Dynamics of the orientational roughening transition

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We use the renormalization-group method to study the dynamics of the sine-Gordon model for the orientational roughening transition. Implications of our results for spatial and temporal behavior of surfaces are discussed.

I. INTRODUCTION

The problems of the static and dynamic behavior of surface transitions have drawn much attention in recent years.[1-4] The roughening, wetting, and reconstruction transitions occurring in the interfaces between crystals and vacuums or between solids and fluids are typical examples of the surface phase transitions. The study of the roughening transition was carried out through the Coulomb gas model or the sine-Gordon model, and it turned out that the roughening transition belongs to a different universality class, the Kosterlitz-Thouless transition. The roughening transition arising both in the Coulomb gas model and in the sine-Gordon model is mainly caused by the two competing effects: the unbinding effect caused by the surface tension energy and the binding effect caused by the lattice structure.

However, in tensionless surfaces like membranes and amphiphilic films, the surface curvature energy plays a much more important role than surface tension in surface transitions.[5,6] In this case, a surface undergoes two successive roughening transitions. Upon increasing temperature, the first transition occurs in which the surface becomes rough by losing the long-range transitional order, but there still remains the quasi-long-range orientational order. At the second transition occurring at the higher temperature, the surface becomes completely rough by losing the quasi-long-range orientational order. Thus this transition is called the orientational roughening (OR) transition,[7-10] or the Laplacian roughening transition, because the curvature energy is proportional to $(\nabla^2\phi)^2$. The OR transition was discovered by Nelson in the study of the two-dimensional melting phenomenon.[7,8] In the original work, the OR transition was studied by introducing the *vector* Coulomb gas model, while the ordinary roughening transition was studied by the *scalar* Coulomb gas model. In the vector Coulomb gas model, the tilt-tilt correlation of the surface height is understood via the Coulomb interaction between *electrical dipole moments,* while in the scalar Coulomb gas model, the height-height correlation of the surface corresponds to the Coulomb interaction between *electrical charges.*

On the other hand, very recently Levin and Dawson[9] developed the sine-Gordon model for the OR transition, which is analogous to the sine-Gordon model for the ordinary roughening transition except for the surface height by the surface normal. They studied the sine-Gordon model through renormalization group (RG) analysis, which produces the same recursion relations as those derived from the vector Coulomb gas model. Thus it is believed that the sine-Gordon model is equivalent to the vector Coulomb gas model within the RG scheme. In addition, the benefit of the sine-Gordon approach enables us to study the dynamic property of the OR transition by using the Langevin equation, but this study has not been carried out yet. In this paper, we shall study the dynamics of the OR transition.

The Hamiltonian of the sine-Gordon model for the OR transition introduced by Levin and Dawson[9] is written as

$$\mathcal{H}=-\frac{1}{2}\kappa\int d^2r[\nabla^2\phi(\mathbf{r})]^2+2y\sum_{k=1}^{3}\int d^2r\cos[\hat{\mathbf{e}}_k\cdot\nabla\phi(\mathbf{r})]\ , \tag{1}$$

where $\phi(\mathbf{r})$ means the height of the interface at position $\mathbf{r}$ on the substrate and $\{\hat{\mathbf{e}}_k\}$ are the basis vectors of the triangular lattice,

$$\{\hat{\mathbf{e}}_k\}=(1,0),\ \left(-\frac{1}{2},\frac{\sqrt{3}}{2}\right),\ \left(-\frac{1}{2},-\frac{\sqrt{3}}{2}\right). \tag{2}$$

These unit vectors satisfy the important rule that $\hat{\mathbf{e}}_1+\hat{\mathbf{e}}_2+\hat{\mathbf{e}}_3=0$, which shall be used later. The dynamic equation from this static Hamiltonian can be obtained via the Langevin equation

$$\eta\frac{\partial\phi(\mathbf{r},t)}{\partial t}=-\Gamma\frac{\delta\mathcal{H}}{\delta\phi}+R(\mathbf{r},t)\ , \tag{3}$$

where η is the friction coefficient and $R(\mathbf{r},t)$ is the thermal random noise satisfying

$$\langle R_k(t)R_{k'}(t')\rangle=G_k\delta(t-t')\ . \tag{4}$$

The solution of the Langevin equation, after a transient period, produces the surface configurations which follow the canonical distribution; the probability for a given height profile $\phi(\mathbf{r})$ is given by

