From Quark to Matter

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Topics in High Energy Physics

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Magnetic Moments

Quark representation of the proton

$$\begin{aligned}
|p\uparrow> \quad &= \quad \sqrt{\frac{1}{18}}\,[uud(\uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow -2\uparrow\uparrow\downarrow) \\
&\quad +udu(\uparrow\uparrow\downarrow + \downarrow\uparrow\uparrow -2\uparrow\downarrow\uparrow) \\
&\quad +duu\,(\uparrow\downarrow\uparrow + \uparrow\uparrow\downarrow -2\downarrow\uparrow\uparrow)] \\
&= \quad \sqrt{\frac{1}{18}}\,[u\uparrow u\downarrow d\uparrow + u\downarrow u\uparrow d\uparrow -2u\uparrow u\uparrow d\downarrow \\
&\quad +\text{permutations}]
\end{aligned}$$

Magnetic moment of the quarks

$$\mu_i = Q_i \left(\frac{e}{2m_i}\right)$$

Magnetic Moments (Cont.)

Magnetic moment of the proton

$$\begin{aligned}\mu_p &= \sum_{i=1}^{3} < p\uparrow |\mu_i (\sigma_3)_i | p\uparrow > \\ &= \frac{1}{3}(4\mu_u - \mu_d)\end{aligned}$$

Similarly for the neutron

$$\mu_n = \frac{1}{3}(4\mu_d - \mu_u)$$

If $m_u = m_d$, $\mu_u = -2\mu_d$ and

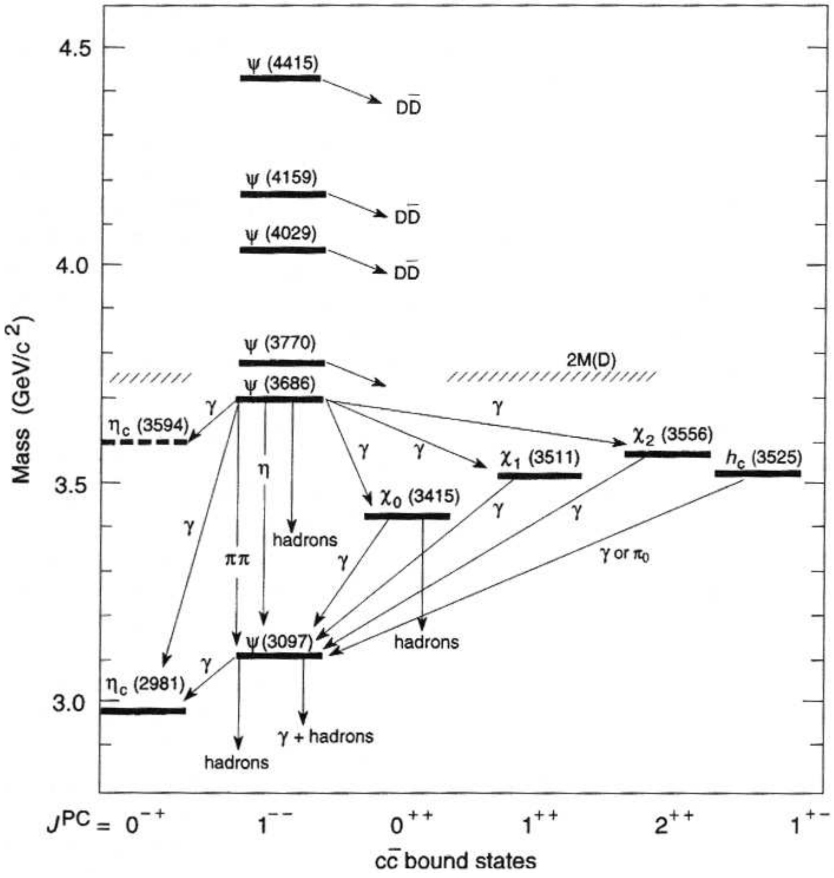
$$\frac{\mu_n}{\mu_p} = -\frac{2}{3}$$

Experiment shows

$$\frac{\mu_n}{\mu_p} = -0.68497945 \pm 0.00000058$$

One more quark - c

- Discovery of J/ψ in 1974 (3.1 GeV)
- Turns out to be $c\bar{c}$ pair (charmonium)



