

미시세계와 거시세계

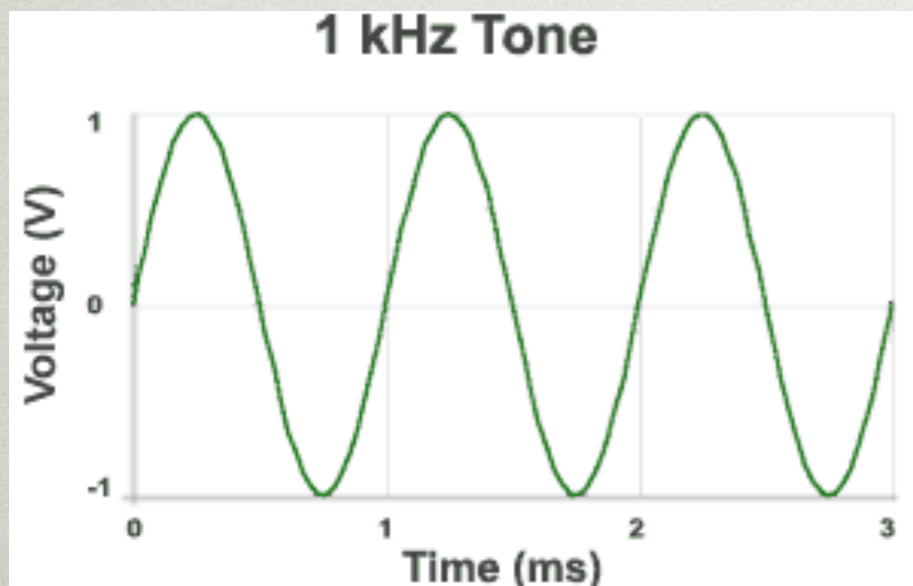
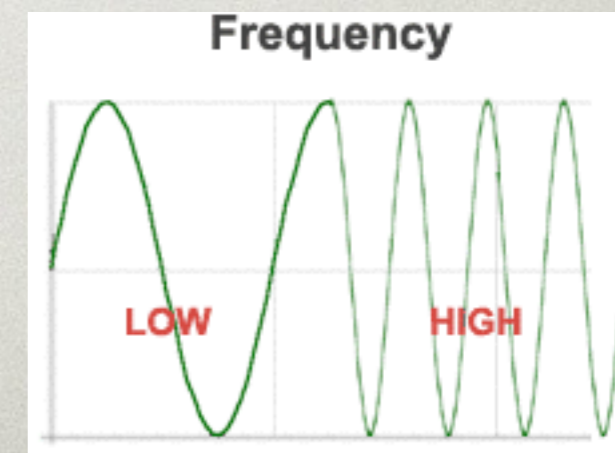
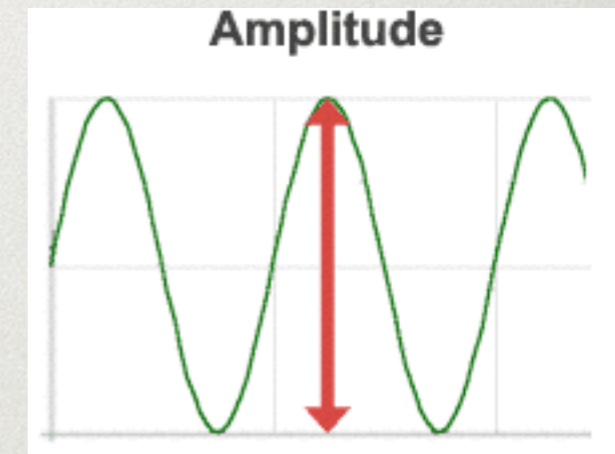
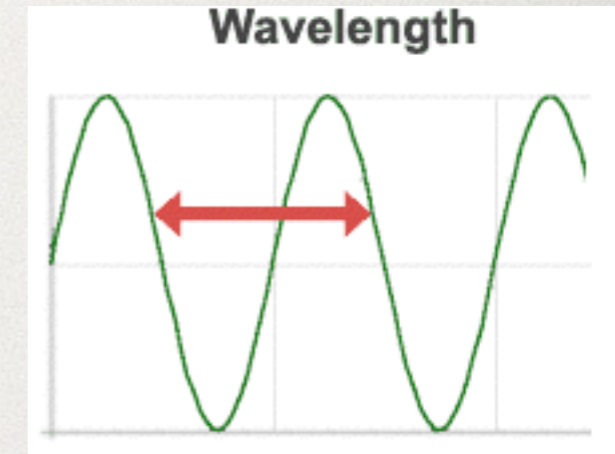
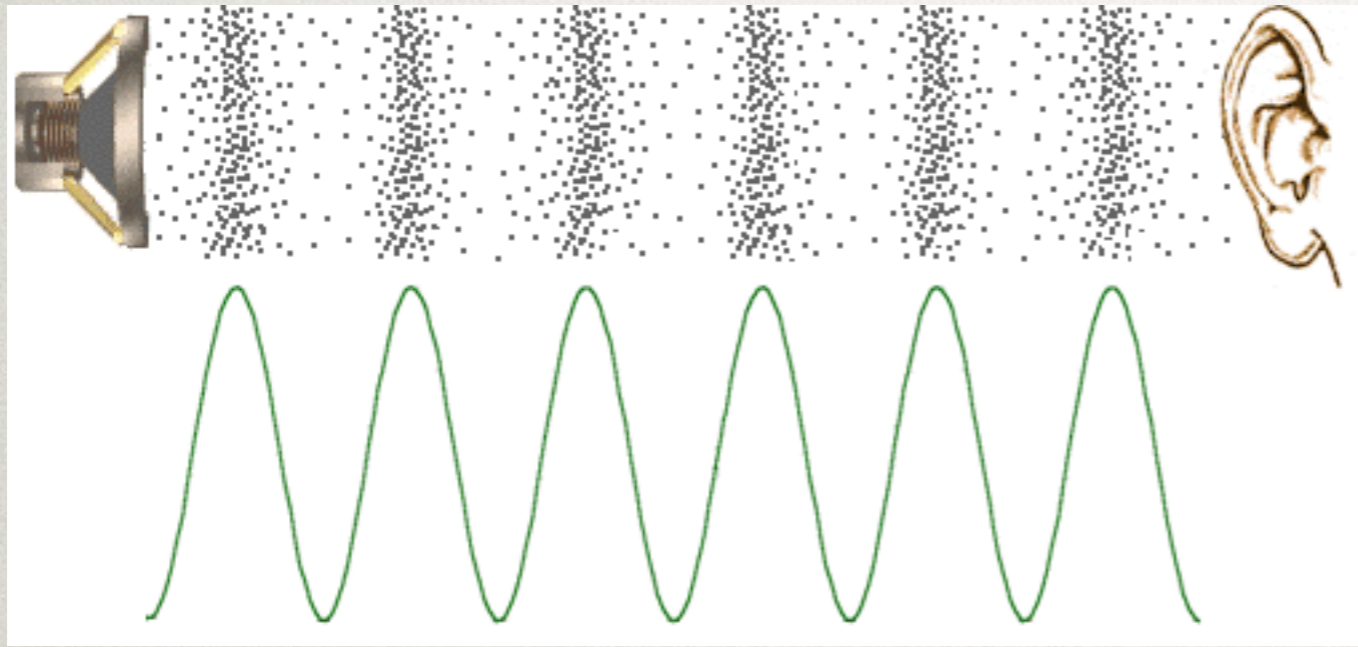
6a. Fourier Series

