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Weighted Scale-Free Network in Financial Correlations

Hyun-Joo KIM[1], Youngki LEE[2], Byungnam KAHNG[3] and In-mook KIM[1*]

[1]*Department of Physics, Korea University, Seoul, 136-701, Korea*
[2]*Yanbian University of Science and Technology, Beishan St. Yanji, Jilin 133000, China*
[3]*School of Physics and Center for Theoretical Physics, Seoul National University, Seoul 151-747, Korea*

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While many scale-free (SF) networks have been introduced recently for complex systems, most of them are binary random graphs. Here we introduce a weighted SF network in associated with the cross-correlations in stock price changes among the S&P 500 companies, where all vertices (companies) are fully connected and each edge has nonuniform weight given by the covariance between the two returns connected, normalized by their volatilities. Influence-strength (IS) is defined as the sum of the weights on the edges incident upon a given vertex. Then the IS distribution in its absolute magnitude $|q|$ exhibits a SF behavior, $P_I(|q|) \sim |q|^{-\eta}$ with the exponent $\eta \approx 1.8(1)$.

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Recently complex systems such as biological, economic, physical, and social systems have received considerable attentions as an interdisciplinary subject.[1)] Such systems consist of many constituents such as individuals, companies, substrates, spins, etc., exhibiting cooperative and adaptive phenomena through diverse interactions between them. In particular, in economic systems, adaptive behaviors of individuals, companies, or nations, play a crucial role in forming macroscopic patterns such as commodity prices, stock prices, exchange rates, etc., which are formed mostly in a self-organized way. Recently there have been considerable efforts to understand such complex systems in terms of random graph, consisting of vertices and edges, where vertices (edges) represent constituents (their interactions).[2,3)] This approach was initiated by Erdös and Rényi (ER).[4)] In the ER model, the number of vertices is fixed, while edges connecting one vertex to another occur randomly with certain probability. The ER model is however too random to describe complex systems in real world. An interesting feature emerging in such complex systems is the scale-free (SF) behavior in the degree distribution, $P_{\mathrm{D}}(k) \sim k^{-\gamma}$, where the degree k is the number of edges incident upon a given vertex. Barabási and Albert (BA)[5)] introduced an evolving model illustrating the SF behavior. In the BA model, the number of vertices increases linearly with time, and a newly introduced vertex is connected to m already existing vertices, following the so-called preferential attachment (PA) rule that the vertices with more edges are preferentially selected for the connection to the new vertex with the probability linearly proportional to the degree of that vertex. Then it is known that the degree distribution follows $P(k) \sim k^{-3}$ for the BA model.

The ER and BA models are binary random graphs where each edge weighs either 1 or 0 depending on whether it is present or not, respectively. One may introduce a weighted random graph, where weight on each edge is not uniform, but distributed in a given interval. For further studies, the random graph for the former (later) case is called binary random graph (BRG) [weighted random graph (WRG)]. While many WRGs can be found in various real-world networks such as neural networks, cardiovascular networks, and respiratory networks in biological systems, acquaintance networks in social systems, etc., they are less studied, compared with BRG.[2,3)] Moreover, it has not been investigated yet if such WRGs exhibit the SF behavior. In this paper, we will introduce a WRG exhibiting the SF behavior in association with the cross-correlations in stock price changes among the S&P 500 companies.

Recently, Yook, Jeong and Barabási (YJB) introduced an evolving WRG.[6)] In that model, a vertex i is newly introduced at each time step, connecting to m vertices existing already according to the so-called preferential attachment rule. The edge connecting from the vertex i to an existing vertex j is assigned a nonuniform weight $w_{i,j}$, depending on the degree of the vertex j. The weight at each vertex is defined as the sum of the weights on the edges incident upon that vertex, which follows a power-law in its distribution, $P_{\mathrm{YJB}}(q) \sim q^{-\eta'}$, where q means the weight at a given vertex. The exponent η' is different from the degree exponent γ, and turns out to depend on the mean degree m strongly. While the YJB model is meaningful as the first step towards relating WRGs to SF networks, it still remains as a theoretical model. The WRG exhibiting SF behavior has not been discovered yet. In this paper, we introduce a WRG showing the SF behavior in association with the cross-correlations in stock price changes. The WRG we will introduce is fully connected, while the YJB graph is sparsely connected.

