High Energy Experiments

prepared by

Seonho Choi

Seoul National University

Topics in High Energy Physics

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Lorentz Transformation

- Energy-momentum 4-vector

$$
\begin{aligned}
p &= (E, \mathbf{p}) \\
p^2 &= E^2 - \mathbf{p}^2 = m^2 \\
\beta &= \mathbf{p}/E
\end{aligned}
$$

- Lorentz Transformation

$$
\begin{pmatrix} E^* \\ p^*_{\|} \end{pmatrix} = \begin{pmatrix} \gamma_f & -\gamma_f \beta_f \\ -\gamma_f \beta_f & -\gamma_f \end{pmatrix} \begin{pmatrix} E \\ p_{\|} \end{pmatrix}, \qquad p^*_T = p_T
$$

- Lorentz invariant

$$
p_1 \cdot p_2 = E_1 E_2 - \mathbf{p}_1 \cdot \mathbf{p}_2
$$

Center of Mass

- It is often convenient to consider collision process in the *center of mass*(CM) frame
- Center of Mass energy

$$
\begin{aligned}
E_{\rm cm} &= \sqrt{(p_1 + p_2)^2} \\
&= \left[(E_1 + E_2)^2 - (\mathbf{p}_1 + \mathbf{p}_2)^2\right]^{1/2} \\
&= \left[m_1^2 + m_2^2 + 2E_1E_2(1 - \beta_1\beta_2\cos\theta)\right]^{1/2}
\end{aligned}
$$

- Special case - Lab frame (one particle is at rest)

$$E_{\rm cm} = (m_1^2 + m_2^2 + 2E_{1\rm lab}m_2)^{1/2}$$

Examples

- LHC (in construction) will collide two protons at 7 TeV each. (1 TeV = 1000 GeV)
- Center of mass energy = 14 TeV
- 7 TeV proton on fixed target

$$E_{\rm cm} \approx 118\,{\rm GeV}$$

- the rest was used for the CM motion
- 1 GeV proton on fixed proton target (KOMAC)

$$E_{\rm cm} = 2.33\,{\rm GeV}$$

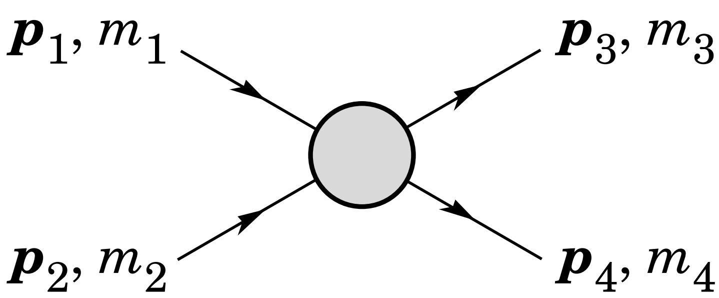
- production of $p + K^+ + \Lambda$ requires

$$m_p + m_{K^+} + m_\Lambda = 2.55\,{\rm GeV} \quad \text{or 1.58 GeV beam}$$

Examples (Cont.)

- Use heavy nuclei such as lead (^{208}Pb)
- Nucleons have Fermi motion inside the nuclei
- If k_F=0.2 GeV, $E_{\rm cm} = 2.48$ GeV
- If k_F=0.3 GeV, $E_{\rm cm} = 2.58$ GeV (barely possible)

Mandelstam Variables



$$
\begin{aligned}
s &= (p_1 + p_2)^2 = (p_3 + p_4)^2 \\
t &= (p_1 - p_3)^2 = (p_2 - p_4)^2 \\
u &= (p_1 - p_4)^2 = (p_2 - p_3)^2
\end{aligned}
$$

and

$$s + t + u = m_1^2 + m_2^2 + m_3^2 + m_4^2$$

Cross Section - Part 1

Mechanics, 3rd edition by Keith R. Symon

If N incident particles strike a thin foil containing n scattering centers per unit area, the average number dN of particles scattered through an angle between Θ and $\Theta + d\Theta$ is given in terms of the cross section $d\sigma$ by the formula

$$\frac{dN}{N} = n\,d\sigma$$

$d\sigma$ is called the cross section for scattering through an angle between Θ and $\Theta + d\Theta$, and can be thought of as the effective area surrounding the scattering center which the incident particle must hit in order to be scattered through an angle between Θ and $\Theta + d\Theta$. For if there is a "target area" $d\sigma$ around each scattering center, then the total target area in a unit area is $n\,d\sigma$. If N particles strike one unit area, the average number striking the target area is $Nn\,d\sigma$, and this,..., is just dN,...