유재준

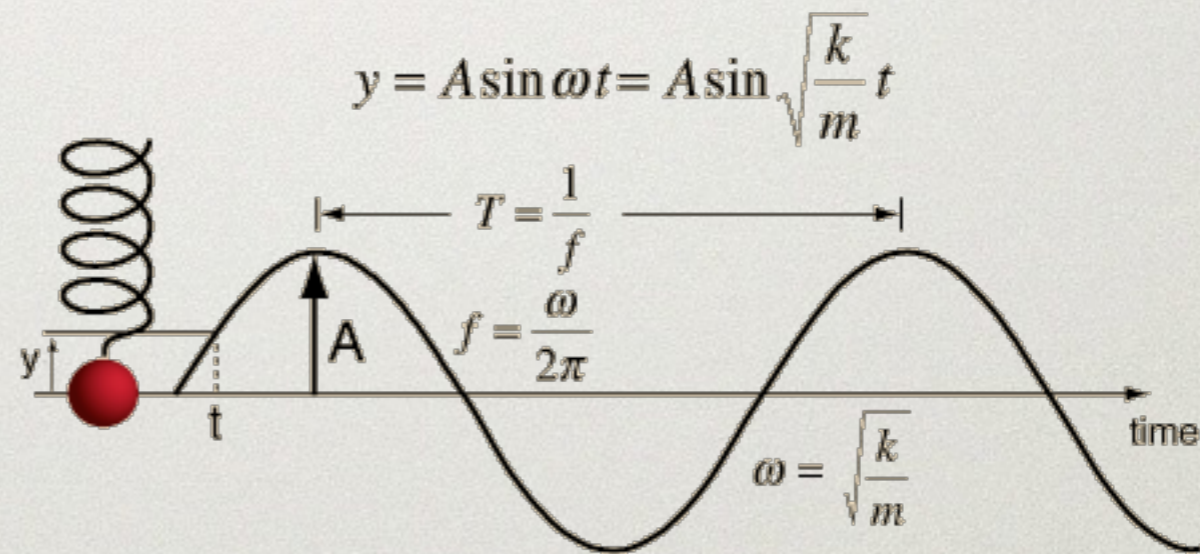
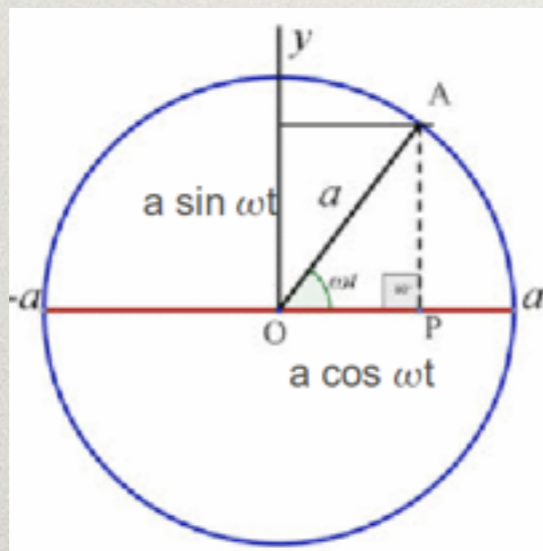
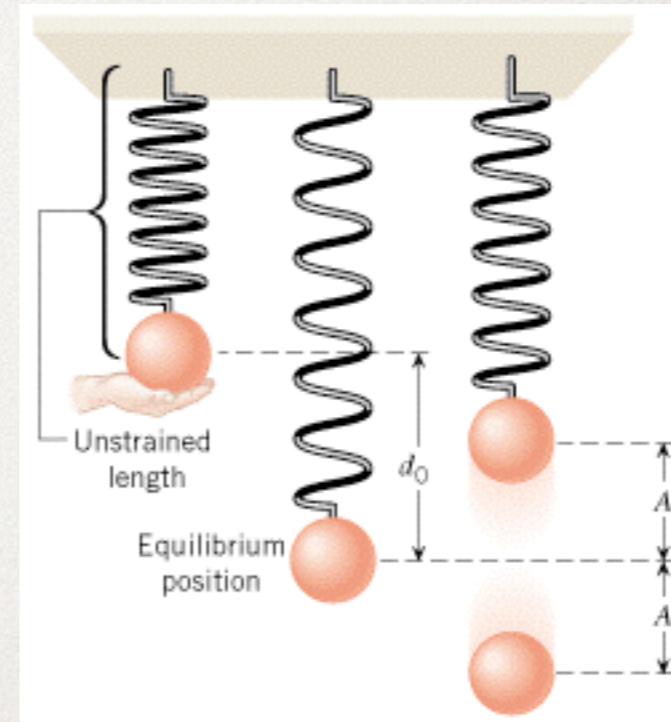
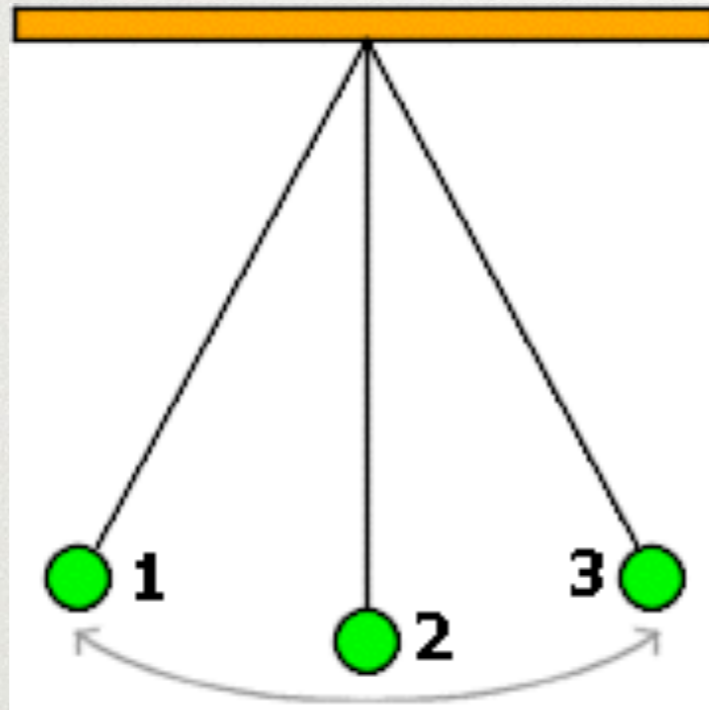
서울대 물리천문학부
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2015/1학기