Recently, many attentions and studies have been focused and performed on the fluctuations and the correlations in stock price changes between different companies in physics communities by applying physics concepts and methods.[7,8)] For the problem of the correlations in stock price changes, each vertex (edge) in the random graph represents a company (the cross-correlation in stock price changes between the companies connected via that edge). Stock price changes of individual companies are influenced by others. Thus, one of the most important quantities in understanding the cooperative behavior in stock market is the cross-correlation coefficient between different companies. Since the stock prices changes depend on various economic environments, it is extremely hard to construct a

*E-mail: imkim@korea.ac.kr

dynamic equation, and predict the evolution of the stock price change in the future. Recently, there have been many efforts to understand the correlations in stock price changes between different companies using random matrix theories. In,[9,10] it was found that large eigenvalues for the cross-correlation matrix are located far away from the bulk part obeying the random matrix theory, reflecting that there exist cooperative behaviors of the entire market.

Let $Y_i(t)$ be the stock-price of a company i $(i = 1, \ldots, N)$ at time t. Then the return of the stock-price after a time interval Δt is defined as

$$S_i(t) = \ln Y_i(t + \Delta t) - \ln Y_i(t), \tag{1}$$

meaning the geometrical change of $Y_i(t)$ during the interval Δt. We take Δt as one day in the following analysis throughout this paper. The cross-correlations between individual stocks are considered in terms of the matrix $\boldsymbol{C}$, whose elements are given by

$$c_{i,j} \equiv \frac{\langle S_i S_j \rangle - \langle S_i \rangle \langle S_j \rangle}{\sqrt{(\langle S_i^2 \rangle - \langle S_i \rangle^2)(\langle S_j^2 \rangle - \langle S_j \rangle^2)}}, \tag{2}$$

where the brackets mean a temporal average over the period we studied. Then $c_{i,j}$ can vary between $[-1, 1]$. The case of $c_{i,j} = 1$ (-1) means that two companies i and j are completely correlated (anti-correlated), while $c_{i,j} = 0$ means that they are uncorrelated. Since the matrix $\boldsymbol{C}$ is symmetric and real, all eigenvalues are real, and the largest eigenvalue is not degenerate. It was found that the eigenvector corresponding to the largest eigenvalue is strongly localized at a few companies which strongly influence other companies in the stock price changes.[9,10]

In this paper, we consider the cross-correlations in stock-price changes between the S&P 500 companies during 5-year period 1993–1997 from the viewpoint of the WRG. The $N = 500$ companies correspond to 500 vertices, which are fully connected to each other through $N(N-1)/2$ edges. Each edge is assigned weight, $w_{i,j}$ $(i,j = 1, \ldots, N)$, slightly modified from the cross-correlation coefficient $c_{i,j}$. Before defining $w_{i,j}$ specifically, we first recall some properties exhibited by $c_{i,j}$. It is known that the distribution of the coefficients $\{c_{i,j}\}$ is of a bell-shape, and the mean value of the distribution depends on time, while the standard deviation remains as almost constant.[11] The time-dependent behavior of the mean value might be caused by external economic environments such as bank interest, inflation index, exchange rate, etc., which fluctuates from time to time. Thus we introduce a quantity,

$$G_i(t) = S_i(t) - \frac{1}{N} \sum_i S_i(t), \tag{3}$$

where $G_i(t)$ means the relative return of a company i to its mean value over the entire 500 companies at time t. The cross-correlation coefficients are redefined in terms of G_i as

$$w_{i,j} \equiv \frac{\langle G_i G_j \rangle - \langle G_i \rangle \langle G_j \rangle}{\sqrt{(\langle G_i^2 \rangle - \langle G_i \rangle^2)(\langle G_j^2 \rangle - \langle G_j \rangle^2)}}. \tag{4}$$

