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Depinning of an anisotropic interface in random media: The tilt effect

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We study the tilt dependence of the pinning-depinning transition for an interface described by the anisotropic quenched Kardar-Parisi-Zhang equation in $2+1$ dimensions, where the two signs of the nonlinear terms are different from each other. When the substrate is tilted by m along the positive sign direction, the critical force $F_c(m)$ depends on m as $F_c(m)-F_c(0)\sim-|m|^{1.9(1)}$. The interface velocity v near the critical force follows the scaling form $v\sim|f|^{\theta}\Psi_{\pm}(m^2/|f|^{\theta+\phi})$ with $\theta=0.9(1)$ and $\phi=0.2(1)$, where $f\equiv F-F_c(0)$ and F is the driving force.

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The pinning-depinning (PD) transition from a pinned to a moving state is of interest due to its relevance to many physical systems. Typical examples include interface growth in porous (disordered) media under external pressure [1,2], dynamics of a domain wall under external field [3–5], dynamics of a charge density wave under external field [6,7], and vortex motion in superconductors under external current [8,9]. In the PD transition, there exists a critical value, F_c, of the external driving force F, such that for $F<F_c$ the interface (or charge, or vortex) is pinned by the disorder, while for $F>F_c$, it moves forward with a constant velocity v, leading to a transition across F_c. The velocity v plays the role of the order parameter and typically behaves as

$$v\sim(F-F_c)^{\theta}, \tag{1}$$

with the velocity exponent θ.

The interface dynamics in disordered media may be described via the Langevin-type continuum equation for the interface position $h(x,t)$,

$$\partial_t h(x,t)=\mathcal{K}[h]+F+\eta(x,h). \tag{2}$$

The first term on the right-hand side of Eq. (2) describes the configuration dependent force, the second is the external driving force, and the last, the quenched random noise, independent of time, describes the fluctuating force due to randomness or impurities in the medium. The random noise is assumed to have the properties, $\langle\eta(x,h)\rangle=0$ and $\langle\eta(x,h)\eta(x',h')\rangle=2D\delta^d(x-x')\delta(h-h')$, where the angular brackets represent the average over different realizations and d is the substrate dimension. When $\mathcal{K}[h]=\nu\nabla^2 h$, the resulting linear equation is called the quenched Edwards-Wilkinson equation (QEW) [10].

Recently, a couple of stochastic models mimicking the interface dynamics in disordered media have been introduced [11,12], displaying the PD transition different from the QEW universality behavior [13]. It was proposed that the models are described by the Kardar-Parisi-Zhang equation with quenched noise (QKPZ) [14], where

$$\mathcal{K}[h]=\nu\nabla^2 h+\frac{\lambda}{2}(\nabla h)^2. \tag{3}$$

The nonlinear term comes from the anisotropic nature of disordered medium, thus nonvanishing at the critical force F_c as shown by Tang *et al.* [15] in the context of vortex dynamics. When effective pinning force in the random impurity takes the form $(\Delta_h+s^2\Delta_x)^{2/3}/(1+s^2)^{2/3}$, where $s\equiv\partial_x h$ is the local slope and $\Delta_h^{1/2}$ and $\Delta_x^{1/2}$ are the amplitudes of random forces in the h and x directions, respectively, the nonlinear term is derived by expanding the random force in power of the slope s, leading to $\lambda\propto(\Delta_h-\Delta_x)$. Therefore, when $\Delta_h>\Delta_x(\Delta_h<\Delta_x)$ i.e., when the interface is driven along the hard (easy) direction, λ is positive (negative), and when the medium is isotropic, $\Delta_h=\Delta_x$, the nonlinear term vanishes. It has been shown [16,17] that the interface dynamics of the QKPZ equation depends on the sign of λ, in contrast to the thermal case where the sign is irrelevant [14].