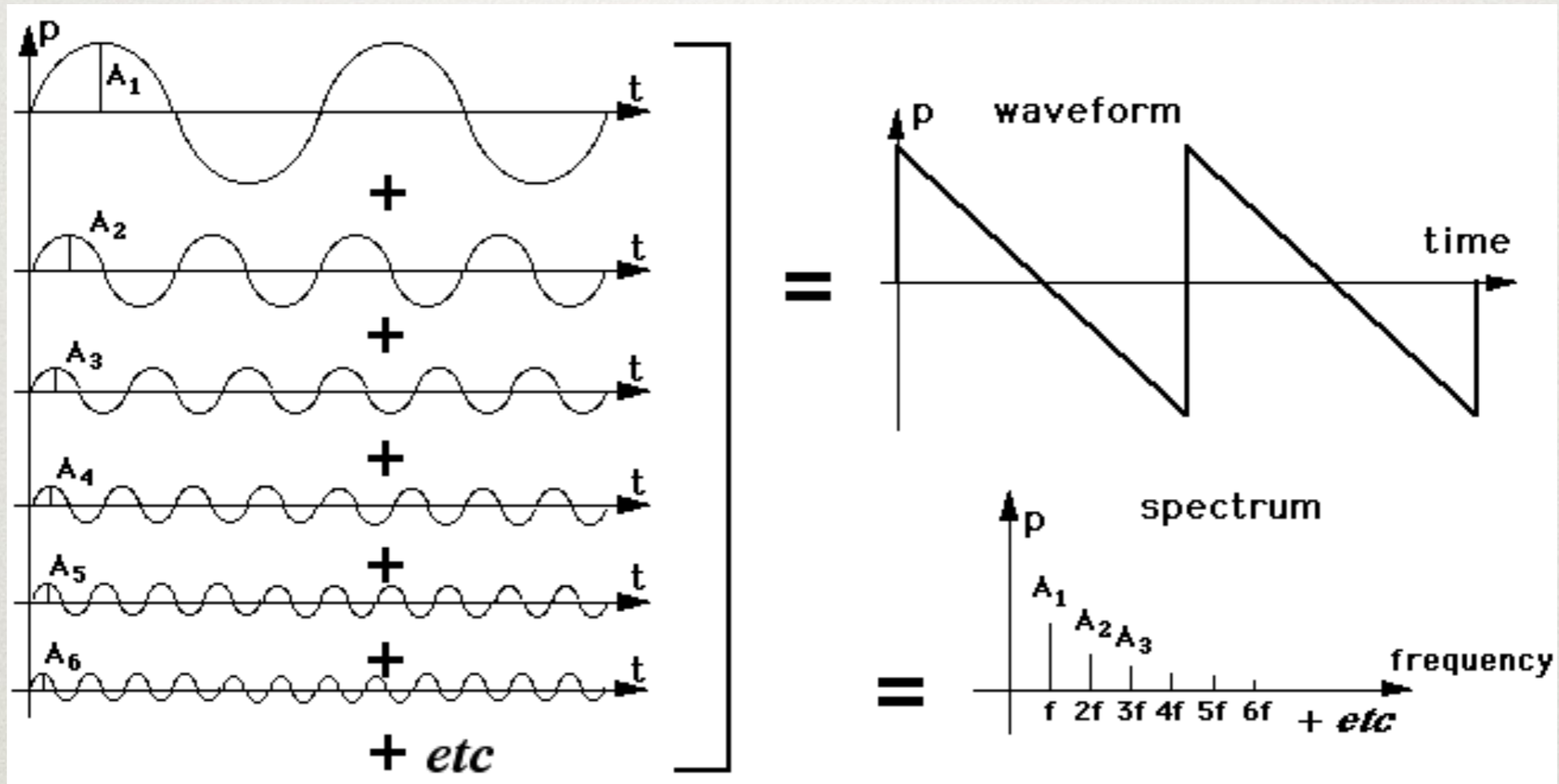
How sound wave propagates



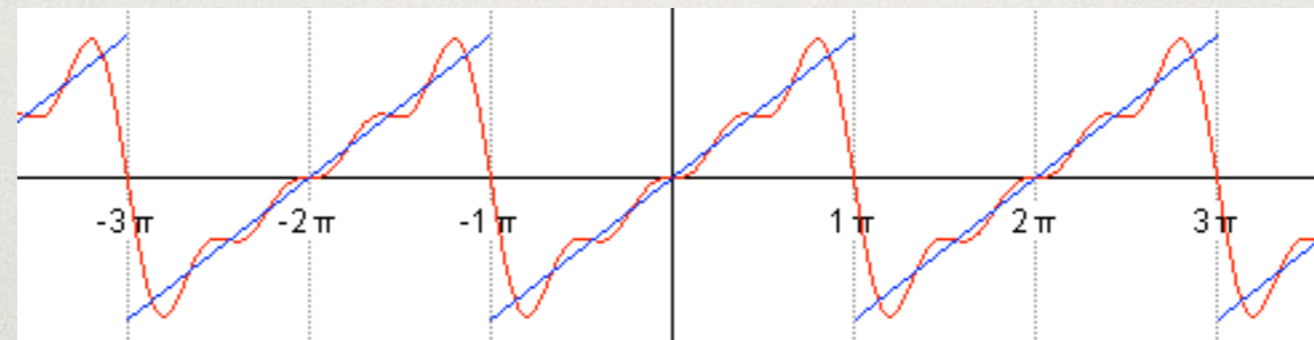
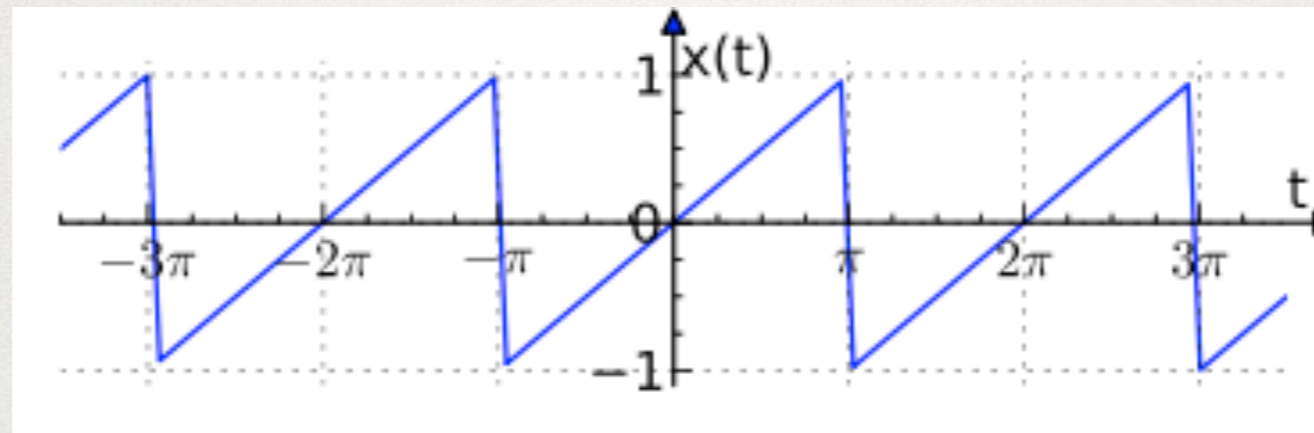
진동이란? What vibrates?



소리 스펙트럼



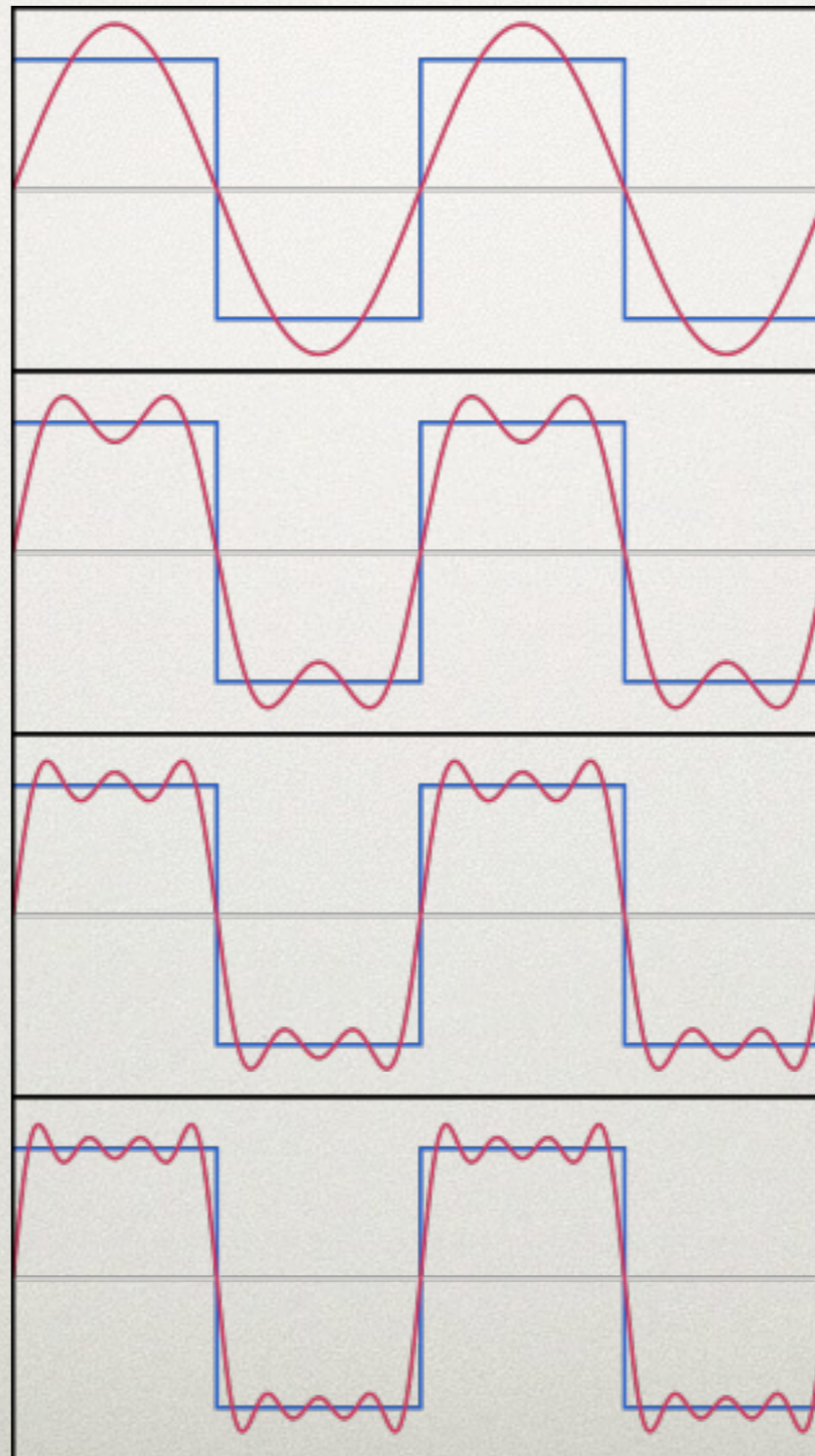
A sawtooth wave



$$\begin{aligned} f(x) &= \frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos(nx) + b_n \sin(nx)] \\ &= 2 \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin(nx), \quad \text{for } x - \pi \notin 2\pi\mathbb{Z}. \end{aligned}$$



The first four Fourier series approximations for a square wave.