Cross Section - Part 2

Dimension of the cross section

$$d\sigma = \frac{dN}{N}\frac{1}{n}$$

$$[d\sigma] = \frac{1}{[n]} = [\text{area}]$$

In real experiments,

- target is specified by *density*(ρ) and *thickness* (Δl)
- beam is specified by *current* (I)

$$n = \rho \cdot \Delta l$$

$$N = \int \frac{I}{e}\, dt$$

Cross Section - Part 3

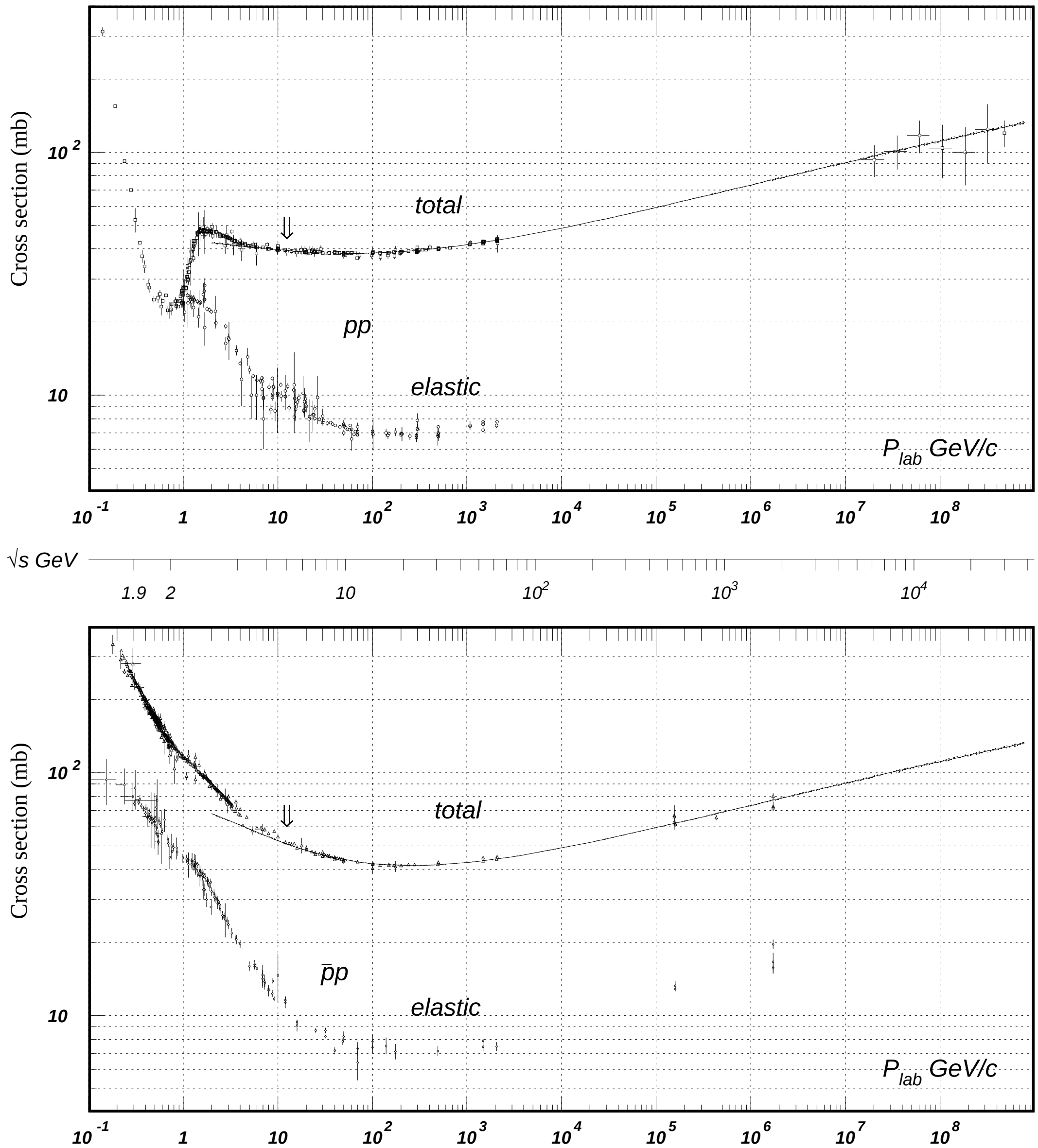
$$d\sigma = \frac{1}{\rho \cdot \Delta l} \frac{dN}{\int \frac{I}{e}\, dt}$$