The cross-correlation coefficient $w_{i,j}$ is assigned to the edge connecting between vertices i and j. Note that $w_{i,j}$ is slightly different from other similar quantities defined in ref. 12 In

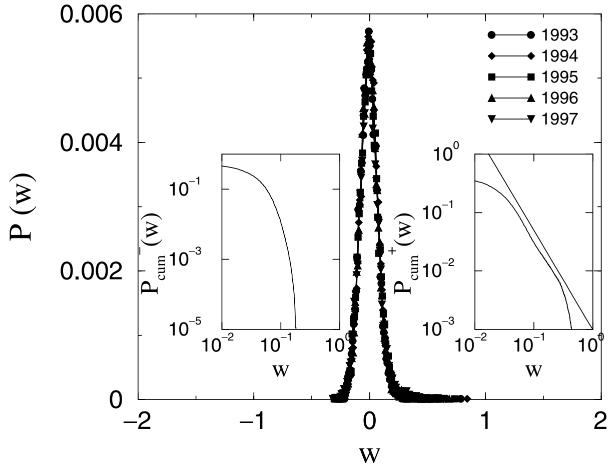

Fig. 1. Plot of the distribution of the cross-correlation coefficients $\{w_{i,j}\}$. The data are obtained by temporal averaging over each year from 1993 to 1997. Left (Right) inset: Plot of the distribution $P(w)$ for $w < 0$ $(w > 0)$ in a cummulative way versus w in double logarithmic scales. The solid line in the right inset is a guideline with slope -1.7.

order to check if the distribution $P(w)$ is time-independent, we take temporal average in eq. (4) over each year from 1993 to 1997. In Fig. 1, we plot the distributions of $\{w_{i,j}\}$ obtained for each year. The data for different years are indeed collapsed, confirming the time-independent behavior. Therefore the cross-correlation coefficients $\{w_{i,j}\}$ are generic for the cross-correlations among the S&P 500 companies. We find that the distribution function $P(w)$ follows a power-law $P^+(w) \sim w^{-1-\mu^+}$ with $\mu^+ \approx 1.7$ for $w > 0$, while $P^-(w)$ for $w < 0$ decays much faster than the power-law decay (the inset of Fig. 1). Those values are comparative to the previous measurements, $\mu^+ \approx 1.78$ for $w > 0$ and $\mu^- \approx 2.18$ for $w < 0$ for the distribution of the covariance of stock price changes defined slightly differently.[12] Since μ^+ is less than 2, the coefficients $\{w_{i,j}\}$ follow the Lévy distribution for $w > 0$.[13]

We introduce a physical quantity to measure how strongly a given vertex influences others, called the influence-strength (IS). Such a quantity is defined as the sum of the weights on the edges incident upon a given vertex i,

$$q_i = \sum_{j \neq i} w_{i,j}, \tag{5}$$

where j denotes the vertices connected to the vertex i. $\{w_{i,j}\}$ are obtained numerically by temporal averaging over the 5 years in eq. (4). Then the IS at a certain vertex i, q_i, is interpreted as the net influence for the company i to affect other companies in stock price changes. Since the weight $w_{i,j}$ is distributed in the range $[-1, 1]$, the IS at a certain vertex can be negative. Thus, we deal with the absolute magnitude of the IS for each vertex. In Fig. 2, we plot the IS distribution $P_I(|q|)$ in the absolute magnitude as a function of $|q|$, which turns out to follow a power-law, $P_I(|q|) \sim |q|^{-\eta}$. The exponent η is estimated to be $\eta \approx 1.8(1)$. Although the power-law regime is rather short due to the small size of the S&P 500 data, we expect it to be extended for larger system size. Thus the WRG for the cross-correlations in stock price changes exhibits the SF behavior. The fat-tail behavior in the IS distribution implies that there