Fourier Transform

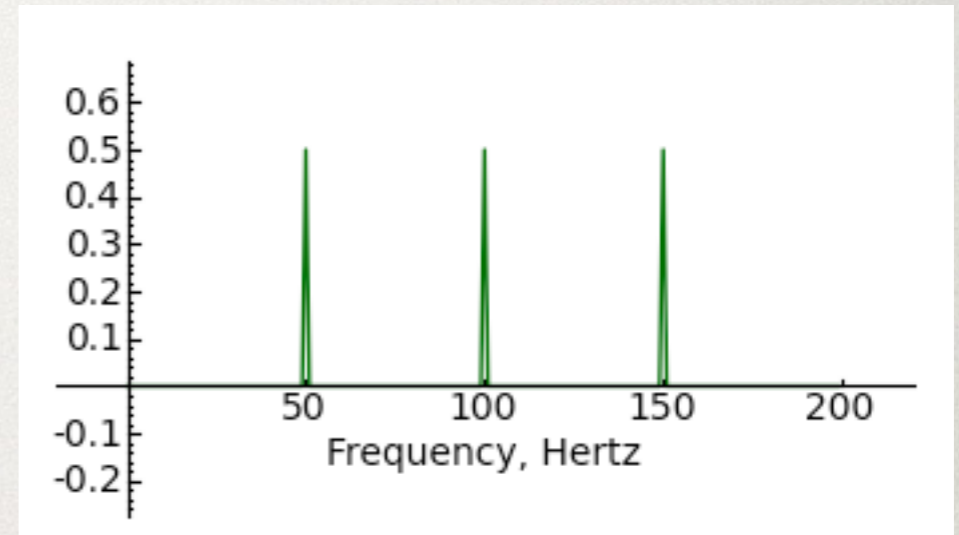
$$f(y) = \int_{-\infty}^{\infty} f(x) e^{-2\pi ixy} dx$$

$$f(x) = \int_{-\infty}^{\infty} f(y) e^{2\pi ixy} dy$$

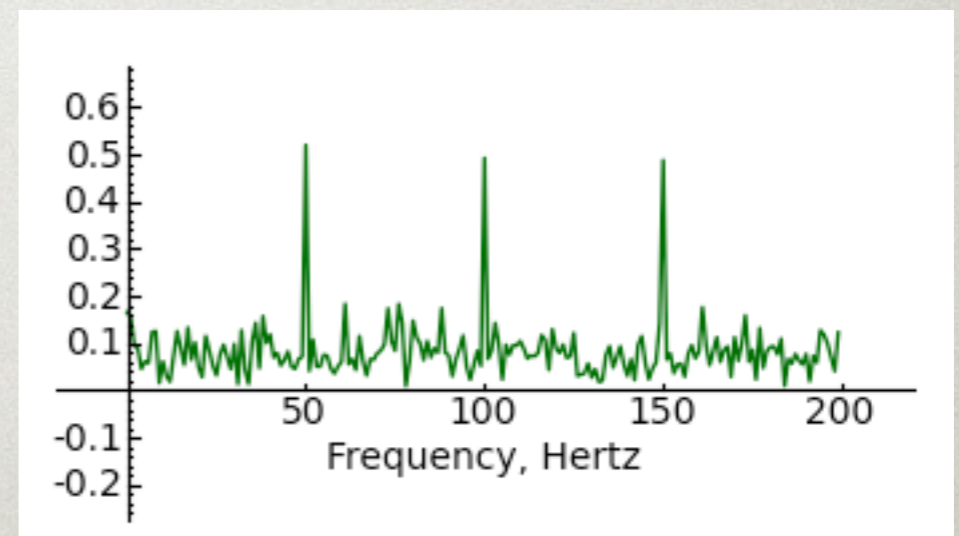
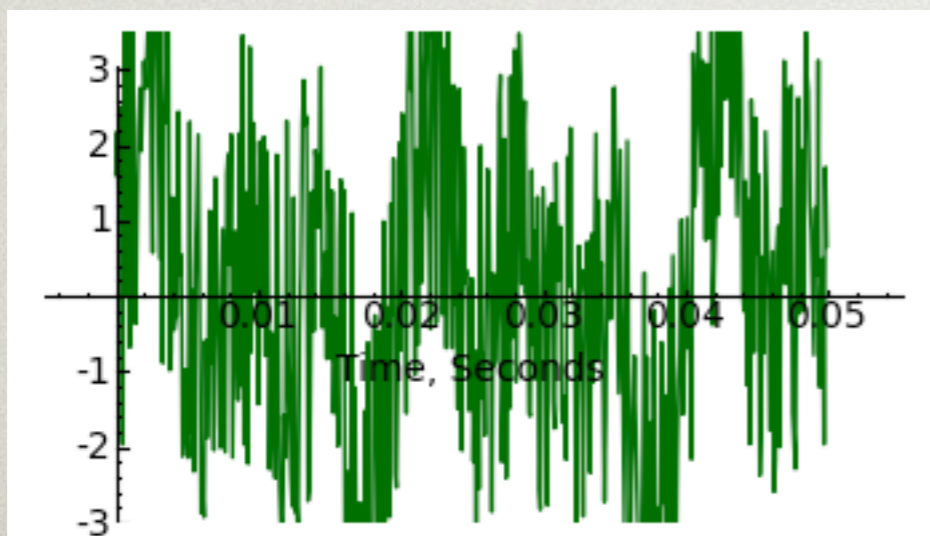
$$e^{2\pi i\theta} = \cos 2\pi\theta + i \sin 2\pi\theta$$



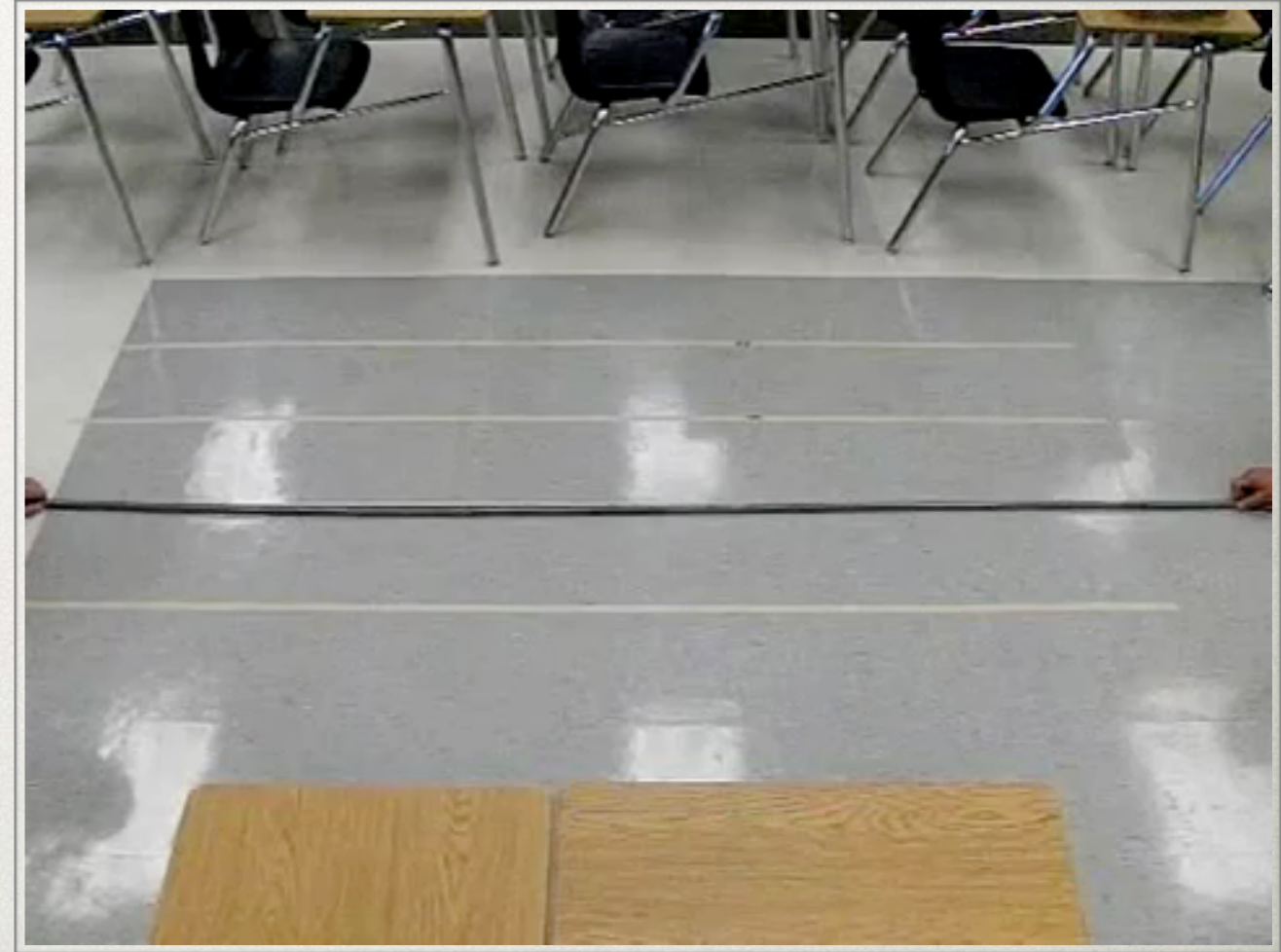
Time Domain



Frequency Domain



Interference (간섭)