- In general, cross sections are written in *barns*, or b
  1 barn $= 10^{-28}\text{m}^2 = 10^{-24}\text{cm}^2$, 1 fm^2 = 10 mb
- Actually, 1 barn is *very* big cross section, usually use smaller units such as μb, nb, pb.
- In the previous expression,

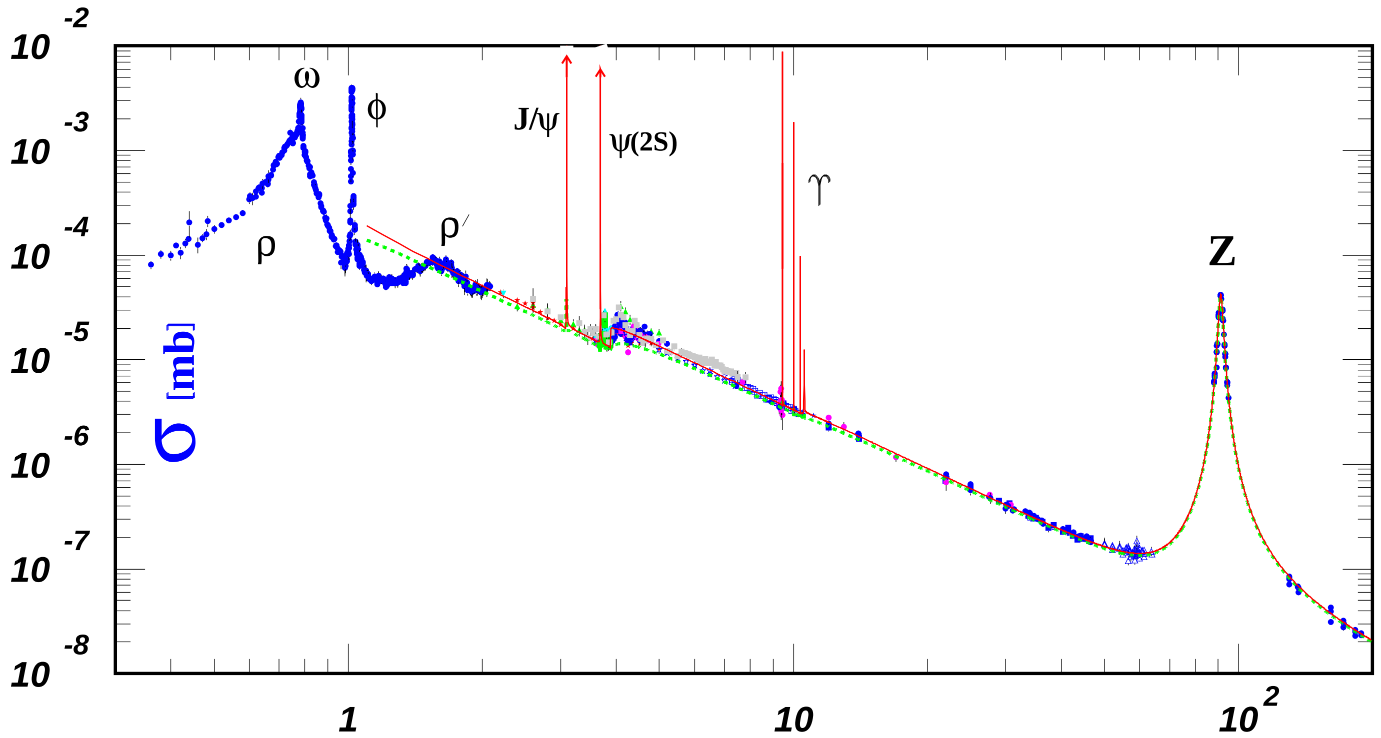
$$(\rho \cdot \Delta l) \cdot \frac{I}{e}$$

  is called *luminosity*, $\mathcal{L}$ with units cm$^{-2}\cdot$sec^{-1}.
- $\mathcal{L} \cdot d\sigma$ gives dN per second (reaction rate, event rate)

Cross Section - Part 4



Cross Section - Part 5



Luminosity for Colliders

- Two bunches each containing N_1 and N_2 particles colliding f times per second
- Each bunch has Gaussian distribution in transverse direction with σ_x and σ_y
- Head-on collision along z direction

$$\mathcal{L} = f \cdot \frac{N_1 \cdot N_2}{4\pi\sigma_x\sigma_y}$$