Interpretation

- ➜ c quark much heavier than other quarks
- ➜ Appropriate to interprete as heavy quarks inside a *potential*
- ➜ A naïve form of potential between $c\ \bar{c}$

$$V(r) = -\frac{4}{3}\frac{\alpha_S}{r} + ar$$

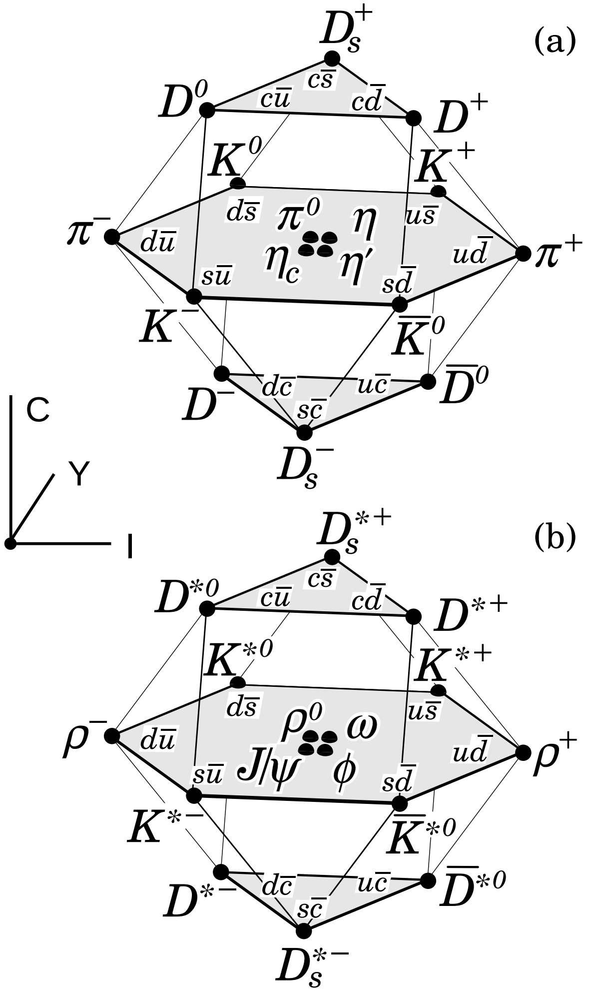
- ➜ At small distance: Coulomb type potential $\sim 1/r$
- ➜ At large distnace: confining potential $\sim r$
- ➜ Too much separation?
  Energy wise, easier to produce $q\bar{q}$ pair in the middle

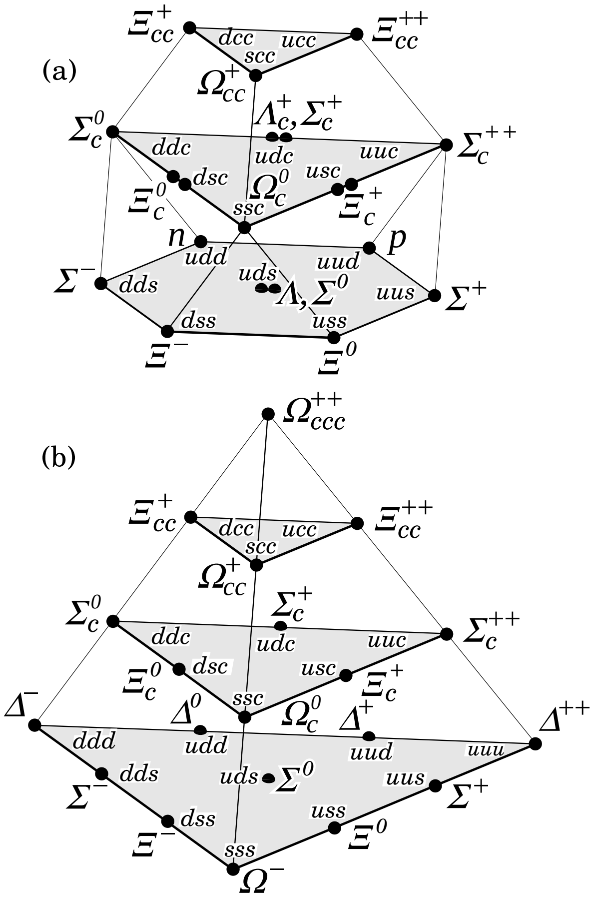
4 quarks instead of 3

- Mass of the c quark much heavier than any other
- Nevertheless, try SU(4)