$$P(\{\phi(\mathbf{r})\})\sim\exp[-(1/kT)\mathcal{H}(\phi)]\ . \tag{5}$$

Using the Langevin equation (3), we shall derive the dynamic equation for the OR transition, and the analysis of it shall be performed by using the dynamic renormalization-group method introduced by Nozières and Gallet.[11] This method is especially useful for studying the sine-Gordon type model. We obtain the recursion relations of the dynamic RG transformation, which reduce to the static ones in the limit of the sharp cutoff and under the large-distance approximation. Thus we can say that the equilibrium dynamics are conservative. The reduction enables us to confirm that the dynamic equation is correct. In the dynamic equation, the term derived from the pinning potential is introduced in this paper. This term should be helpful in understanding the orientational order in the study of the epitaxial growth of a crystalline film. It is found that the tilt-tilt correlation function diverges logarithmically in space and time above the OR transition temperature, which makes it likely that the height-height correlation function diverges logarithmically in the roughening transition.[11,12] This paper is organized as follows: we shall first derive the dynamic equation for the OR transition, and shall analyze it by using the dynamic renormalization-group technique in Sec. II. Section III will be devoted to conclusions and discussions.

II. THE DYNAMIC RENORMALIZATION-GROUP ANALYSIS

Let us begin this section by deriving the dynamic equation of the OR transition. We first perform the integration by part of Eq. (1), and insert that into Eq. (3). Then after redefining the coefficients, the dynamic equation is obtained as

$$\eta\frac{\partial\phi(\mathbf{r},t)}{\partial t}=-\kappa\nabla^4\phi-2y\sum_{k=1}^{3}(\hat{\mathbf{e}}_k\cdot\nabla)\sin(\hat{\mathbf{e}}_k\cdot\nabla\phi(\mathbf{r},t))+R(\mathbf{r},t)\ , \tag{6}$$

where R satisfies the condition, Eq. (4). The dynamic equation can be solved exactly when $y=0$. In this case, it is straightforward to obtain the solution

$$\phi(r,t)=\int_{-\infty}^{t}dt'\int d^2r'\chi_0(r-r',t-t')R(r',t')\ , \tag{7}$$

where χ_0 is the diffusion response function,

$$\chi_0(r,t)=\frac{1}{\eta}\int d^2k\,\exp\left[-\frac{\kappa k^4 t}{\eta}+i\mathbf{k}\cdot\mathbf{r}\right]\ . \tag{8}$$

But the closed form of $\chi_0(r,t)$ cannot be obtained exactly due to the non-Gaussian property of Eq. (8).

However, when $y\neq 0$, an exact solution is not possible. Thus we invoke the dynamic RG method to study the time-dependent behavior. The dynamic RG analysis shall follow a perturbative RG method introduced by Nozières and Gallet.[9] Originally the method was applied to the problem of the dynamics of the roughening transition, and the detailed calculations of it were presented. Since the basic scheme of the RG calculations for the OR transition is the same as that which appeared in the original paper, we will present the essential steps of the dynamic RG transformations in this paper. First, we split R into two parts, $R=\bar{R}+\delta R$ which are statistically independent. Here we may regard δR as the effect of the short wavelength, rapidly varying degrees of freedom left out of the coarse-grained description. Thus we shall take the average over δR. For comparison, the standard dynamic RG transformation[13] is carried out by eliminating fluctuations in the momentum shell $\Lambda/b<k<\Lambda$ (with Λ the momentum cutoff), while the current method is done by doing the random force $\mathbf{R}(k)$. Accordingly, the noise spectrum is readily divided into $G_k=\bar{G}_k+\delta G_k$, contributed by $\bar{\mathbf{R}}$ and $\delta\mathbf{R}$, respectively. Moreover, we obtain the average height $\bar{\phi}=\langle\phi(\bar{\mathbf{R}}+\delta\mathbf{R})\rangle_{\delta\mathbf{R}}$ by performing a partial average over $\delta\mathbf{R}$, defining $\delta\phi\equiv\phi-\bar{\phi}$. Then the dynamic equations of $\bar{\phi}$ and $\delta\phi$ are

$$\eta\frac{\partial\bar{\phi}}{\partial t}=-\kappa\nabla^4\bar{\phi}-2y\left\langle\sum_{k=1}^{3}(\hat{\mathbf{e}}_k\cdot\nabla)\sin(\hat{\mathbf{e}}_k\cdot\nabla\phi)\right\rangle+\bar{R}\ , \tag{9a}$$

$$\eta\frac{\partial\delta\phi}{\partial t}=-\kappa\nabla^4\delta\phi-2y\left[\sum_{k=1}^{3}(\hat{\mathbf{e}}_k\cdot\nabla)\sin(\hat{\mathbf{e}}_k\cdot\nabla\phi)-\left\langle\sum_{k=1}^{3}(\hat{\mathbf{e}}_k\cdot\nabla)\sin(\hat{\mathbf{e}}_k\cdot\nabla\phi)\right\rangle\right]+\delta R\ , \tag{9b}$$

where the bracket $\langle\ \rangle$ means the average over δR. We first solve for $\delta\phi$ by iterating and expanding in powers of y. In the zeroth order, the solution is

$$\delta\phi^{(0)}(r,t)=\int d^2r'\,dt'\chi_0(r-r',t-t')\delta R(r',t')\ . \tag{10}$$