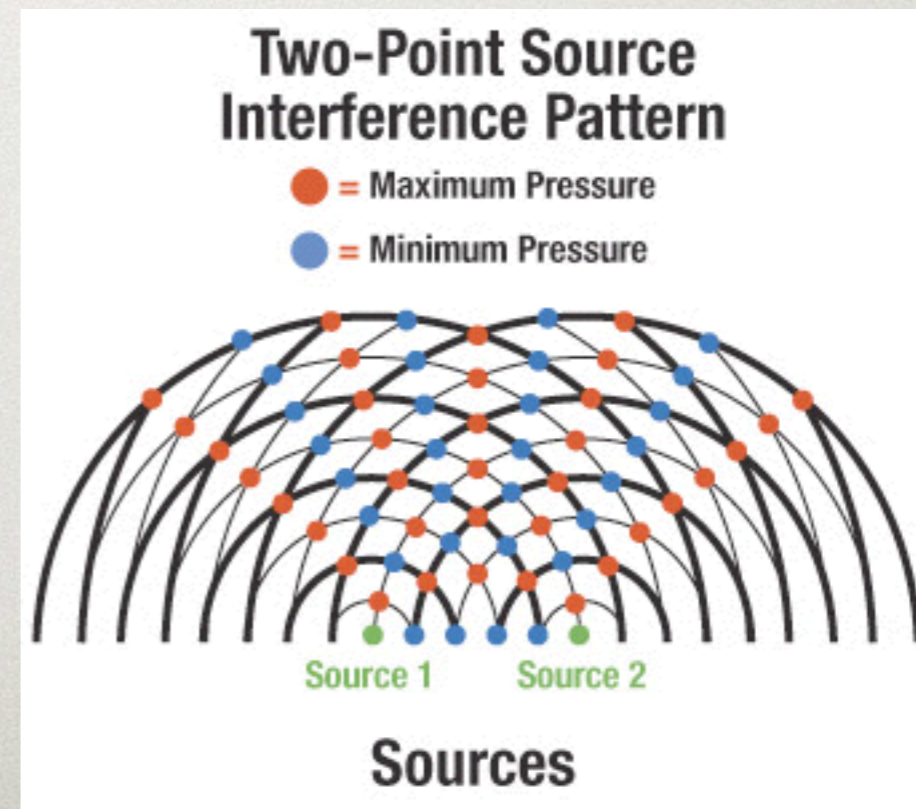
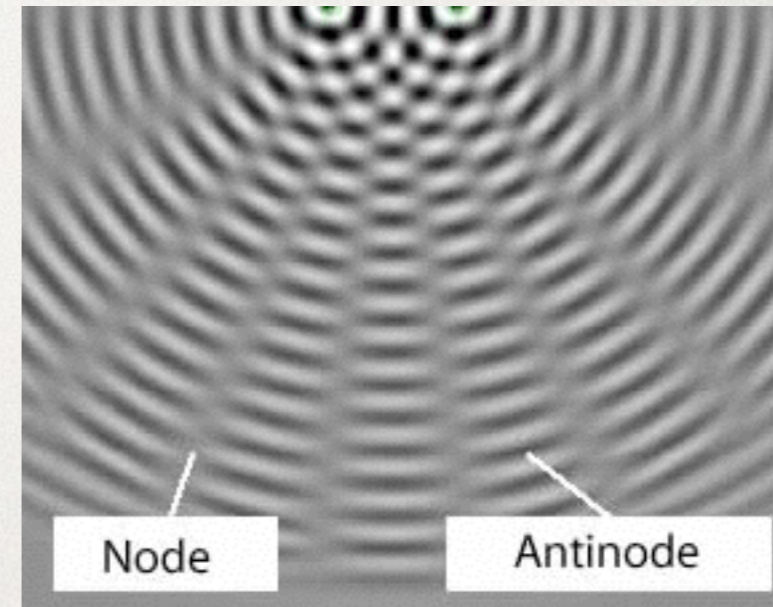
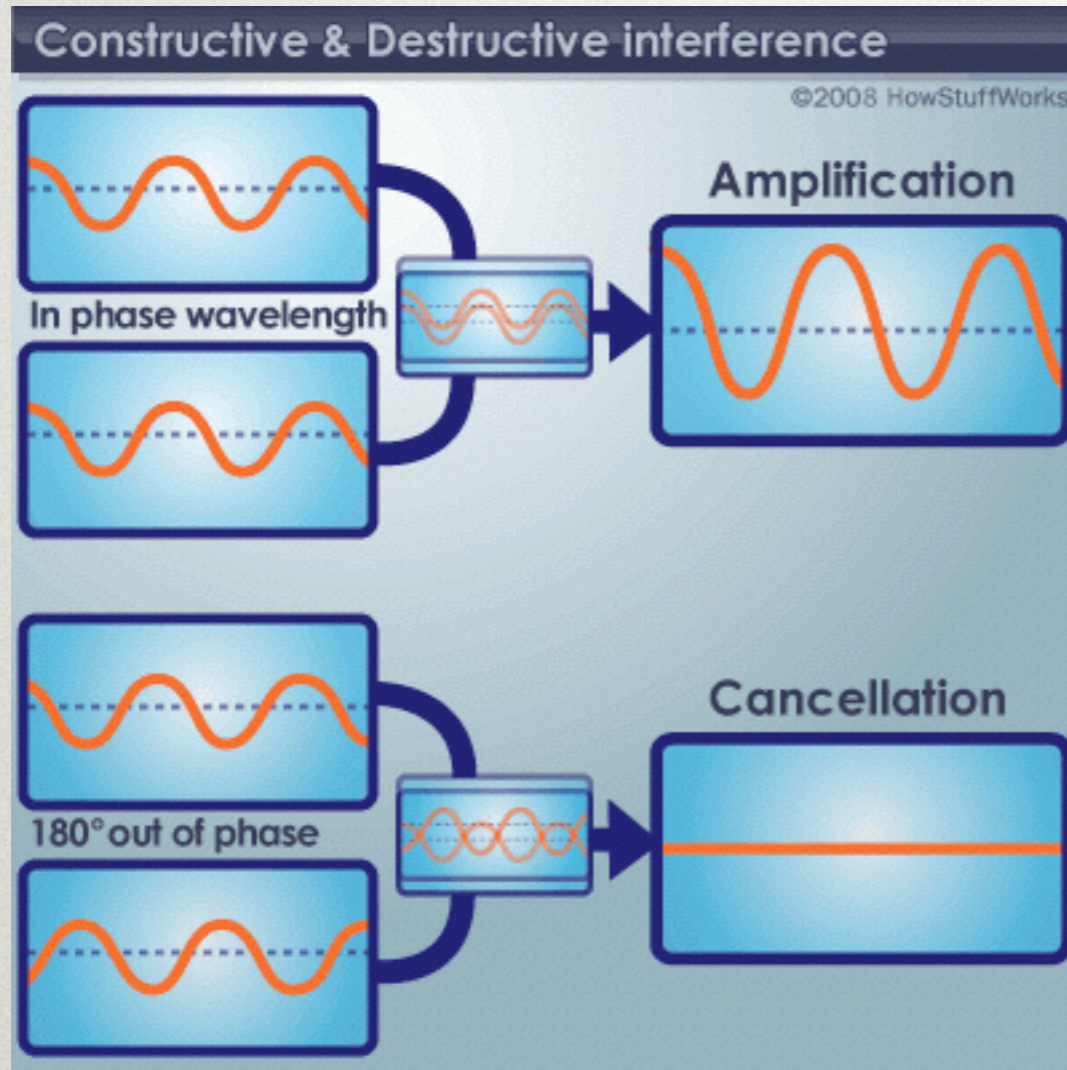


