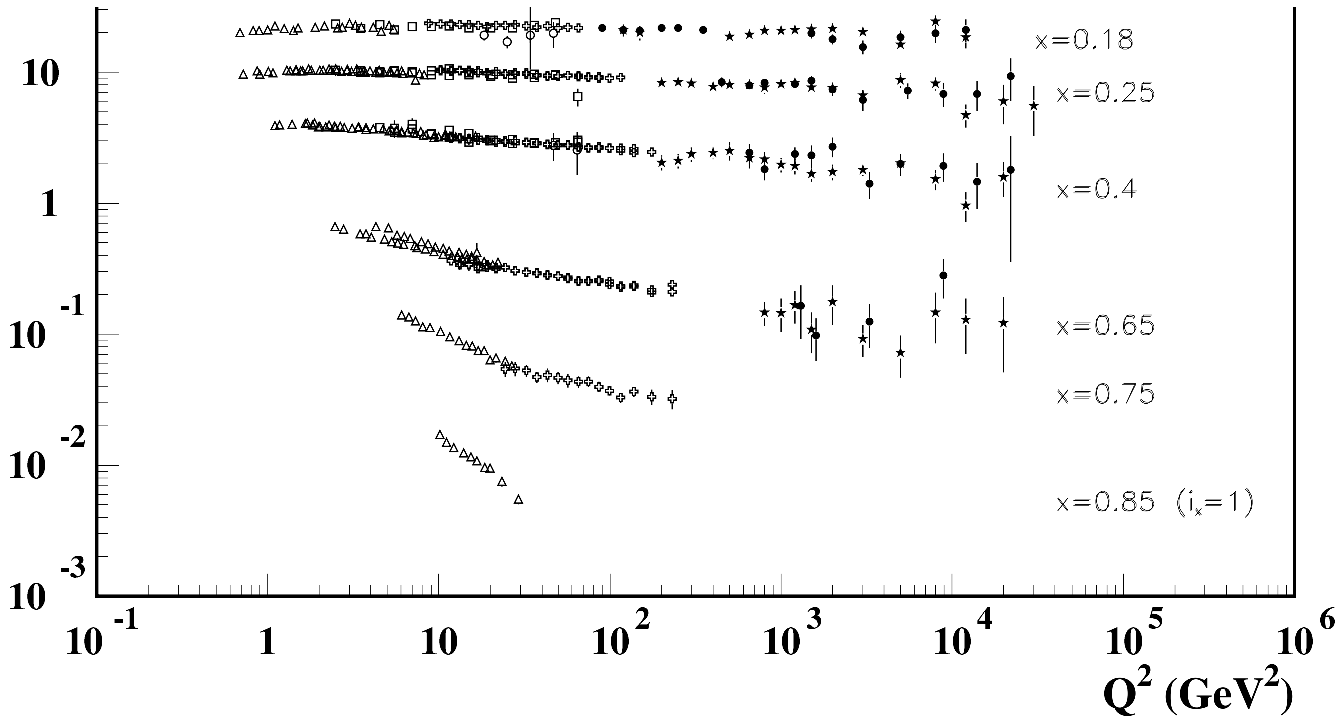
where

$$x = \frac{Q^2}{2M\nu}$$

In reverse, the observation of the *scaling* (independence of MW_1 and νW_2 as a function of Q^2) is an evidence of *point* charge inside the nucleons.

Scaling from Experiment

Measurement of $F_2(x, Q^2)$



For example, F_2 at $x = 0.25$ is almost constant over wide range of Q^2

Parton Model

- Electron (*or* virtual photon) interacts with a *parton* inside the nucleon
- The struck parton carries a fraction of the nucleon's momentum (xp)
- The momentum distribution of the parton is given by $f_i(x)$

$$\sum_i \int dx\, x\, f_i(x) = 1 \quad i \text{ runs over each parton}$$

- Electron scatters off the parton *elastically*

	Proton	**Parton**
Energy	E	xE
Momentum	$p_L = p$	$xp_L = xp$
	$p_T = 0$	$xp_T = 0$
Mass	M	$m = \sqrt{x^2 E^2 - x^2 p_L^2} = xM$

