From Quark to Matter

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High Energy Physics

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Appetizer - Units

- Necessary units: length(L), mass(M), time(T)
- Two fundamental constants

$$\hbar \equiv \frac{h}{2\pi} = 1.055 \times 10^{-34}\,\mathrm{J\,sec} \quad (\mathrm{ML^2/T})$$
$$c \equiv 2.998 \times 10^{8}\,\mathrm{m/sec} \quad (\mathrm{L/T})$$

- New system of units: $\hbar = c = 1$
- We need one more independent relation to define the system of units
- Commen choice: energy ($\mathrm{ML^2/T^2}$)
- High energy physics: GeV (1 GeV$\equiv 10^9$ electron volts)

Units (Cont.)

- Coupling constant

$$\alpha_{\text{em}} = \frac{e^2}{4\pi\hbar c} \approx \frac{1}{137} \quad \text{(dimensionless)}$$

- Under new system of units

Conversion Factor	$\hbar = c = 1$ Units	Actual Dimension
1 kg $= 5.61\times 10^{26}$ GeV	GeV	$\frac{\text{GeV}}{c^2}$
1 m $= 5.07\times 10^{15}$ GeV^{-1}	GeV^{-1}	$\frac{\hbar c}{\text{GeV}}$
1 sec $= 1.52\times 10^{24}$ GeV^{-1}	GeV^{-1}	$\frac{\hbar}{\text{GeV}}$
$e = \sqrt{4\pi\alpha}$	—	$(\hbar c)^{1/2}$

A Little Exercise

- Mass of the proton
  $1 \text{ GeV} = 10^3 \text{ MeV} = 10^6 \text{ KeV} = 10^9 \text{ eV}$
- Size of the proton
  $1 \text{ fm} = 10^{-13} \text{ cm} \approx 5 \text{ GeV}^{-1}$
- Cross section of the proton
  $1 \text{ fm}^2 = 10 \text{ mb} = 10^4\ \mu\text{b} = 10^7 \text{ nb} = 10^{10} \text{ pb}$
- $(1 \text{ MeV})^{-1} = 197 \text{ fm}$ (or $\hbar c = 1 = 197 \text{ MeV fm}$)
- $(1 \text{ GeV})^{-2} = 0.389 \text{ mb}$ (or $(\hbar c)^2 = 1 = 0.389 \text{ GeV mb}$)

High Energy Physics

- Current maximum energy of the electron beam at Jefferson National Lab: 6 GeV

$$\begin{aligned} 6\text{GeV} &= 10^{-9}\,\text{J} \\ &= 10^{-16}\,\text{kWh} \end{aligned}$$

- Energy of 60W light bulb for 1 second $= 1.7 \times 10^{-5}$ kWh
- Energy of an electron in a car running at 100 km/h $\approx 2 \times 10^{-21}$ GeV
- Electrons at 6 GeV will need 0.66 seconds more to cover the distance of 1 light year.
- Building cost of Jefferson Lab: 600 M$ (약 7천억원)
- Operating cost of Jefferson Lab: 72 M$ (약 860억원)
- 1 hour of electron beam time $\approx$ 10000$ (1200만원)

A Brief History

1911 Rutherford experiment - existence of *nucleus*

1932 Discovery of the neutron

1936 Yukawa's prediction of *meson*

1947 Discovery of the pion (π)

50's-60's Discovery of numerous *new* particles

- mesons: π, ρ, K, η, ω, ϕ etc.
- baryons: Δ, Σ, Λ, Ξ, Ω etc.

➜ All these new particles, are they *fundamental*?

➜ More fundamental building blocks - *Quarks*

Naïve Quark Model

- ➜ Start from *two* quarks (u and d) for n, p, π's and Δ's
- ➜ Add one more *strange* quark for Λ, K's and Σs
- ➜ Three quarks make baryons
- ➜ Pair of one *quark* and *anti*-quark makes mesons
- ➜ *Three* different quarks (u, d, s)
  - Are they completely different objects? (as sun and moon)
  - Or something similar? (as electrons with two different spins)
  - Search for symmetry

Symmetry

- Symmetries in nature

Crystals rotational/translational symmetry

Snowflakes rotational symmetry by $60°$

Human body mirror symmetric

Electrons spin up and spin down (SU(2))

Particles with respect to spin in general

Proton-Neutron almost symmetric (isospin 1/2)

Pions again almost symmetric (isospin 1)

Quarks quite symmetric (SU(3))

People symmetric?