<http://www.youtube.com/watch?v=1gcps37L0r4>

<http://www.youtube.com/watch?v=J4qFPComzoo>

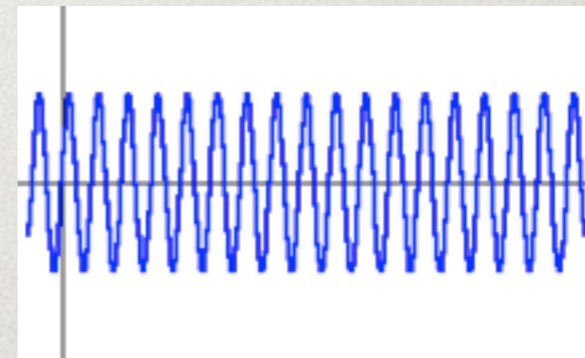
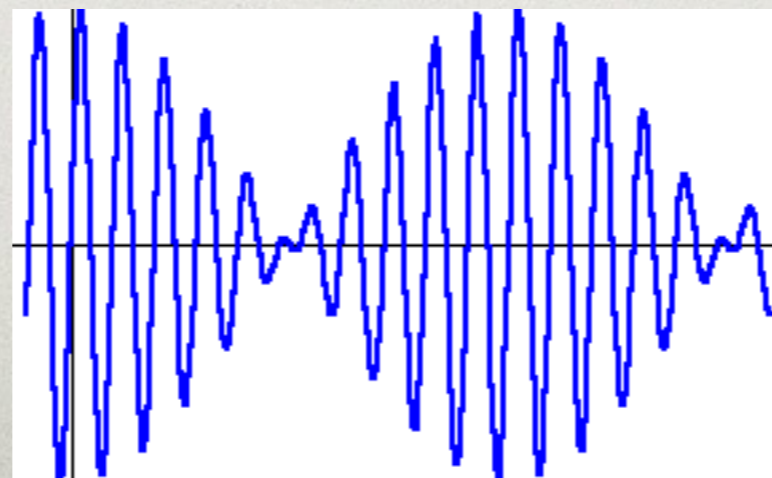
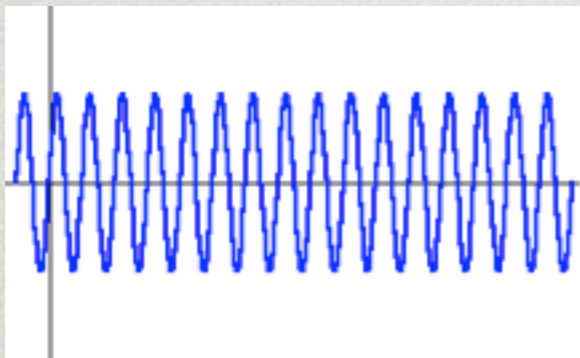


Interference (간섭)

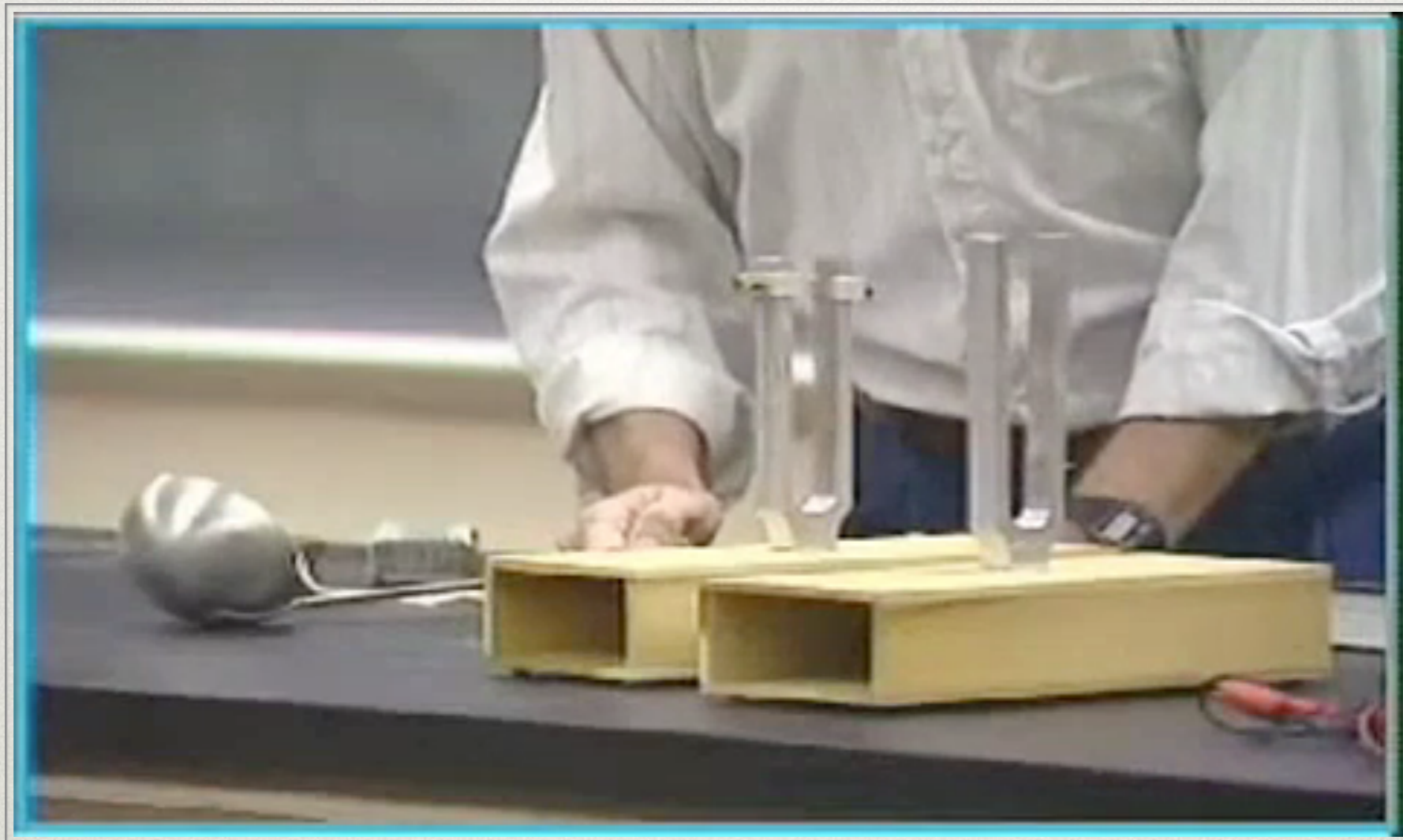


조율 (Tuning)

