Probing Internal Structures

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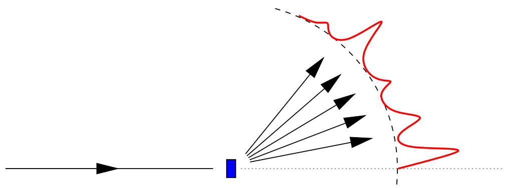
Study of the Internal Structure

Study of X-tal structure (one of many ways)

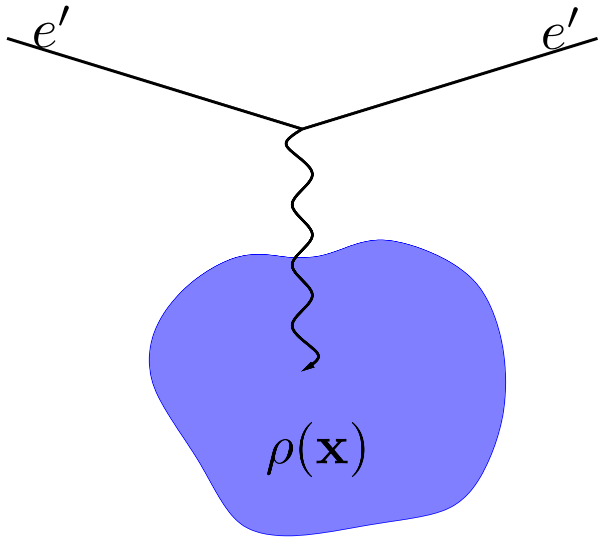
- ➜ X-ray scattering on X-tal
- ➜ Intensity pattern contains information on X-tal structure
- ➜ Reciprocity between length(d) and momentum(k)



Study of nucleon structure (one of many ways)

- ➜ Electron scattering on the nucleon
- ➜ Cross section contains information on internal structure
- ➜ Fourier transform of spatial distribution

Probing Charge Distribution



$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{point}} |F(q)|^2$$

$$F(\mathbf{q}) = \int \rho(\mathbf{x}) e^{i\mathbf{q}\cdot\mathbf{x}}\, d^3x \quad \text{or}$$

$$\rho(\mathbf{x}) = \int F(\mathbf{q}) e^{-i\mathbf{q}\cdot\mathbf{x}}\, d^3q \quad \text{or}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{point}} \equiv \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} = \frac{(Z\alpha)^2 \cos^2\dfrac{\theta}{2}}{4E^2 \sin^4\dfrac{\theta}{2}}$$

Proton Form Factors

Cross section for electron scattering on spin 1/2 Dirac particle

$$\frac{d\sigma}{d\Omega}_{\rm lab} = \left(\frac{\alpha^2}{4E^2 \sin^4 \frac{\theta}{2}} \right) \frac{E'}{E} \left\{ \cos^2 \frac{\theta}{2} - \frac{q^2}{2M^2} \sin^2 \frac{\theta}{2} \right\}$$

- Internal structure of the proton: *two* new unknown functions (form factors)

$$F_1(q^2), \quad F_2(q^2) \qquad \text{Sach's Form Factors}$$

$$\begin{aligned} \frac{d\sigma}{d\Omega}_{\rm lab} &= \left(\frac{\alpha^2}{4E^2 \sin^4 \frac{\theta}{2}} \right) \frac{E'}{E} \\ &\times \left\{ \left(F_1^2 + \frac{\kappa^2 Q^2}{4M^2} F_2^2 \right) \cos^2 \frac{\theta}{2} + \frac{Q^2}{2M^2} (F_1 + \kappa F_2)^2 \sin^2 \frac{\theta}{2} \right\} \end{aligned}$$