Review of Spin - SU(2)

- Stern-Gerlach experiment - discovery of two *different* electrons
- Almost identical (same mass, charge etc) except magnetic moments
- Same particle with *different* spin
- Transformation from one spin to another spin governed by SU(2) symmetry group
- Famous commutators

$$[J_j, J_k] = i\varepsilon_{jkl} J_l$$

- Step-up, step-down operators

$$\begin{aligned} J_{\pm} &= J_1 \pm iJ_2 \\ J_+|\text{spin down}> &= |\text{spin up}> \\ J_-|\text{spin up}> &= |\text{spin down}> \end{aligned}$$

Application to p-n system

- the proton and the neutron *are* different
- $m_p = 0.93827203$ GeV, $m_n = 0.93956536$ GeV
- $\Delta m/\bar{m} = 0.07\% \approx 0$
- Same algebra as electron spin

$$[I_j, I_k] = i\varepsilon_{jkl} I_l$$

- Electric charge of the baryons

$$\begin{aligned} Q &= I_3 + \frac{Y}{2} & \quad Y &= \text{hyper-charge} \\ Y &= B + S & \quad B &= \text{baryon number} \quad S = \text{strangeness} \end{aligned}$$

- Experimentally, isospin is a very good symmetry

What about three quarks?

- Quite different masses

$$
\begin{aligned}
m(u) &\approx 1.5 \sim 4\,\mathrm{MeV} \\
m(d) &\approx 4 \sim 8\,\mathrm{MeV} \\
m(s) &\approx 80 \sim 130\,\mathrm{MeV}
\end{aligned}
$$

- Reasonably good symmtery with u and d quarks (SU(2))
- Approximate symmetry with all three quarks (SU(3))
- Separate consideration of the effects from SU(3) symmetry breaking

Exercise with u and d

- Similar to combining two spin 1/2 (hydrogen atom)
- Add one quark and one *anti*-quark to form mesons

$$\begin{pmatrix} u \\ d \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} -\bar{d} \\ \bar{u} \end{pmatrix}$$

- *iso*-triplet

$$\begin{aligned} |I=1, I_3=1> &= -u\bar{d} \quad (\pi^+) \\ |I=1, I_3=0> &= \sqrt{1/2}(u\bar{u} - d\bar{d}) \quad (\pi^0) \\ |I=1, I_3=-1> &= d\bar{u} \quad (\pi_-) \end{aligned}$$

- *iso*-singlet

$$|I=0, I_3=0>= \sqrt{1/2}(u\bar{u} + d\bar{d}) \quad (\omega)$$

To 3 quarks

- mesons with u and d quarks using SU(2) terminology

$$\mathbf{2} \otimes \bar{\mathbf{2}} = \mathbf{3} \oplus \mathbf{1}$$

- extension to 3 quarks to form baryons with SU(3)
- systematic method with Young diagrams

$$\begin{aligned} \mathbf{3} \otimes \mathbf{3} &= \mathbf{6} \oplus \bar{\mathbf{3}} \\ \mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} &= (\mathbf{6} \otimes \mathbf{3}) \oplus (\bar{\mathbf{3}} \otimes \mathbf{3}) \\ &= \mathbf{10} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{1} \end{aligned}$$

- Quarks with spins: SU(3) and SU(2) $\rightarrow$ SU(6)
- Getting more complicated