$$\begin{aligned} \mathbf{4} \otimes \bar{\mathbf{4}} &= \mathbf{15} \oplus \mathbf{1} \\ \mathbf{4} \otimes \mathbf{4} \otimes \mathbf{4} &= \mathbf{20} \oplus \mathbf{20} \oplus \mathbf{20} \oplus \bar{\mathbf{4}} \end{aligned}$$

Baryons and Mesons





Hadron Masses

- If SU(4) were *exact*, single meson mass and single hadron mass
- However, great variety of hadron masses (broken SU(4) symmetry)
- Constituent quark mass

$$\left.\begin{aligned} m_\omega \approx m_\rho(u\bar{u}) &= 0.78\text{GeV} \\ m_\phi(s\bar{s}) &= 1.02\text{GeV} \\ m_{K^*}(s\bar{u}) &= 0.89\text{GeV} \\ m_{D^*}(c\bar{u}) &= 2.01\text{GeV} \\ m_{F^*}(c\bar{s}) &= 2.11\text{GeV} \\ m_{J/\psi}(c\bar{c}) &= 3.1\text{GeV} \end{aligned}\right\} \rightarrow \begin{aligned} & m_u \approx m_d \approx 0.39\,\text{GeV} \\ & m_s \approx 0.51\,\text{GeV} \\ & m_c \approx 1.6\,\text{GeV} \end{aligned}$$

Hadron Masses (Cont.)

- Surprising success in explaining mass hierarchies and mass differences between hadrons
- Effect of relative spins of the quarks
- Remind from hydrogen atom (hyperfine structure)

$$\Delta E_{hf} = -\frac{2}{3}\boldsymbol{\mu_1} \cdot \boldsymbol{\mu_2} \left|\psi(0)\right|^2 = \frac{2\pi\alpha_{\mathrm{em}}}{3} \frac{\boldsymbol{\sigma_1} \cdot \boldsymbol{\sigma_2}}{m_1 m_2} \left|\psi(0)\right|^2$$