The critical behavior of the PD transition for the QKPZ equation has been thoroughly studied in $1+1$ dimensions. For $\lambda>0$, the PD transition is continuous, and the interface at F_c is characterized in terms of the directed percolation (DP) cluster [18] spanning in the perpendicular-to-the-growth direction [11,12]. In this case, the effective nonlinear coefficient diverges as $\sim(F-F_c)^{-\phi}$ as $F\to F_c^+$, and the critical force F_c depends on the substrate-tilt m as

$$F_c(m)-F_c(0)\sim-|m|^{1/\nu(1-\alpha)}, \tag{4}$$

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where ν and α are the correlation length and the roughness exponent, respectively. These exponents are related to one another as [15]

$$\phi=2\nu(1-\alpha)-\theta. \tag{5}$$

For $\lambda<0$, the surface at F_c forms a facet with a characteristic slope s_c. The effective nonlinear coefficient is insensitive to F. When the substrate-tilt m is smaller than s_c, the PD transition is discontinuous, and F_c is independent of m, while the transition is continuous and F_c increases with m for $m>s_c$. The discontinuous transition is caused by the presence of a critical pinning force due to both the negative nonlinear term and random noise, which is localized. Once this pinning force is overcome by increasing external force F, the surface moves forward abruptly, yielding the velocity jump at F_c. The amount of the velocity jump decreases with increasing m, and vanishes at the characteristic tilt $m_c=s_c$. For $m>m_c$, the velocity increases continuously from zero, and the PD transition is continuous. Accordingly, the characteristic substrate-tilt m_c is a multicritical point [16,17].

Since the sign of the nonlinear term is relevant in the quenched case, it would be interesting to consider the case of the anisotropic QKPZ equation in $2+1$ dimensions, where the signs of the nonlinear terms are alternative. Thus, we consider

$$\partial_t h=\nu_x\partial_x^2 h+\nu_y\partial_y^2 h+\frac{\lambda_x}{2}(\partial_x h)^2+\frac{\lambda_y}{2}(\partial_y h)^2 +F+\eta(x,y,h), \tag{6}$$

where $\lambda_x>0$ and $\lambda_y<0$. The anisotropic case is in particular interesting due to its application to the vortex motion in disordered system [9] and the adatom motion on step edge in epitaxial surface [19]. It has been shown [19] that for the thermal case, the two nonlinear terms cancel each other effectively, thereby the anisotropic KPZ equation is reduced to the linear equation, the EW equation. For the quenched case, however, the interface dynamics in each direction are different from each other and the surface morphology is anisotropic: the surface is gently sloping in the positive-sign direction (x-direction), and is of the shape of a mountain range with steep slope in the negative-sign direction (y-direction). In spite of the facet shape in the negative-sign direction, the PD transition is continuous due to the critical behavior in the positive-sign direction. Consequently, one may expect that the critical force and the effective nonlinear coefficient along the positive-sign direction can be described by the scaling theory introduced for $\lambda>0$ in $1+1$ dimensions. In this brief report, we show, from extensive numerical simulations, that this is indeed the case and determine the scaling exponents ϕ and $\nu(1-\alpha)$ independently.

Direct numerical integration has been carried out using standard discretization techniques [20,21], in which we choose the parameters, $\nu_x=\nu_y=1$, $\lambda_x=-\lambda_y=1$, and a temporal increment $\Delta t=0.01$. The noise is discretized as $\eta(x,y,[h])$, where $[\cdots]$ means the integer part, and η is uniformly distributed in $[-a/2,a/2]$ with $a=(10)^{2/3}$. In order to consider the tilt-dependence, we tilted the substrate as $h(x,y,0)=mx$ along the positive-sign direction, and used the helicoidal boundary condition, $h(L+x,y,t)=h(x,y,t)+Lm$.