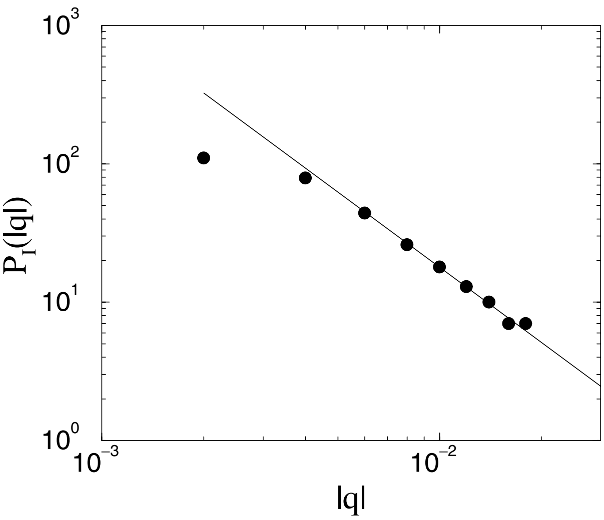

Fig. 2. Plot of the influence-strength distribution $P_I(|q|)$ versus the absolute magnitude of the influence-strength $|q|$ in double-logarithmic scales. The solid line is a guide line with slope -1.8.

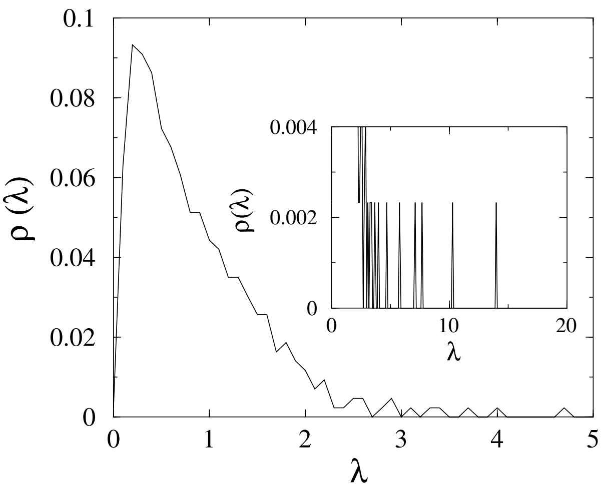

Fig. 3. Plot of the eigenvalue distribution of the matrix $\boldsymbol{W}$. Inset: The same plot as the main panel closed-up in the long-tail region.

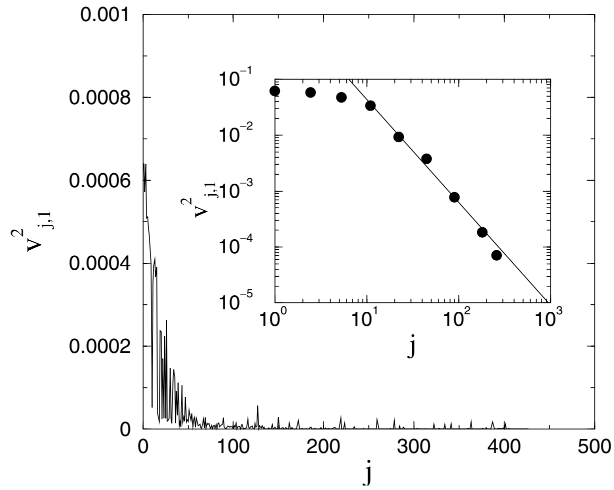

Fig. 4. Plot of the component-squares $v_{j,1}^2$ of the eigenfunction corresponding to the largest eigenvalue versus the vertex index j. The vertices are ordered following the absolute magnitudes of their influence-strengths. Inset: The same plot as the main panel but in double-logarithmic scales using log-bin. The solid line is a guid line representing $v_{j,1}^2 \sim j^{-1.8}$.