In the first order, the solution is

$$\delta\phi^{(1)}=-2y\int d^2r'dt'\chi_0(r-r',t-t')\left[\sum_{k=1}^{3}(\hat{\mathbf{e}}_k\cdot\nabla)\sin(\hat{\mathbf{e}}_k\cdot\nabla\phi+\hat{\mathbf{e}}_k\cdot\nabla\delta\phi)-\left\langle\sum_{k=1}^{3}(\hat{\mathbf{e}}_k\cdot\nabla)\sin(\hat{\mathbf{e}}_k\cdot\nabla\phi+\hat{\mathbf{e}}_k\cdot\nabla\delta\phi)\right\rangle\right]\ . \tag{11}$$

Inserting $\delta\phi=\delta\phi^{(0)}+\delta\phi^{(1)}$ into Eq. (9a), we obtain the following equation, up to the order $\mathcal{O}(y^2)$:

$$\eta\frac{\partial\bar{\phi}(r)}{\partial t}=-\kappa\nabla^4\bar{\phi}(r)-2y\sum_{k=1}^{3}(\hat{\mathbf{e}}_k\cdot\nabla)\sin(\hat{\mathbf{e}}_k\cdot\nabla\phi)e^{-(1/2)\delta g_{kk}(0,0)}+Y(y^2)+K(y^2)+\bar{R}\ , \tag{12}$$

where

$$Y(y^2)=-2y^2\sum_{i,j}e^{-(1/2)\delta g_{ii}(0,0)-(1/2)\delta g_{jj}(0,0)}(\hat{\mathbf{e}}_i\cdot\nabla_\tau)\int d^2r'\,dt'[(\hat{\mathbf{e}}_i\cdot\nabla_\tau)(\hat{\mathbf{e}}_j\cdot\nabla_\tau)\chi_0(r-r',t-t')]\times\{\sin[\hat{\mathbf{e}}_i\cdot\nabla_\tau\phi(r,t)+\hat{\mathbf{e}}_j\cdot\nabla_\tau\phi(r',t')](e^{-\delta g_{ij}(\rho,\tau)}-1)\} \tag{13}$$

and

$$K(y^2)=2y^2\sum_{i,j}e^{-(1/2)\delta g_{ii}(0,0)-(1/2)\delta g_{jj}(0,0)}(\hat{\mathbf{e}}_i\cdot\nabla_\tau)\int d^2r'\,dt'[(\hat{\mathbf{e}}_i\cdot\nabla_\tau)(\hat{\mathbf{e}}_j\cdot\nabla_\tau)\chi_0(r-r',t-t')]$$
$$\times\{\sin[\hat{\mathbf{e}}_i\cdot\nabla_\tau\phi(r,t)-\hat{\mathbf{e}}_j\cdot\nabla_{\tau'}\phi(r',t')](e^{\delta g_{ij}(\rho,\tau)}-1)\}\ . \tag{14}$$

In order to derive the above equations, we have performed integration by part and have used the property $\nabla_{\tau'}\chi_0=-\nabla_\tau\chi_0$. In the above equations, the correlation function $\delta g_{ij}(\rho,\tau)$ is defined as

$$\delta g_{ij}(\rho,\tau)\equiv\langle(\hat{\mathbf{e}}_i\cdot\nabla_\tau)\delta\phi^{(0)}(r,t)(\hat{\mathbf{e}}_j\cdot\nabla_{\tau'})\delta\phi^{(0)}(r',t')\rangle,$$
$$=\frac{1}{4\pi^2\kappa}\int\frac{(\hat{\mathbf{e}}_i\cdot\mathbf{q})(\hat{\mathbf{e}}_j\cdot\mathbf{q})e^{i\mathbf{q}\cdot\rho-\kappa q^4\tau/\eta}}{q^4}q\frac{df(q)}{dq}d^2q \tag{15}$$

where $f(q)$ is a cutoff function and $\rho=r-r'$ and $\tau=t-t'$. The second-order term $Y(y^2)$ shall contribute to the renormalized correction Δy, and $K(y^2)$ shall do the same to $\Delta\kappa$ and $\Delta\eta$ later. The derivations of the renormalized corrections are based on the Kadanoff's operator product expansion method,[14] and are presented in Appendix A. After the rescaling, $\Lambda\to(1-\epsilon)\Lambda$ with $\Lambda=1$ and $\rho\to(1+\epsilon)\rho$, the recursion relations of the RG transformation in the differential form are