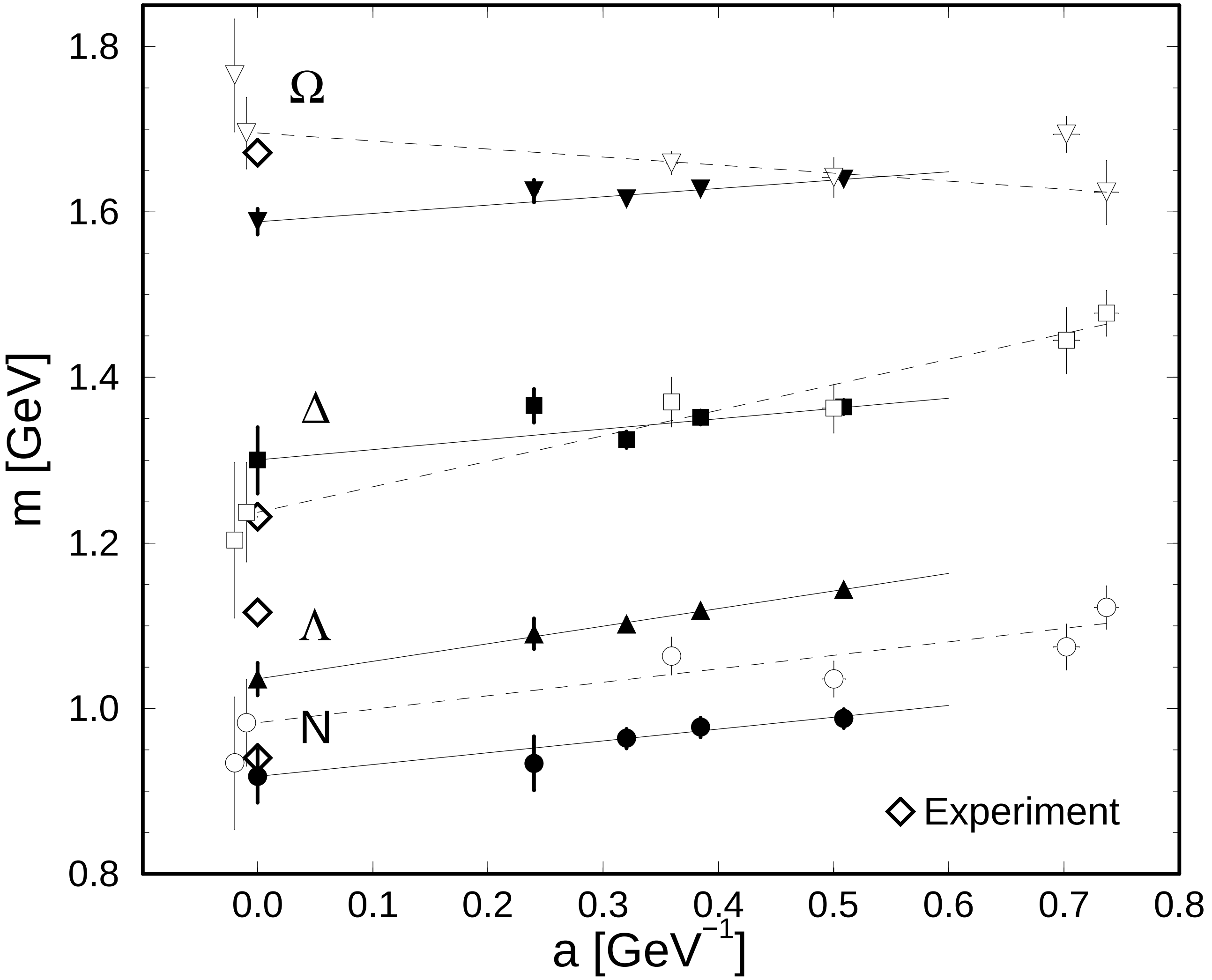
- Similary for mesons and baryons

$$\begin{aligned} m(q_1\bar{q}_2) &= m_1 + m_2 + \left[\frac{a(\boldsymbol{\sigma_1} \cdot \boldsymbol{\sigma_2})}{m_1 m_2}\right] \\ m(q_1 q_2 q_3) &= m_1 + m_2 + m_3 + \left[\frac{a'}{2}\sum_{i>j}\frac{a(\boldsymbol{\sigma_i} \cdot \boldsymbol{\sigma_j})}{m_i m_j}\right] \end{aligned}$$

Comments on the Mass

- Hadron masses: one of the fundamental questions *unanswered* in QCD
- No *a priori* calculation from QCD Lagrangian
- Some hope in lattice QCD
- Non-zero quark mass breaks chiral symmetry: problem in effective theories
- In hindsight, Standard Model has no predictive power for masses.
- Higg's mechanism gives mass for particles.

Lattice QCD


Ω
Δ
Λ
N
◇ Experiment
m [GeV]
0.8
1.0
1.2
1.4
1.6
1.8
a [GeV^{-1}]
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8

Particle Mixing

- Addendum to yesterday's colloquium by Prof. Subong Kim
- Particles in Standard Model

Quarks	$+\frac{2}{3}$	u	c	t
	$-\frac{1}{3}$	d	s	b
Leptons	-1	e	μ	τ
	0	ν_e	ν_μ	ν_τ

- (u,c,t) and (e,μ,τ): model particles with good behavior
- (d,s,b) and (ν_e,ν_μ,ν_τ): strange behavior from time to time
- Weak interaction (W^+,W^-) couples within the same *family* (most of the time)

Particle Mixing

- However, occasionally, coupling between different families
- Linear combinations of (d,s,b) or (ν_e,ν_μ,ν_τ) give mass eigenstate which does not change with time
- Mixing matrix (similar for both quarks and leptons)

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

- Definite flavor state (*e.g.* ν_e) contains a mixture of different mass eigenstates
- With time, composition of different mass eigenstates changes
- When observed, flavor changes