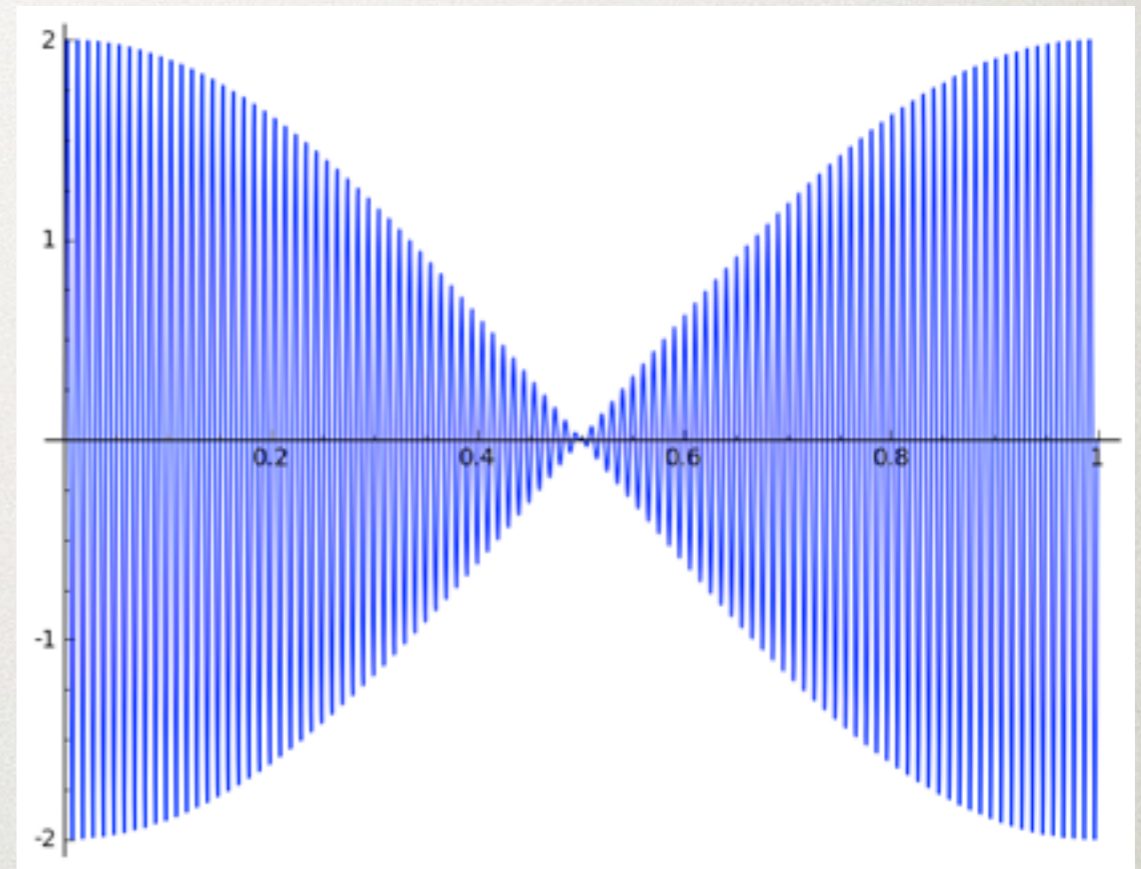
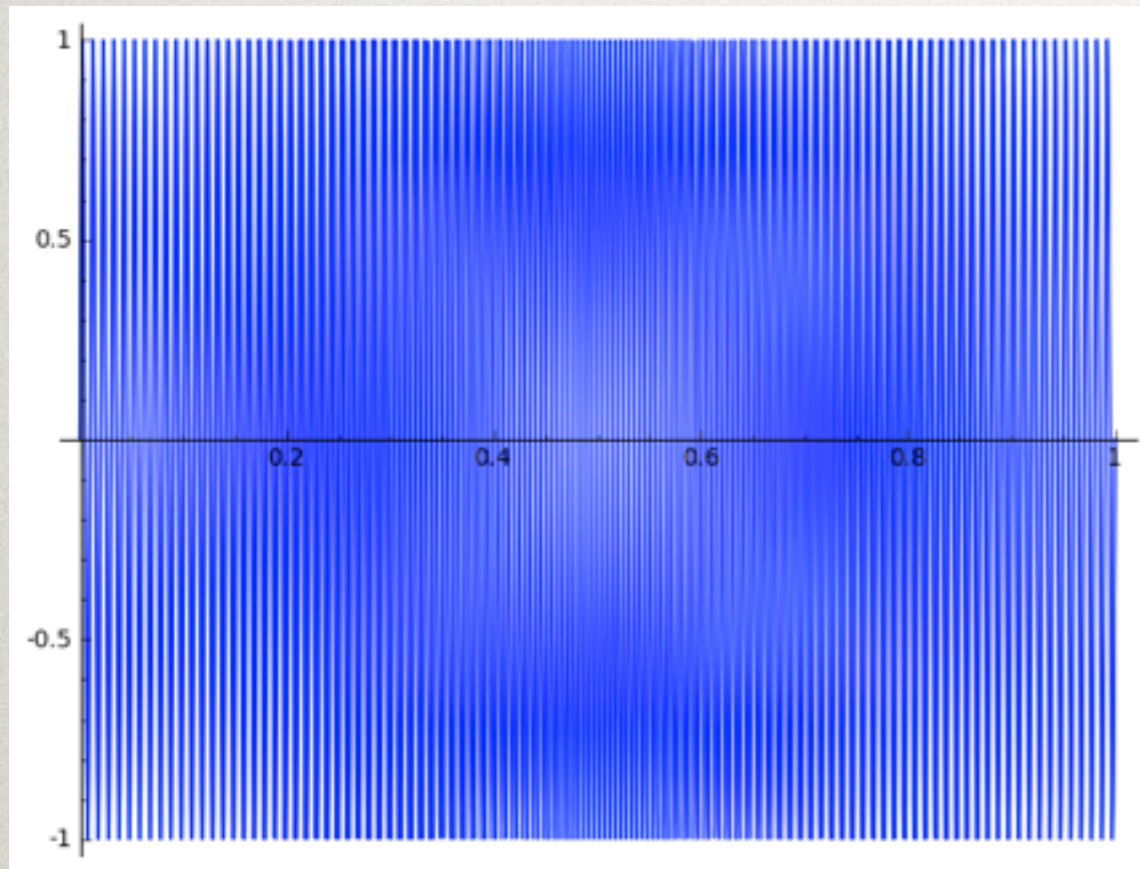
- **각 음간의 간격은 일정한 비율로 정해짐:**
 - Two notes separated by a perfect fifth have a frequency ratio of 3:2.
 - Notice that 2nd and 3rd harmonic on string are perfect 5th
- **맥놀이 (beating)**