$$\frac{dy}{d\epsilon}=\left[2-\frac{c}{2}\right]y+2y^2\sum_{\substack{i<j\\ i+j=-k}}\int d^2\rho\,d\tau[(\hat{\mathbf{e}}_i\cdot\nabla)(\hat{\mathbf{e}}_j\cdot\nabla)\chi_0(\rho,\tau)]\rho^{1+\hat{\mathbf{e}}_i\cdot\hat{\mathbf{e}}_j/4\pi\kappa}\frac{d}{d\rho}\left[\exp\left[-V_{ij}(\rho,x)-\frac{\hat{\mathbf{e}}_i\cdot\hat{\mathbf{e}}_j}{4\pi\kappa}\ln\rho\right]\right]\ , \tag{16}$$

$$\frac{d\kappa}{d\epsilon}=2y^2\sum_i\int d^2\rho\,d\tau[(\hat{\mathbf{e}}_i\cdot\nabla_\rho)^2\chi_0(\rho,\tau)]\rho^{3-1/4\pi\kappa}\frac{d}{d\rho}\left[\exp\left[V_{ii}(\rho,x)+\frac{1}{4\pi\kappa}\ln\rho\right](\hat{\rho}\cdot\hat{\mathbf{n}})^2(\hat{\mathbf{e}}_i\cdot\hat{\mathbf{n}})^2\right]\ , \tag{17}$$

$$\frac{d\eta}{d\epsilon}=2y^2\sum_i\int d^2\rho\,d\tau\,\tau[(\hat{\mathbf{e}}_i\cdot\nabla_\rho)^4\chi_0(\rho,\tau)]\rho^{1-1/4\pi\kappa}\frac{d}{d\rho}\left[\exp\left[V_{ii}(\rho,x)+\frac{1}{4\pi\kappa}\ln\rho\right]\right]\ , \tag{18}$$

where c is a positive function of κ defined as

$$c\equiv-\frac{1}{4\pi\kappa}\int dq\frac{1}{q}\frac{df}{dq}\ , \tag{19}$$

and x is a dimensionless parameter, $x=\kappa\tau/\eta\rho^4$, which replaces τ. The derivation of the recursion relations appear in Appendix A. We note that this result is exact within the second order of y.

The above recursion relations look much more complicated than those obtained from the static Hamiltonian, due to the differential kernel. But the recursion relations Eqs. (16), (17) for the static variables y and κ reduce to those derived from the static Hamiltonian in the limit of the sharp cutoff and under the large-distance approximation. The detailed derivation of it is presented in Appendix B. Within those conditions, the recursion relations for the static variables after performing the angular part integration are

$$\frac{dy}{d\epsilon}=\left[2-\frac{c}{2}\right]y+2\pi y^2\int d\rho\,\rho^{2-1/8\pi\kappa}\frac{d}{d\rho}\times\left[\exp\left[\frac{1}{2}A+\frac{1}{8\pi\kappa}\ln\rho\right]\times I_0(B)\right]\ , \tag{20}$$

$$\frac{d\kappa}{d\epsilon}=\frac{3}{2}\pi y^2\int d\rho\,\rho^{4-1/4\pi\kappa}\frac{d}{d\rho}\left[\exp\left[A+\frac{1}{4\pi\kappa}\ln\rho\right]\times\left[I_0(B)-\frac{1}{2}I_1(B)\right]\right]\ , \tag{21}$$

where c was defined in Eq. (19). Especially at the transition temperature $\kappa=1/16\pi$, the flow equations become

$$\frac{dy}{d\epsilon}=\left[2-\frac{1}{8\pi\kappa}\right]y+2\pi y^2 I_0(2)\ , \tag{22}$$

$$\frac{d\kappa}{d\epsilon}=\frac{3}{2}y^2\left[I_0(2)-\frac{1}{2}I_1(2)\right]\ . \tag{23}$$

Equations (20)–(23) are equivalent to the ones derived from the static Hamiltonian.[9]

So far we have derived the recursion relations and have shown that the static variables reduce to the static case. Thus the dynamics are conservative. The reduction implies indirectly that the dynamic equation is correct. On the other hand, the recursion relation for the dynamic variable η remains complicated due to nonintegrability over x. But it is obvious that Eq. (18) can be written simply as

$$\frac{d\ln\eta}{d\epsilon}=2y^2D(\kappa)\ , \tag{24}$$

where $D(\kappa)$ is a positive function of κ. Thus as $y\to 0$, the friction coefficient $\eta\to 0$ undergoes the RG transformation.