Parton Structure Functions

$$2mW_1 = \frac{Q^2}{2m\nu}\,\delta\left(1 - \frac{Q^2}{2m\nu}\right)$$

$$\nu W_2 = \delta\left(1 - \frac{Q^2}{2m\nu}\right)$$

$$2xMW_1^{\text{parton}} = \frac{Q^2}{2xM\nu}\,\delta\left(1 - \frac{Q^2}{2xM\nu}\right)$$

$$\nu W_2^{\text{parton}} = \delta\left(1 - \frac{Q^2}{2xM\nu}\right)$$

From $F_1 = MW_1$ and $F_2 = \nu W_2$

$$F_1^{\text{parton}} = MW_1^{\text{parton}} = \frac{Q^2}{2M\nu}\frac{1}{2x^2}\,\delta\left(1 - \frac{Q^2}{2xM\nu}\right)$$

$$F_2^{\text{parton}} = \nu W_2^{\text{parton}} = \delta\left(1 - \frac{Q^2}{2xM\nu}\right)$$

Proton Structure Functions

$$
\begin{aligned}
F_{1,2}^{\mathrm{proton}} &= \sum \int dx\, e_i^2 f_i(x) F_{1,2}^{\mathrm{parton}} \\
F_1^{\mathrm{proton}} &= \sum \int dx\, e_i^2 f_i(x) \frac{Q^2}{2M\nu} \frac{1}{2x^2}\, \delta \left(1 - \frac{Q^2}{2xM\nu}\right) \\
&= \sum \int dx\, e_i^2 f_i(x) \frac{Q^2}{2M\nu} \frac{1}{2x}\, \delta \left(x - \frac{Q^2}{2M\nu}\right) \\
&= \frac{1}{2} \sum e_i^2 f_i \left(\frac{Q^2}{2M\nu}\right) \\
F_2^{\mathrm{proton}} &= \sum \int dx\, e_i^2 f_i(x) \delta \left(1 - \frac{Q^2}{2xM\nu}\right) \\
&= \sum \int dx\, e_i^2 f_i(x) x \delta \left(x - \frac{Q^2}{2M\nu}\right) \\
&= \sum e_i^2 f_i \left(\frac{Q^2}{2M\nu}\right) \frac{Q^2}{2M\nu}
\end{aligned}
$$