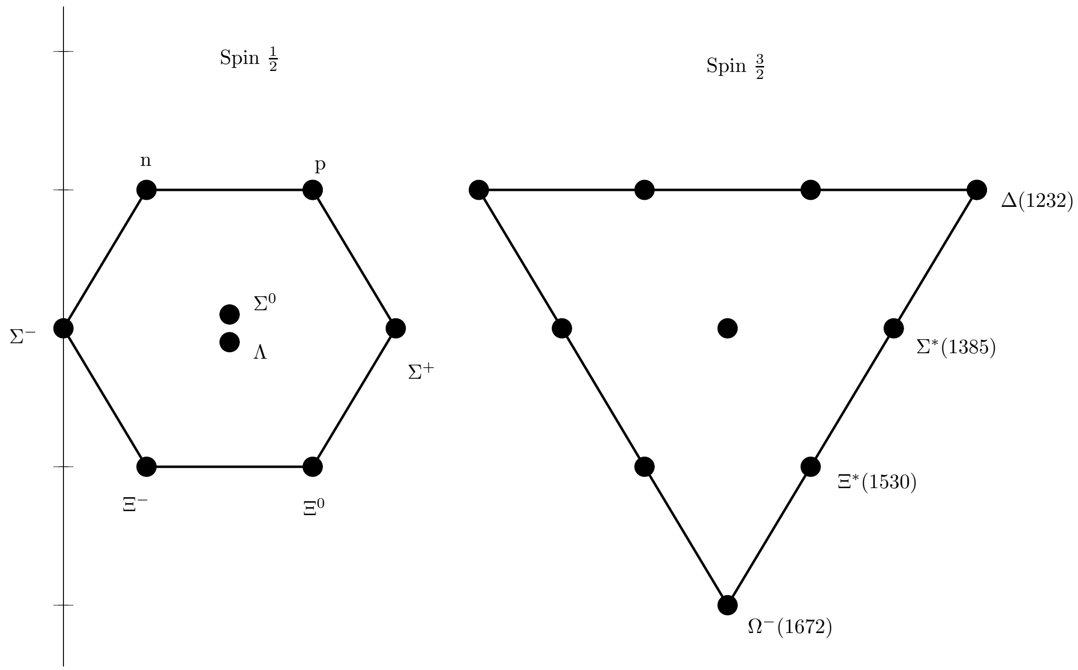
$$\mathbf{6} \otimes \mathbf{6} \otimes \mathbf{6} = \mathbf{56} \oplus \mathbf{70} \oplus \mathbf{70} \oplus \mathbf{20}$$

Baryons

➜ Completely symmetric state

$$\mathbf{56} = (\mathbf{10} \otimes \mathbf{4}) \oplus (\mathbf{8} \otimes \mathbf{2})$$

➜ Nice fit with lowest mass baryons



A little problem

- Quark representation for Δ^{++} of $J_3 = \frac{3}{2}$

$$u\uparrow\; u\uparrow\; u\uparrow$$

- *symmetric* under the exchange of identical quarks
- solution: additional quantum number, *color*

u,d,s	:	Flavor SU(3)
$\uparrow$,$\downarrow$	:	Spin SU(2)
Red, Green, Blue	:	Color SU(3)

- All hadrons are postulated to be *colorless* - color singlet

$$(qqq)_{\text{color singlet}} = \sqrt{\frac{1}{6}}(RGB - RBG + BRG - BGR + GBR - GRB)$$

- Overall symmetric for space×spin×flavor

Magnetic Moments

Quark representation of the proton

$$
\begin{aligned}
|p\uparrow> &= \sqrt{\frac{1}{18}}\,[uud(\uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow - 2\uparrow\uparrow\downarrow) \\
&\quad + udu(\uparrow\uparrow\downarrow + \downarrow\uparrow\uparrow - 2\uparrow\downarrow\uparrow) \\
&\quad + duu\,(\uparrow\downarrow\uparrow + \uparrow\uparrow\downarrow - 2\downarrow\uparrow\uparrow)] \\
&= \sqrt{\frac{1}{18}}\,[u\uparrow u\downarrow d\uparrow + u\downarrow u\uparrow d\uparrow - 2u\uparrow u\uparrow d\downarrow \\
&\quad + \text{permutations}]
\end{aligned}
$$

Magnetic moment of the quarks

$$\mu_i = Q_i\left(\frac{e}{2m_i}\right)$$

Magnetic Moments (Cont.)

Magnetic moment of the proton

$$\mu_p = \sum_{i=1}^{3} < p\uparrow |\mu_i(\sigma_3)_i| p\uparrow > = \frac{1}{3}(4\mu_u - \mu_d)$$

Similarly for the neutron

$$\mu_n = \frac{1}{3}(4\mu_d - \mu_u)$$

If $m_u = m_d$, $\mu_u = -2\mu_d$ and

$$\frac{\mu_n}{\mu_p} = -\frac{2}{3}$$

Experiment shows

$$\frac{\mu_n}{\mu_p} = -0.68497945 \pm 0.00000058$$