III. CONCLUSIONS AND DISCUSSIONS

We have derived the dynamic equation for the OR transition from the static Hamiltonian via the Langevin equation. In the static OR transition, the tilt-tilt correlation function of the surface height is known to diverge as

$$G(r)\equiv\langle[\nabla\phi(r)-\nabla\phi(0)]^2\rangle\sim\ln r\ , \tag{25}$$

where r is less than the correlation length ξ.[7,8] The correlation length diverges as $\ln\xi\sim|T-T_c|^{-\nu}$ with $\nu=0.3696$. The value of ν is different from the value of the ordinary roughening transition $\nu=1/2$. For the dynamics, we note that the recursion relations Eqs. (22)–(24) have the familiar look of the sine-Gordon model for the roughening transition. The dynamic variable goes to zero as $y\to 0$ above the OR transition point. Thus the dynamics of the OR transition are conventional[11,12] and the dynamic behavior of the OR transition is similar to the roughening transition, which means that the tilt-tilt correlation is logarithmically divergent in space and time above the transition temperature,

$$G(r,t)\equiv\langle[\nabla\phi(r,t)-\nabla\phi(0,0)]^2\rangle\sim\ln t \tag{26}$$

for $t<r^4$ and it is saturated as $G(r,t)\sim\ln r$ for $t>r^4$. Below the transition temperature, it decays exponentially.

Recently there have been many studies in surface growth, which are either conservative or nonconservative.[15] Among them, many growth models have been introduced to study the epitaxial growth of a crystalline film. In those models, in order to consider the effect of the surface diffusion of adatoms in the molecular beam deposition, recently many authors[16] introduced dynamic equations which commonly include $\nabla^4\phi$. The growth driven by $\nabla^4\phi$ has the orientational order. In this case, we have shown in this paper that the relevant term reflecting the lattice pinning potential is of the form

$$\sum_k(\hat{\mathbf{e}}_k\cdot\nabla)\sin(\hat{\mathbf{e}}_k\cdot\nabla\phi(r,t))\ .$$

Thus it would be interesting to study nonlinear dynamic equations including the flux of the pinning potential term, such as

$$\eta\frac{\partial\phi(\mathbf{r},t)}{\partial t}=-\kappa\nabla^4\phi-2y\sum_{k=1}^{3}(\hat{\mathbf{e}}_k\cdot\nabla)\sin(\hat{\mathbf{e}}_k\cdot\nabla\phi(\mathbf{r},t))+\frac{\lambda}{2}(\nabla\phi(\mathbf{r},t))^2+R(\mathbf{r},t)\ . \tag{27}$$

In summary, we have derived the conservative dynamic equation for the orientational roughening transition through the Langevin equation from the sine-Gordon Hamiltonian. In the dynamic equation, the term due to "lattice pinning potential" has been introduced. This term plays an important role in the OR transition in crystalline film. Through the dynamic RG transformation, it is found that the dynamics are conventional and reduce to the harmonic case above the OR transition point. The tilt-tilt correlation function is logarithmically divergent in space and time above the OR transition point, and the characteristic time is proportional to the system size as L is to L^4.

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APPENDIX A: DERIVATIONS OF RECURSION RELATIONS

We begin with $Y(y^2)$ to derive the renormalized correction Δy. Let us first consider the following replacement as $r\to r'$ and $t\to t'$:

$$\sin[\hat{\mathbf{e}}_i\cdot\nabla\phi(r,t)+\hat{\mathbf{e}}_j\cdot\nabla\phi(r',t')]\to a(r-r',t-t')\sin(\hat{\mathbf{e}}_k\cdot\nabla\phi(r,t))+\text{irrelevant part}\ , \tag{A1}$$

where the indices $\{i,j,k\}$ are cyclic with $\hat{\mathbf{e}}_i+\hat{\mathbf{e}}_j=-\hat{\mathbf{e}}_k$. The replacement is based on the Kadanoff's operator product expansion, which yields

$$a(\rho,\tau)=e^{-V_{ij}(\rho,\tau)}\ , \tag{A2}$$

where $V_{ij}(\rho,\tau)$ was given in Eq. (19). Thus $Y(y^2)$ becomes

$$Y(y^2)=2y^2\sum_k\sum_{\substack{i,j\\ i+j=-k}}e^{-(1/2)\delta g_{ii}(0,0)-(1/2)\delta g_{jj}(0,0)}(\hat{\mathbf{e}}_i\cdot\nabla_\tau)\sin(\hat{\mathbf{e}}_k\cdot\nabla\phi(r,t))\times\int d^2\rho\,d\tau[(\hat{\mathbf{e}}_i\cdot\nabla_\rho)(\hat{\mathbf{e}}_j\cdot\nabla_\rho)\chi_0(\rho,\tau)](e^{-\delta g_{ij}(\rho,\tau)}-1)e^{-V_{ij}(\rho,\tau)}\ . \tag{A3}$$