Electric & Magnetic Form Factors

$$G_E \equiv F_1 + \frac{\kappa q^2}{4M^2} F_2$$

$$G_M \equiv F_1 + \kappa F_2$$

$$\frac{d\sigma}{d\Omega}_{\text{lab}} = \left(\frac{\alpha^2}{4E^2 \sin^4 \frac{\theta}{2}} \right) \frac{E'}{E}$$

$$\times \left(\frac{G_E^2 + \tau G_M^2}{1+\tau} \cos^2 \frac{\theta}{2} - 2\tau G_M^2 \sin^2 \frac{\theta}{2} \right)$$

$$\tau \equiv \frac{Q^2}{4M^2}$$

G_E distribution of charge inside the proton

G_M distribution of magnetization inside the proton

Proton Form Factor

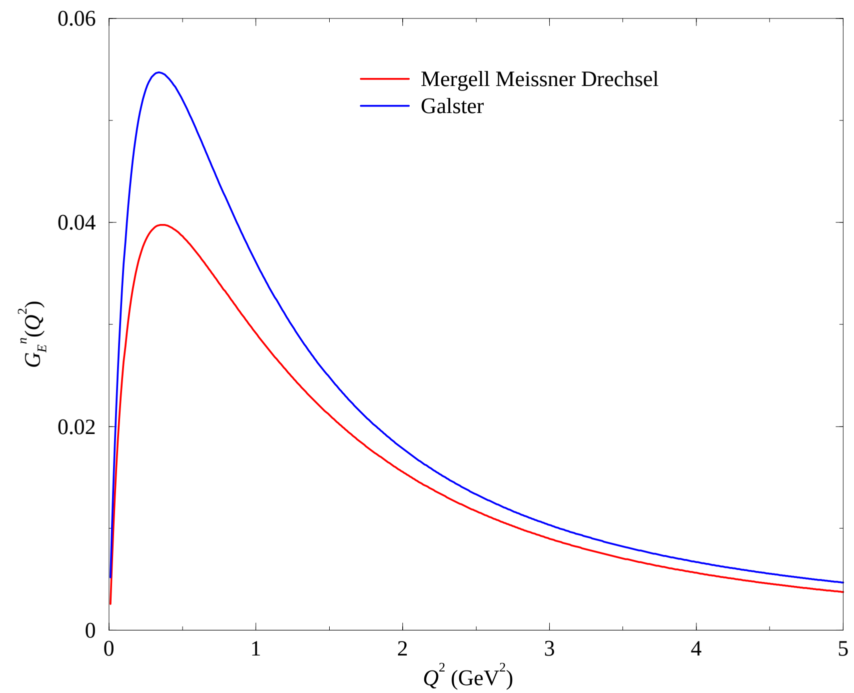
$$G_E = G_M/\mu = \frac{1}{\left(1 + \frac{Q^2}{0.71}\right)^2}$$

- Inverse Fourier transform of G_E and G_M has exponential shape in space
- Mean square proton charge radius

$$< r^2 >= 6 \left(\frac{dG_E(q^2)}{dq^2} \right)_{q^2=0} = (0.81\,\mathrm{fm})^2$$

Neutron Form Factors

- Magnetic form factor G_M^n has the same Q^2 dependence as G_E^p or G_M^p
- Electric form factor G_E^n is *not* trivially zero



Neutron Charge Radius

- The slope of G_E^n near $Q^2 = 0$ is about 0.26 to 0.36 GeV$^-$2.
- $< r_n^2 >= (0.25 \text{ to } 0.29 \text{ fm})^2$
- Charge distribution is exponential
- There are *non-zero* charge distribution inside the neutron
- Compared to proton elastic form factor, neutron form factors are *poorly* known

Generalization to Inelastic Scattering

$$\left.\frac{d\sigma}{dE'd\Omega}\right|_{\text{lab}} = \frac{\alpha^2}{4E^2\sin^4\frac{\theta}{2}}$$

$$\times \left\{W_2(\nu,Q^2)\cos^2\frac{\theta}{2} + 2W_1(\nu,Q^2)\sin^2\frac{\theta}{2}\right\}$$

$$= \Gamma(\sigma_T + \varepsilon\sigma_L)$$

$$\Gamma = \frac{\alpha K}{2\pi^2 Q^2}\frac{E'}{E}\frac{1}{1-\varepsilon} \quad \text{virtual photon flux}$$

$$\varepsilon = \left(1 + 2\frac{\mathbf{q}^2}{Q^2}\tan^2\frac{\theta}{2}\right)^{-1} \quad \text{polarization of the virtual photon}$$

$$K = \nu - \frac{Q^2}{2M} \quad \text{equivalent photon momentum}$$

As $Q^2 \to 0$ (real photon),

$$\sigma_T \to \sigma^{\text{tot}}(\gamma p), \qquad \sigma_L \to 0$$