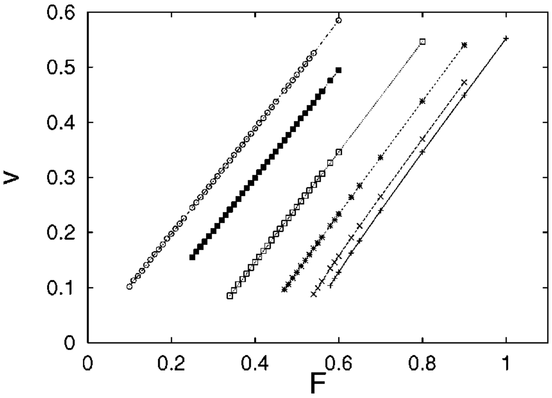

FIG. 1. The interface velocity as a function of F for various m. The curves correspond to the cases of $m=0.0$, 0.2, 0.4, 0.6, 0.8, and 0.9, from right to left.

We measured the growth velocity as a function of the external force F for several values of substrate-tilt m, which is shown in Fig. 1. For $m=0$, we found that $\theta=0.9(1)$ as in Ref. [16]. But for nonzero m, the exponent $\theta(m)$ is generically 1, different from its $m=0$ value [15]. This picture can be seen in Fig. 1, which shows straight lines for large m.

The critical force F_c is estimated as the maximum value of F for which all samples of 10 are pinned until large Monte Carlo steps, typically $10^5\Delta t$. In this way, we find $F_c(0)\approx 0.51(1)$. This value is slightly larger than that obtained by extrapolating the velocity curve, ≈ 0.50. Also, we estimated the critical force $F_c(m)$ as a function of the substrate-tilt m. The critical force $F_c(m)$ decreases with increasing substrate-tilt m with the exponent $1/\nu(1-\alpha)\approx 1.9(1)$ as shown in Fig. 2.

The PD transition is continuous due to the positive nonlinear term. To determine the exponent ϕ independently, we assume the scaling form for the interface velocity $v(F,m)$ as [13,15]

$$v\sim|f|^{\theta}\Psi_{\pm}\left(\frac{m^2}{|f|^{\theta+\phi}}\right). \tag{7}$$

Here, $f\equiv F-F_c(0)$ and the subscript $+(-)$ denotes the positive (negative) f branch. To be consistent with Eqs. (1) and (4) and the fact that $\theta(m)=1$ for $m\neq 0$, the scaling function should behave as

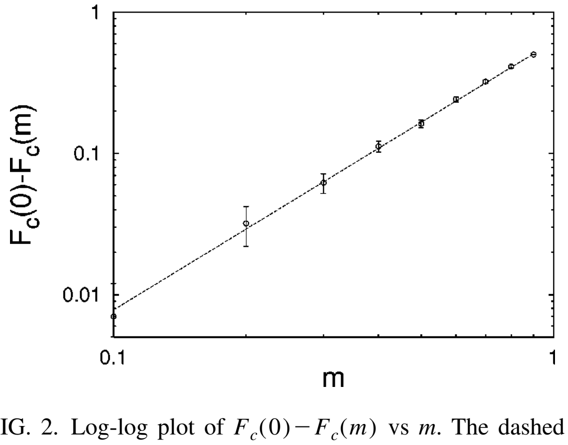

FIG. 2. Log-log plot of $F_c(0)-F_c(m)$ vs m. The dashed line has slope 1.9, drawn for the eye.

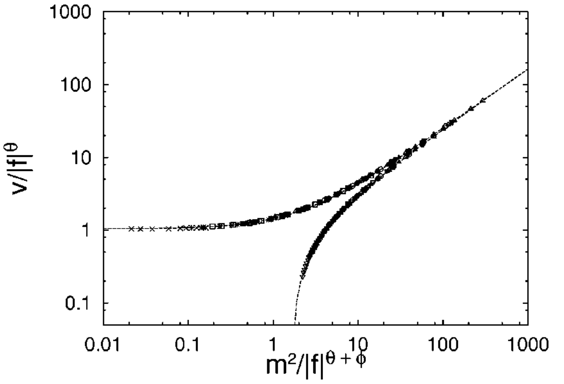

FIG. 3. Data collapse for the interface velocity using Eq. (7) with the exponents $\theta = 0.9(1)$ and $\phi = 0.2(1)$. The dashed lines are drawn for the approximate form of the scaling functions, Eqs. (8) and (9).