Using the property $\hat{\mathbf{e}}_i+\hat{\mathbf{e}}_j=-\hat{\mathbf{e}}_k$, we obtain

$$Y(y^2)=2y^2\sum_k(\hat{\mathbf{e}}_k\cdot\nabla_\tau)\sin(\hat{\mathbf{e}}_k\cdot\nabla\phi(r,t))\sum_{\substack{i<j\\ i+j=-k}} e^{-(1/2)\delta g_{ii}(0,0)-(1/2)\delta g_{jj}(0,0)}$$
$$\times\int d^2\rho\, d\tau[(\hat{\mathbf{e}}_i\cdot\nabla)(\hat{\mathbf{e}}_j\cdot\nabla)\chi_0(\rho,\tau)](e^{-\delta g_{ij}(\rho,\tau)}-1)e^{-V_{ij}(\rho,\tau)}\ . \tag{A4}$$

Thus

$$\Delta y=2y^2e^{-(1/2)\delta g_{ii}(0,0)-(1/2)\delta g_{jj}(0,0)}\int d^2\rho\, d\tau[(\hat{\mathbf{e}}_i\cdot\nabla)(\hat{\mathbf{e}}_j\cdot\nabla)\chi_0(\rho,\tau)](e^{-\delta g_{ij}(\rho,\tau)}-1)e^{-V_{ij}(\rho,\tau)} \tag{A5}$$

for the index k.

Next let us derive the renormalized corrections $\Delta\gamma$ and $\Delta\eta$ from $K(y^2)$. To proceed, we again consider the following replacement as $\rho\to 0$ and $\tau\to 0$:

$$\sin[\hat{\mathbf{e}}_i\cdot\nabla_r\phi(r,t)-\hat{\mathbf{e}}_j\cdot\nabla_{r'}\phi(r',t')]\to[\hat{\mathbf{e}}_i\cdot\nabla_r\phi(r,t)-\hat{\mathbf{e}}_j\cdot\nabla_{r'}\phi(r',t')]e^{V_{ij}(\rho,\tau)}+\text{irrelevant parts}\ . \tag{A6}$$

This derivation is also based on the Kadanoff's operator product expansion, which was presented in Refs. 9 and 14. When $i=j$, we expand $\hat{\mathbf{e}}_j\cdot\nabla_{r'}\phi(r',t')$ with respect to $\hat{\mathbf{e}}_i\cdot\nabla_r\phi(r,t)$. Then $K(y^2)$ becomes

$$K(y^2)=2y^2\sum_i e^{-\delta g_{ii}(0,0)}\int d^2\rho d\tau[(\hat{\mathbf{e}}_i\cdot\nabla_\rho)^2\chi_0(\rho,\tau)](e^{\delta g_{ii}(\rho,\tau)}-1)e^{V_{ii}(\rho,\tau)}(\hat{\mathbf{e}}_i\cdot\nabla_r)^2\frac{1}{2}(\rho\cdot\nabla_r)^2\phi(r,t)$$
$$+2y^2\sum_i e^{-\delta g_{ii}(0,0)}\int d^2\rho\, d\tau\,\tau[(\hat{\mathbf{e}}_i\cdot\nabla_\rho)^4\chi_0(\rho,\tau)](e^{\delta g_{ii}(\rho,\tau)}-1)e^{V_{ii}(\rho,\tau)}\frac{\partial\phi}{\partial t}\ . \tag{A7}$$

The second part of Eq. (A6) was obtained after performing integrations by part twice. Thus $\Delta\kappa$ becomes

$$\Delta\kappa=2y^2\sum_i e^{-\delta g_{ii}(0,0)}\int d^2\rho\, d\tau[(\hat{\mathbf{e}}_i\cdot\nabla_\rho)^2\chi_0(\rho,\tau)](e^{\delta g_{ii}(\rho,\tau)}-1)e^{V_{ii}(\rho,\tau)}\frac{1}{2}(\rho\cdot\hat{\mathbf{n}})^2(\hat{\mathbf{e}}_i\cdot\hat{\mathbf{n}})^2\ , \tag{A8}$$

where $\hat{\mathbf{n}}=\nabla\phi(r,t)/|\nabla\phi(r,t)|$, and $\Delta\eta$ becomes

$$\Delta\eta=2y^2\sum_i e^{-\delta g_{ii}(0,0)}\int d^2\rho\, d\tau\,\tau[(\hat{\mathbf{e}}_i\cdot\nabla_\rho)^4\chi_0(\rho,\tau)](e^{\delta g_{ii}(\rho,\tau)}-1)e^{V_{ii}(\rho,\tau)}\ . \tag{A9}$$