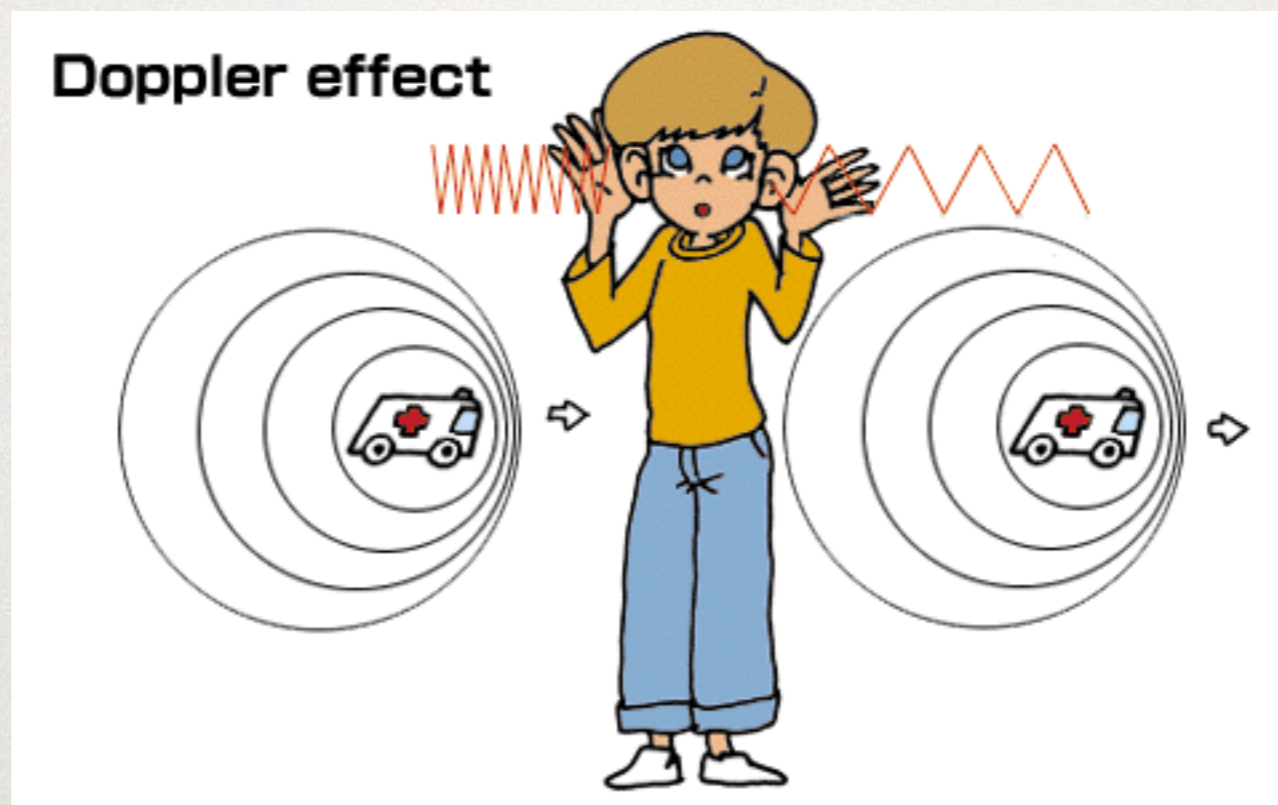
Beating: an interference in time



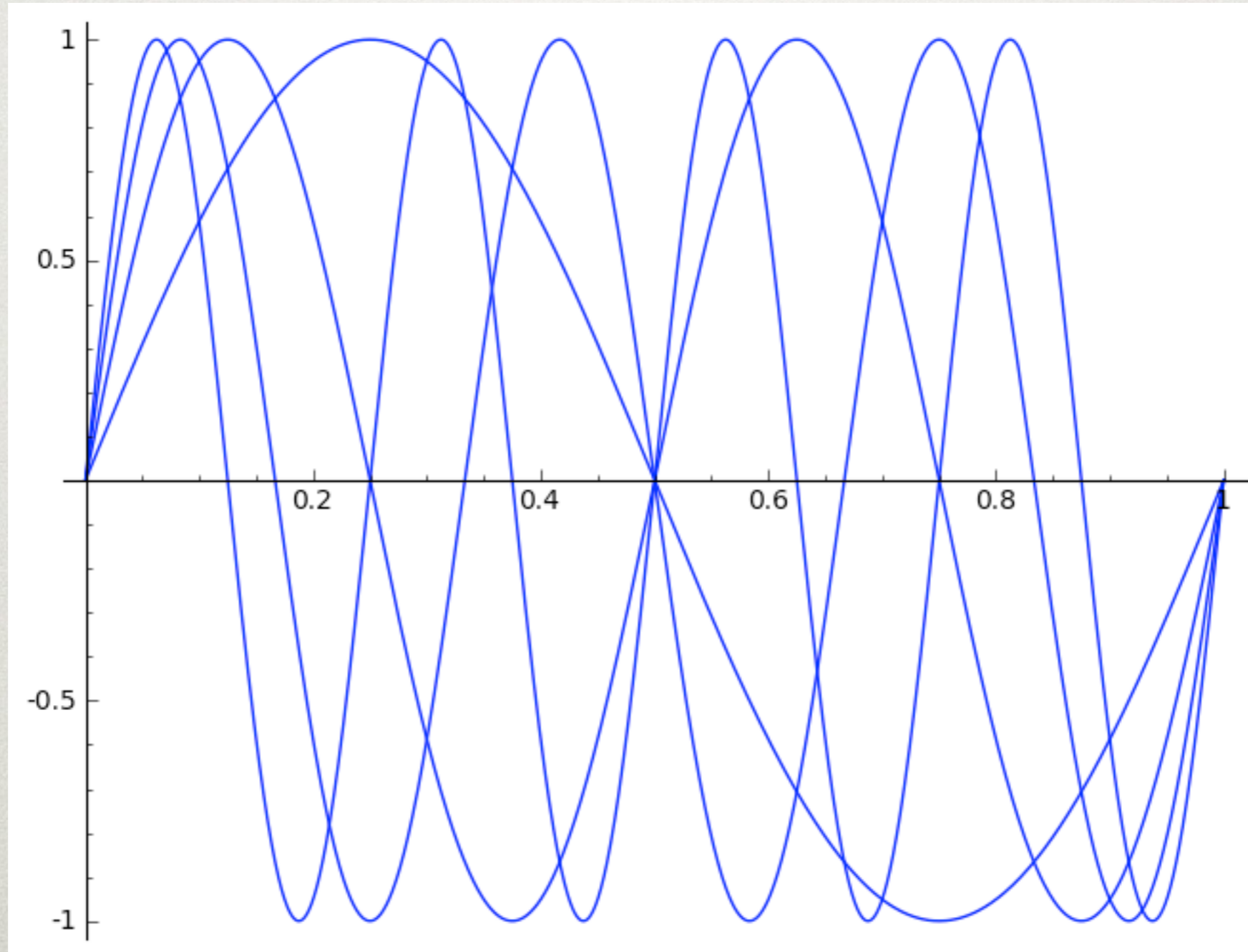
$$\sin(2\pi*99t) + \sin(2\pi*100t)$$



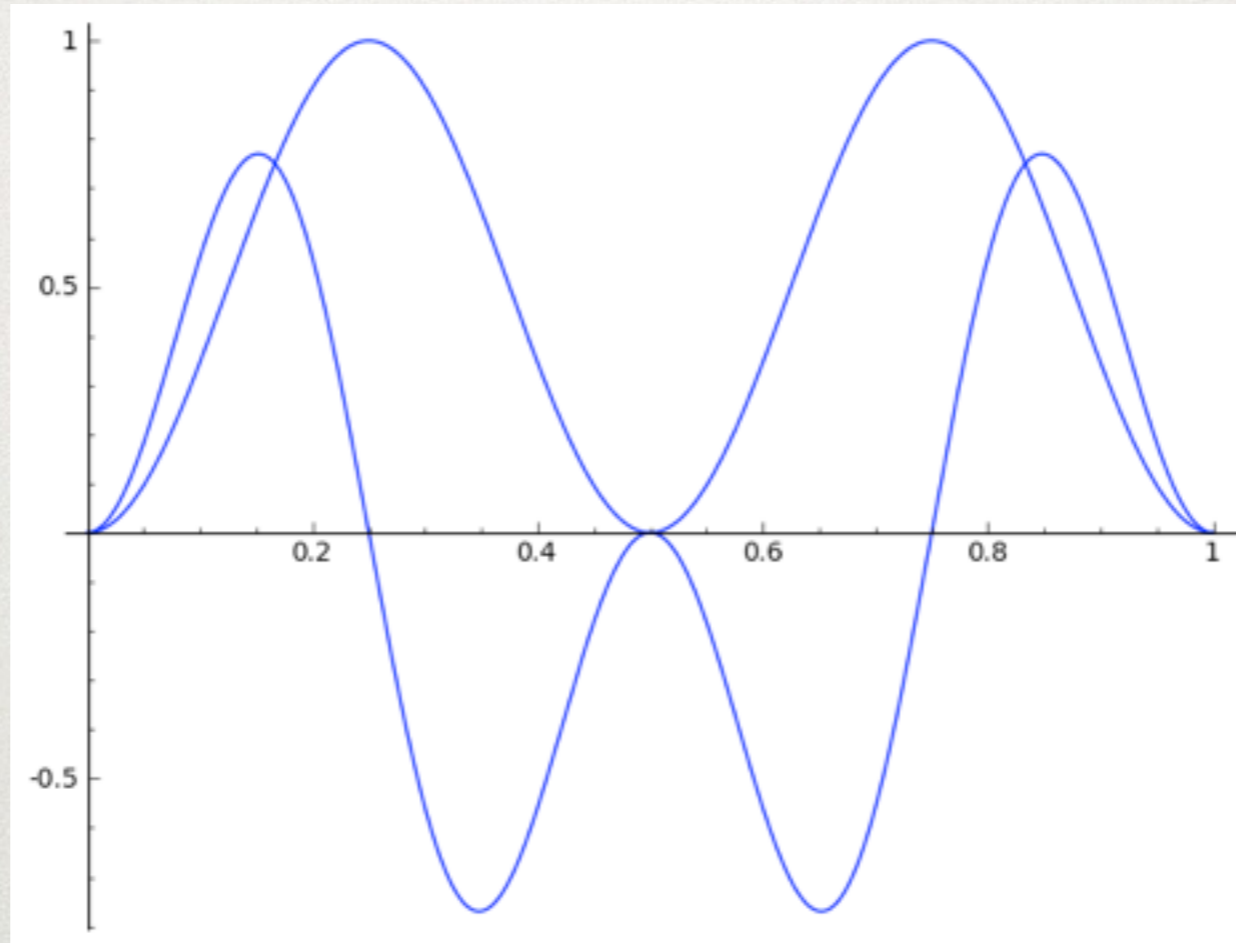
Doppler Effect



Fourier Series and vectors



norm and orthogonality

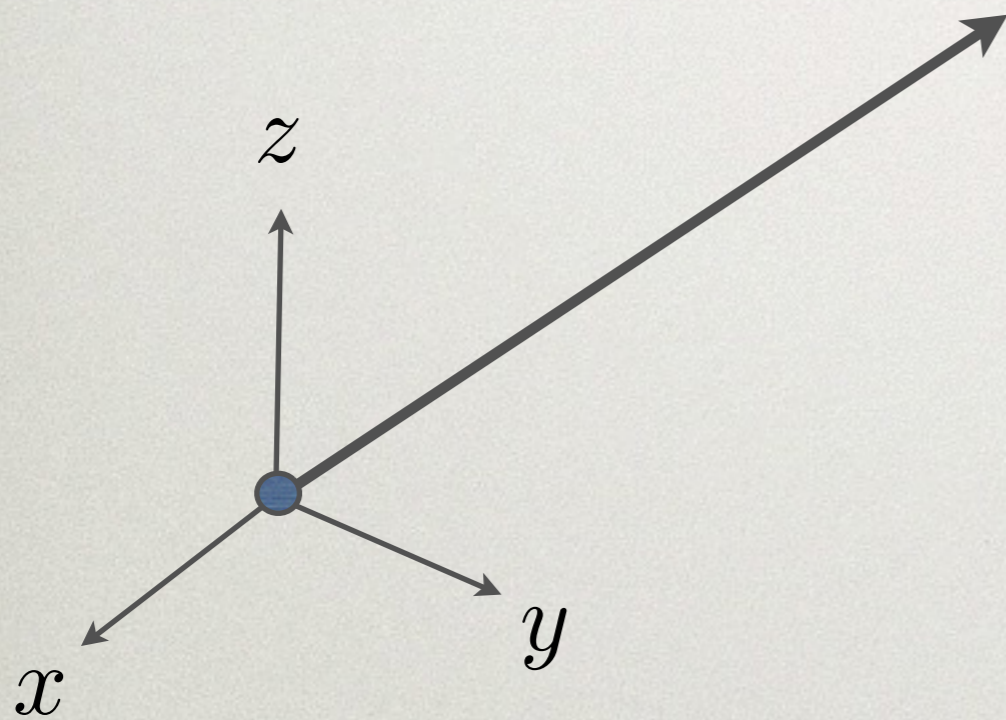


$$\int_0^{2\pi} \sin mt \cdot \sin nt = \pi \delta_{mn} \quad (m, n \neq 0)$$



Vector space

$$\mathbf{r} = x\hat{x} + y\hat{y} + z\hat{z}$$



$$\hat{x} \cdot \hat{x} = 1$$

$$\hat{x} \cdot \hat{y} = 0$$

$$\mathbf{r} = \sum_{i=1,2,3} x_i \hat{e}_i$$

vector space

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \sin nt$$

$$f(x) = (a_0, a_1, a_2, \dots)$$

$$f(x) = \sum_{n=0}^{\infty} a_n \hat{e}_n$$