$$\Psi_{\pm}(x \to \infty) \sim x^{\theta/(\theta+\phi)},$$

$$\Psi_{+}(0^{+}) = \text{constant},$$

and for some positive constant x_0,

$$\Psi_{-}(x_0) = 0,$$

$$\Psi'_{-}(x_0) = \text{constant},$$

where the prime denotes the derivative with respect to x. We find that the velocity data $v(F,m)$ near the transition can be collapsed onto a single curve consistent with the scaling form. The best collapse is achieved with $F_c(0) \approx 0.50$ and the exponents $\theta \approx 0.9(1)$ and $\phi \approx 0.2(1)$, as shown in Fig. 3. From a simple fit to the data, we obtain an approximate functional form of the scaling function $\Psi_{\pm}(x)$ as

$$\Psi_{+}(x) \approx A(x+B)^{\theta/(\theta+\phi)}, \tag{8}$$

and

$$\Psi_{-}(x) \approx A(x^{\theta/(\theta+\phi)} - C), \tag{9}$$

with constants $A \approx 0.57(1)$, $B \approx 2.1(1)$, and $C = x_0^{\theta/(\theta+\phi)} \approx 1.5(1)$. Here we assume that $\Psi_{+}(x)$ is analytic at $x=0$. In Fig. 3, we also plot Eqs. (8) and (9) with dashed lines. The exponents thus obtained satisfy the scaling relation, Eq. (5), within the errors.

Alternatively, one may put the scaling form as

$$v \sim m^{2\theta/(\theta+\phi)} \Phi\left(\frac{f}{m^{2/(\theta+\phi)}}\right), \tag{10}$$

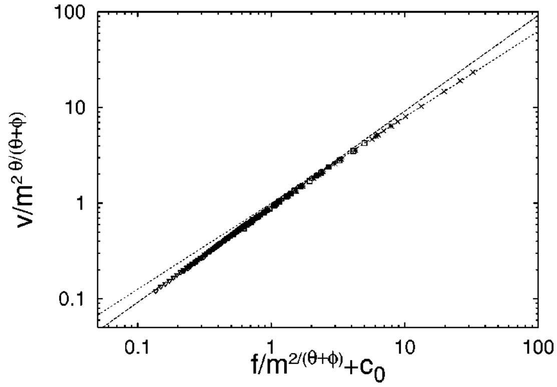

FIG. 4. Data collapse for the interface velocity using Eq. (10) with $\theta = 0.9(1)$ and $\phi = 0.2(1)$. The dotted (dashed) line has slope 0.9 (1.0), drawn for the eye.

with

$$\Phi(x \to \infty) \sim x^{\theta},$$

$$\Phi(-c_0) = 0,$$

$$\Phi'(-c_0) = \text{constant},$$

where the positive constant c_0 is related to x_0 via $c_0 = x_0^{-1/(\theta+\phi)}$. With this form, the data can be described by a single scaling function $\Phi(x)$. The scaling plot using the scaling form Eq. (10) is shown in Fig. 4. There, we have shifted the argument x of the scaling function $\Phi(x)$ by c_0 to make the argument positive. The slope of the log-log plot of the scaling function shows a crossover from 0.9 to 1.0 with increasing the substrate-tilt m, which confirms the fact that $\theta(m) = 1$ for $m \neq 0$.

In summary, we have investigated the critical behaviors of the tilted anisotropic QKPZ equation at the PD transition. The PD transition is continuous when it is tilted along the positive sign direction. It is shown that the data can be collapsed onto a single curve, with the exponents $\theta = 0.9(1)$ and $\phi = 0.2(1)$. The functional form of the scaling function is also numerically determined. We have measured, independently, the exponent which describes the variation of F_c with respect to m, and confirmed their consistency.

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