Next, as the final step of the dynamic RG transformation, the variables are rescaled: $k\to(1-\epsilon)k,\rho\to(1+\epsilon)\rho$. Then it is easily known that δg_{ij} is of the order $\mathcal{O}(\epsilon)$. Since δg is small, the exponential function of δg can be expanded, and then Eqs. (A5), (A8), and (A9) become much simpler. Furthermore, δg_{ij} has the following property when the parameter τ is replaced by $x=\kappa\tau/\eta q^4$:

$$\delta g_{ij}=-\frac{\hat{\mathbf{e}}_i\cdot\hat{\mathbf{e}}_j}{4\pi\kappa}-\rho\frac{\partial V_{ij}(\rho,x)}{\partial\rho}\ . \tag{A10}$$

Then the recursion relations of the RG transformation in the differential form are

$$\frac{dy}{d\epsilon}=\left[2-\frac{c}{2}\right]y+2y^2\sum_{\substack{i<j\\ i+j=-k}}\int d^2\rho\, d\tau[(\hat{\mathbf{e}}_i\cdot\nabla)(\hat{\mathbf{e}}_j\cdot\nabla)\chi_0(\rho,\tau)]\rho^{1+\hat{\mathbf{e}}_i\cdot\hat{\mathbf{e}}_j/4\pi\kappa}\frac{d}{d\rho}\left[\exp\left[-V_{ij}(\rho,x)-\frac{\hat{\mathbf{e}}_i\cdot\hat{\mathbf{e}}_j}{4\pi\kappa}\ln\rho\right]\right]\ , \tag{A11}$$

$$\frac{d\kappa}{d\epsilon}=2y^2\sum_i\int d^2\rho\, d\tau[(\hat{\mathbf{e}}_i\cdot\nabla_\rho)^2\chi_0(\rho,\tau)]\rho^{3-1/4\pi\kappa}\frac{d}{d\rho}\left[\exp\left[V_{ii}(\rho,x)+\frac{1}{4\pi\kappa}\ln\rho\right](\hat{\rho}\cdot\hat{\mathbf{n}})^2(\hat{\mathbf{e}}_i\cdot\hat{\mathbf{n}})^2\right]\ , \tag{A12}$$

$$\frac{d\eta}{d\epsilon}=2y^2\sum_i\int d^2\rho\, d\tau\,\tau[(\hat{\mathbf{e}}_i\cdot\nabla_\rho)^4\chi_0(\rho,\tau)]\rho^{1-1/4\pi\kappa}\frac{d}{d\rho}\left[\exp\left[V_{ii}(\rho,x)+\frac{1}{4\pi\kappa}\ln\rho\right]\right]\ , \tag{A13}$$

where c is a positive function of κ defined as

$$c\equiv-\frac{1}{4\pi\kappa}\int dq\frac{1}{q}\frac{df}{dq}\ . \tag{A14}$$

APPENDIX B: THE REDUCTION TO THE STATIC CASE

First we will show that when the sharp cutoff $f(q)=\theta(1-q)$ and the large-ρ approximation are used, the differential Green function, $[(\hat{\mathbf{e}}_i\cdot\nabla)(\hat{\mathbf{e}}_j\cdot\nabla)\chi_0(\rho,\tau)]$, becomes

$$(\hat{\mathbf{e}}_i\cdot\nabla)(\hat{\mathbf{e}}_j\cdot\nabla)\chi_0(\rho,\tau)|_{\tau=x}=\frac{\kappa}{\eta\rho^4}\frac{dV_{ij}(\rho,x)}{dx}\ , \tag{B1}$$

with $x=\kappa\tau/\eta\rho^4$.