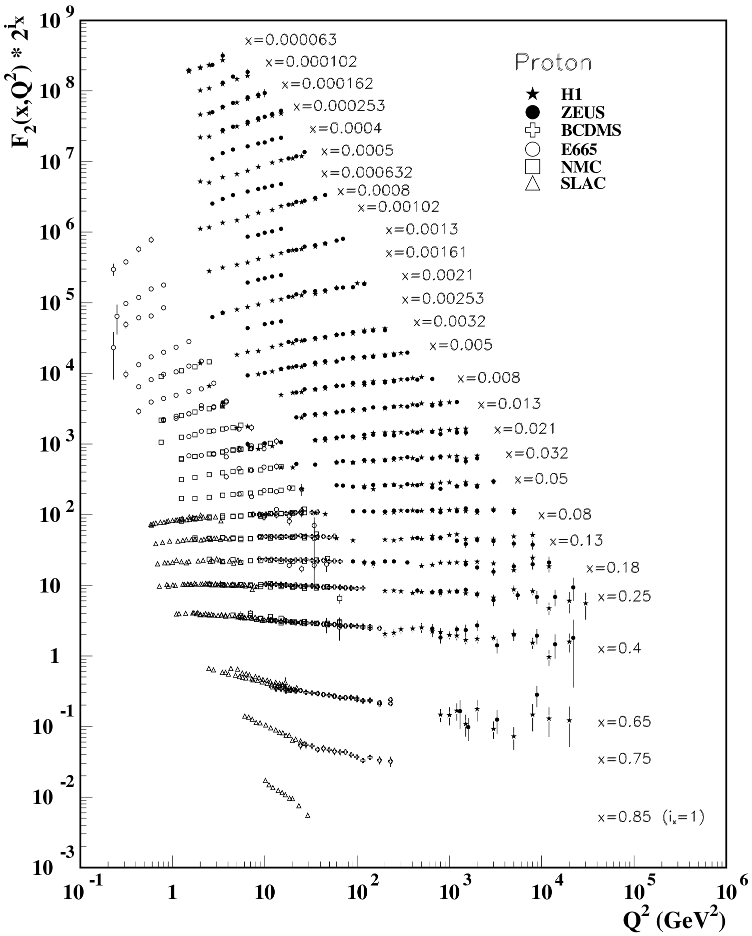
Clean up

Using $x = \dfrac{Q^2}{2M\nu}$

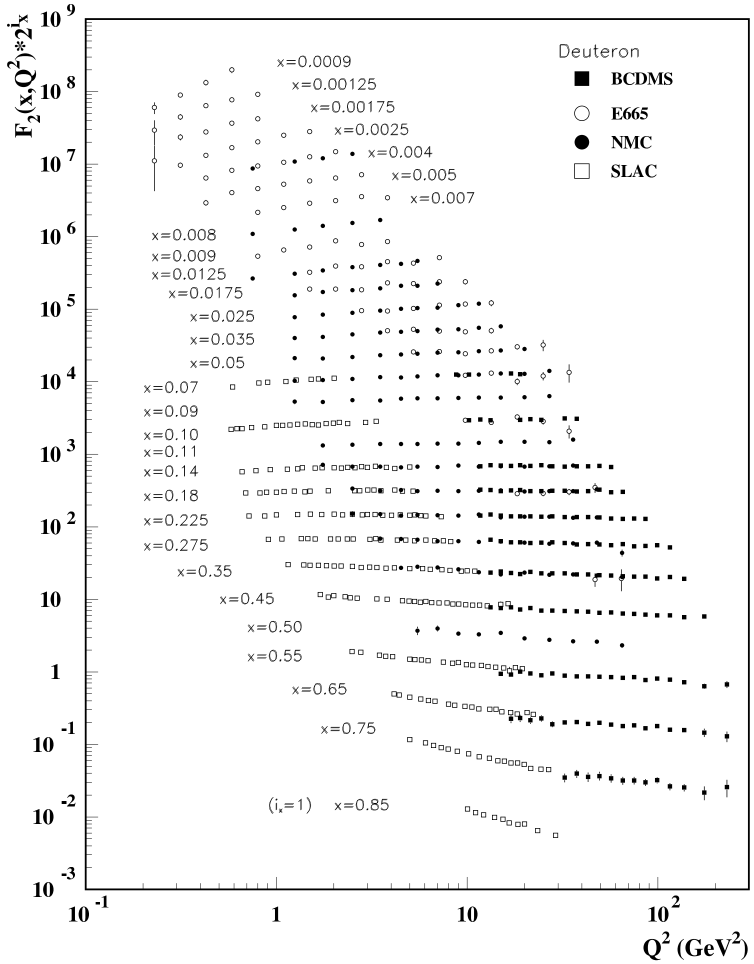
$$
\begin{aligned}
F_2(x) &= \sum_i e_i^2 \, x \, f_i(x) \\
F_1(x) &= \frac{1}{2} \sum_i e_i^2 \, f_i(x) \\
&= \frac{1}{2x} F_2(x) \\
2xF_1(x) &= F_2(x) \quad \text{Callan-Gross relation}
\end{aligned}
$$

Note that Callan-Gross relation is a consequence of partons having spin 1/2

Measured F_2



Proton
H1
ZEUS
BCDMS
E665
NMC
SLAC
F2(x,Q2) * 2^ix
Q2 (GeV2)
x=0.000063
x=0.000102
x=0.000162
x=0.000253
x=0.0004
x=0.0005
x=0.000632
x=0.0008
x=0.00102
x=0.0013
x=0.00161
x=0.0021
x=0.00253
x=0.0032
x=0.005
x=0.008
x=0.013
x=0.021
x=0.032
x=0.05
x=0.08
x=0.13
x=0.18
x=0.25
x=0.4
x=0.65
x=0.75
x=0.85 (ix=1)