exist a few companies influencing strongly other companies in stock price dynamics. This result is reminiscent of that the eigenvector of the largest eigenvalue for the matrix, eq. (2) is strongly localized at a few companies. On the other hand, it is known that as the degree exponent is smaller in SF networks, the connectivity of a vertex with large degree becomes higher, so that the network tends to be more centralized to a few vertices. Since the IS exponent $\eta \approx 1.8$ is smaller than the degree exponent values for SF networks in real world, mostly in the range $2 < \gamma < 3$, the company with the largest IS among the S&P 500 companies plays a much more important role in affecting stock price changes compared with the magnitude of the role of the hub in the Internet[14] and the World-Wide Web[15] in transporting informations. We think that this result reflects that economic systems are more correlated and adaptive themselves to achieve high profits, so that each constituent is likely to behave following the vertex with the highest IS. In contrast, we can expect that a simple drop in the stock price occurring in one of the most influential companies can lead to a crash in the entire stock market, which is reminiscent of that SF networks are vulnerable to the attacks.[16]

We investigate the spectral property of the matrix $\boldsymbol{W}$ with the elements $\{w_{i,j}\}$. The density $\rho(\lambda)$ of the eigenvalues of the matrix $\boldsymbol{W}$ is shown in Fig. 3, which behaves similarly to that for the matrix $\boldsymbol{C}$.[9,10] However, it is found that the gap between the first and second largest eigenvalues for the matrix $\boldsymbol{W}$ is smaller by the factor 10 than the one for the matrix $\boldsymbol{C}$. Thus we think that the huge gap observed in the spectrum for the matrix $\boldsymbol{C}$ is caused by some external effects rather than the generic nature of the cross-correlations among the 500 companies. Next, we investigate the components $\{v_{j,1}\}$ of the eigenvector corresponding to the largest eigenvalue for the matrix $\boldsymbol{W}$, where the vertices are ordered following the absolute magnitude of influence-strength. According to the matrix theory, the square of each component $v_{j,1}^2$ means the relative contribution of the vertex j to the largest eigenvalue. As shown in the plot of $v_{j,1}^2$ versus the index j (Fig. 4), $\{v_{j,1}^2\}$ are strongly localized at the vertices with strong influence-strength (with small index j). Moreover, it is found that $v_{j,1}^2$ is scaled as $\sim j^{-1.8(2)}$ for large j (the inset of Fig. 4). The exponent value coincides with the influence-strength exponent η. Note that the corresponding eigencomponents behave $v_{j,1}^2 \sim j^{-1/(\gamma-1)}$ for SF networks where the index j is ordered according to the degree.[17,18] Therefore, the economic system is much localized at a few companies.

Finally, it would be interesting to compare our result with the tree structure constructed by the minimum spanning tree algorithm by Vandewalle *et al.*[19] They measured the degree distribution for this BRG, following a power-law, $P_D(k) \sim k^{-\gamma}$, with $\gamma \approx 2.2$, which is obviously different from our result obtained from the influence-strength, $\eta \approx 1.8(1)$.

In conclusions, we have considered the cross-correlations in stock price changes among the S&P 500 companies by introducing a weighted random graph (WRG). The vertices of the WRG representing the 500 companies are fully connected via weighted edges. The edge between vertices i and j has a weight given by the correlation coefficient $w_{i,j}$, defined as the normalized covariance of modified returns of the two companies i and j. Here the modified return means the return substracted by the mean over all companies. This modification yields the effect of excluding the overall behavior of the entire stock prices fluctuating from time to time. The distribution $P(w)$ of the correlation coefficients obtained in this way turns out to be time-independent, and the coefficients themselves describe generic correlations between different companies excluding the effect of external environments. It is found that the distribution $P(w)$ follows the Lévy distribution, i.e., $P(w) \sim 1/w^{1+\mu}$ with the index $\mu \approx 1.7$ for large $w > 0$. Next, we defined the influence-strength at each vertex as the sum of the weights assigned to the edges incident upon that vertex. It is found that the influence-strength distribution follows a power-law $P_{\mathrm{I}}(|q|) \sim |q|^{-\eta}$ with $\eta \approx 1.8(1)$, where q means influence-strength. The exponent η is close to the Lévy index μ. The fact that the exponent η is smaller than 2 implies that the stock price changes of the 500 companies are much strongly correlated, compared with the Internet topology, or the world-wide web, reflecting that cooperative and adaptive phenomena appear much dominantly in economic systems.

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