The function $V_{ij}(\rho,x)$ can be rewritten with the functions $A(\rho,x)$ and $B(\rho,x)$ as

$$V_{ij}(\rho,x)=-\frac{1}{2}A(\rho,x)-2(\hat{\mathbf{e}}_i\cdot\hat{\rho})\times(\hat{\mathbf{e}}_j\cdot\hat{\rho})B(\rho,x)-\frac{1}{2}B(\rho,x)\ , \tag{B2}$$

where A and B are defined in Eqs. (22) and (23). But when the sharp cutoff and the large-ρ approximation are used, A and B become

$$A\equiv\frac{1}{4\pi\kappa}\int_0^\infty dy\,[J_0(y)e^{-y^4x}-1]\frac{1}{y}\ , \tag{B3}$$

$$B\equiv\frac{1}{4\pi\kappa}\int_0^\infty dy\,J_2(y)e^{-y^4x}\frac{1}{y}\ . \tag{B4}$$

Then the differential Green function is rewritten as

$$(\hat{\mathbf{e}}_i\cdot\nabla)(\hat{\mathbf{e}}_j\cdot\nabla)\chi_0(\rho,\tau)=-\frac{\kappa}{\eta\rho^4}\left[(\hat{\mathbf{e}}_i\cdot\hat{\rho})(\hat{\mathbf{e}}_j\cdot\hat{\rho})\left[\frac{\partial A}{\partial x}-\frac{\partial B}{\partial x}\right]+(\hat{\mathbf{e}}_j\cdot\hat{\theta})\left[\frac{d}{d\theta}\hat{\mathbf{e}}_i\cdot\hat{\theta}\right]\left[\frac{\partial A}{\partial x}+\frac{\partial B}{\partial x}\right]\right] \tag{B5}$$

where $\hat{\theta}$ is a unit vector perpendicular to $\hat{\rho}$. Therefore, Eq. (B1) holds.

Plugging the result, Eq. (B1), into Eq. (16), and performing the integration over x, we obtain

$$\frac{dy}{d\epsilon}=\left[2-\frac{c}{2}\right]y+2y^2\sum_{\substack{i<j\\ i+j=-k}}\int d^2\rho\,\rho^{1+\hat{\mathbf{e}}_i\cdot\hat{\mathbf{e}}_j/4\pi\kappa}\frac{d}{d\rho}\left[\exp\left[-V_{ij}(\rho,0)-\frac{\hat{\mathbf{e}}_i\cdot\hat{\mathbf{e}}_j}{4\pi\kappa}\ln\rho\right]\right]\ . \tag{B6}$$

Then performing the angular part integration yields Eq. (20). A similar manipulation is applied to Eq. (17), which generates Eq. (21).

[1]Recent reviews in *Phase Transitions and Critical Phenomena,* edited by C. Domb and J. L. Lebowitz (Academic, New York, 1986), Vol. 10.

[2]E. Bauer, in *Structure and Dynamics of Surface II,* edited by W. Shommers and P. von Blanckenhagen (Springer-Verlag, Berlin, 1987).

[3]Recent review by J. D. Weeks, in *Ordering in Strongly Fluctuating Condensed Matter Systems,* edited by T. Riste (Plenum, New York, 1980).

[4]J. D. Weeks, G. H. Gilmer, and H. J. Leamy, Phys. Rev. Lett. **31**, 549 (1973).

[5]Recent reviews in *Statistical Mechanics of Membranes and Surfaces,* edited by D. Nelson, T. Piran, and S. Weinberg (World Scientific, Singapore, 1989).

[6]B. Kahng, A. Berera, and K. A. Dawson, Phys. Rev. A **42**, 6093 (1990).

[7]D. R. Nelson, Phys. Rev. B **18**, 2318 (1978); **26**, 269 (1982).

[8]D. R. Nelson and B. I. Halperin, Phys. Rev. B **19**, 2457 (1979); A. P. Young, *ibid.* **19**, 1855 (1979).

[9]Y. Levin and K. A. Dawson, Phys. Rev. A **42**, 3507 (1990).

[10]K. J. Strandburg, S. A. Solla, and G. V. Chester, Phys. Rev. B **28**, 2717 (1983).

[11]P. Nozières and F. Gallet, J. Phys. (Paris) **48**, 353 (1987).

[12]S. T. Chui and J. D. Weeks, Phys. Rev. Lett. **40**, 733 (1978).

[13]P. C. Hohenberg and B. I. Halperin, Rev. Mod. Phys. **49**, 435 (1977).

[14]L. P. Kadanoff, Ann. Phys. (NY) **120**, 39 (1978); H. J. F. Knops and L. W. J. den Ouden, Physica A **103**, 579 (1980).

[15]Recent reviews in *Dynamics of Fractal Surfaces,* edited by F. Family and T. Vicsek (World Scientific, Singapore, 1991).

[16]D. E. Wolf and J. Villain, Europhys. Lett. **13**, 389 (1990); L. H. Tang and T. Nattermann, Phys Rev. Lett. **66**, 2899 (1991); L. Golubović and R. Bruinsma, *ibid.* **66**, 321 (1991); S. Das Sarma and P. Tamborenea, *ibid.* **66**, 325 (1991).