Deuteron
BCDMS
E665
NMC
SLAC
F2(x,Q2)*2^ix
Q2 (GeV2)
x=0.0009
x=0.00125
x=0.00175
x=0.0025
x=0.004
x=0.005
x=0.007
x=0.008
x=0.009
x=0.0125
x=0.0175
x=0.025
x=0.035
x=0.05
x=0.07
x=0.09
x=0.10
x=0.11
x=0.14
x=0.18
x=0.225
x=0.275
x=0.35
x=0.45
x=0.50
x=0.55
x=0.65
x=0.75
(ix=1) x=0.85

Scattering off the parton

One more check: elastic scattering off the parton

$$\gamma^*(q) + \text{parton}(xp) \to \text{parton}(xp)$$

Elastic condition

$$(q + xp)^2 = (xp)^2$$

From which, we get $2p \cdot q + q^2 = 0$ or

$$x = \frac{-q^2}{2p \cdot q} = \frac{Q^2}{2M\nu}$$

Quarks inside the Nucleons

$$\frac{1}{x}F_2^p(x) = \left(\frac{2}{3}\right)^2 [u^p(x) + \bar{u}^p(x)] + \left(\frac{1}{3}\right)^2 [d^p(x) + \bar{d}^p(x)]$$
$$+ \left(\frac{1}{3}\right)^2 [s^p(x) + \bar{s}^p(x)] + (\cdots)$$

$$\frac{1}{x}F_2^n(x) = \left(\frac{2}{3}\right)^2 [u^n(x) + \bar{u}^n(x)] + \left(\frac{1}{3}\right)^2 [d^n(x) + \bar{d}^n(x)]$$
$$+ \left(\frac{1}{3}\right)^2 [s^n(x) + \bar{s}^n(x)] + (\cdots)$$

Isospin symmetry between the proton and the neutron

$$u^p(x) = d^n(x) \equiv u(x)$$

$$d^p(x) = u^n(x) \equiv d(x)$$

$$s^p(x) = s^n(x) \equiv s(x)$$

Further Constraints

$$u_s(x) = \bar{u}_s(x) = d_s(x) = \bar{d}_s(x) = s_s(x) = \bar{s}_s(x) = S(x)$$

$$u(x) = u_v(x) + u_s(x)$$

$$d(x) = d_v(x) + d_s(x)$$

Then,

$$\begin{aligned}
\frac{1}{x}F_2^p &= \frac{1}{9}[4u_v + d_v] + \frac{4}{3}S \\
\frac{1}{x}F_2^n &= \frac{1}{9}[u_v + 4d_v] + \frac{4}{3}S
\end{aligned}$$

Sum Rules

Protons are made of *two* u quarks and *one* d quark.

$$\int_0^1 u_v(x)\,dx = 2$$

$$\int_0^1 d_v(x)\,dx = 1 \quad \text{or}$$

$$\int_0^1 [u(x) - \bar{u}(x)]\,dx = 2$$

$$\int_0^1 [d(x) - \bar{d}(x)]\,dx = 1$$

$$\int_0^1 [s(x) - \bar{s}(x)]\,dx = 0$$

Limits of F_2

As $x \to 0$, sea quarks $S(x)$ dominates, and

$$\frac{F_2^n(x)}{F_2^p(x)} \xrightarrow{x \to 0} 1$$

As $x \to 1$, valence quarks dominates $(S(x) \to 0)$

$$\frac{F_2^n(x)}{F_2^p(x)} \xrightarrow{x \to 1} \frac{u_v + 4d_v}{4u_v + d_v}$$

Take the difference

$$\frac{1}{x}[F_2^p(x) - F_2^n(x)] = \frac{1}{3}[u_v(x) - d_v(x)]$$

Observation of valence quarks without sea quarks

Gluons?

$$\int dx\, F_2^p(x) \quad = \quad \int dx\, x \left[\frac{4}{9}(u+\bar{u}) + \frac{1}{9}(d+\bar{d})\right] = 0.18$$

$$\int dx\, F_2^n(x) \quad = \quad \int dx\, x \left[\frac{1}{9}(u+\bar{u}) + \frac{4}{9}(d+\bar{d})\right] = 0.12$$

Then

$$\int dx\, x\, [u+\bar{u}+d+\bar{d}+s+\bar{s}] = 0.54$$

implying that 46% of the proton momentum is carried by *gluons*