

Randall-Sundrum Model for Self-Tuning the Cosmological Constant

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(Received 14 November 2000)

The vanishing cosmological constant in the four-dimensional space-time is obtained in a 5D Randall-Sundrum model with a brane (B1) located at $y = 0$. The matter fields can be located at the brane. For settling any vacuum energy generated at the brane to zero, we need a three-index antisymmetric tensor field A_{MNP} with a specific form for the Lagrangian. For the self-tuning mechanism, the bulk cosmological constant should be negative.

DOI: 10.1103/PhysRevLett.86.4223

PACS numbers: 11.10.Kk, 11.25.Mj, 12.10.Dm, 98.80.Cq

The cosmological constant problem is the most serious hierarchy problem and has been known for more than two decades [1]. So far, there have been several attempts [2] toward understanding this hierarchy problem, but there has not yet appeared a fully accepted solution. The hierarchy in this problem is, “Why is the scale for the cosmological constant Λ so small compared to the Planck mass scale $M_P \equiv 2.44 \times 10^{18}$ GeV?” This hierarchy problem has become even more difficult with the recent observation of the small but nonvanishing vacuum energy of order $(0.003 \text{ eV})^4$ [3]. The quintessences have been considered in order to explain the smallness of this tiny vacuum energy [4], but the bottom line of these ideas is that there does exist a solution to the cosmological constant problem.

The cosmological constant was introduced by Einstein in 1917 to make the universe static, since it appeared at that time that the universe did not seem to be not evolving. But the discovery of the expanding universe in 1929 no longer required the assumption of a static universe, and the cosmological constant has become another parameter to be determined by further observations in general relativity. We expect that if a theory describes physics at the mass scale of order m , parameters in the theory are expected to be of order m . Gravitation is described at the Planck scale (or inverse Newton’s constant) of order 10^{19} GeV. However, the cosmological constant appearing in the gravity equation is phenomenologically very strongly bounded $<(0.01 \text{ eV})^4$, which implies that there is a hierarchy of order 10^{-120} between parameters in the gravity theory. This hierarchy problem could have been questioned even at the time of Einstein, not from the static universe condition but as a hierarchy problem.

In theoretical physics, the cosmological constant problem has become a serious one in view of the need for spontaneous symmetry breaking (SSB) in particle physics [1], since the vacuum energy (which is another name for cosmological constant) in SSB is not fixed by any symmetry principle. The electroweak symmetry breaking and QCD chiral symmetry breaking can introduce vacuum energies. Thus, the difficulty of solving the cosmological constant problem in four-dimensional (4D) space-time lies

in the fact that the limit $\Lambda \rightarrow 0$ does not introduce any new symmetry. Thus, it may be necessary to go beyond the 4D space-time or introduce a more general form of the Lagrangian.

In this Letter, we consider a solution of the cosmological constant problem with one extra dimension. In particular, we work with one brane located at $y = 0$ (B1 brane), where y is the fifth dimension (5D), which is the so-called Randall-Sundrum-II model (RSII) [5]. This RSII model is an alternative to compactification of the extra dimension y , but we can obtain an effective 4D flat theory if gravity is localized at the B1 brane. If we stay at B1 and the graviton wave function is sufficiently damped at large y , the bulk of this extra y space is not of much relevance to us even though the 5D space is not compactified. Within this type of setup there may exist a possibility to attack the cosmological constant problem again, since the 4D flat space solutions are obtained with fine tuning(s) even though the 5D cosmological constant is nonvanishing and negative. (Note that in 4D there is no possibility of a flat space solution if the cosmological constant is nonzero.)

In the RS-type models, the idea for self-tuning of the cosmological constant has already been suggested [6]. We define *the self-tuning model as a model allowing a flat space solution without fine-tuning between parameters in the Lagrangian*. This definition is consistent with the one adopted by Hawking and Witten in the early 1980s [7,8]. They did not care about the existence of de Sitter or anti-de Sitter space solutions, but needed the existence of a flat space solution with one undetermined integration constant. This constant can be used to adjust so that the cosmological constant becomes zero, given the parameters in the Lagrangian. But in 4D space-time their’s was just an idea, not a working model, since their 3-form field was not a dynamical field. In recent papers [6], the self-tuning idea has been revived in five-dimensional space-time. Arkani-Hamed *et al.* and Kachru *et al.* [6] used a specific potential for a bulk scalar field in the RSII model [5] and obtained a flat space solution for a finite range of input parameters in the Lagrangian. Thus, it seemed that they realized the self-tuning idea. However, their solution contained an essential singularity, where they just cut off

the bulk for a finite 4D Planck mass. But one cannot ignore this singularity. If we ignore it, we would not obtain a vanishing effective cosmological constant in an effective 4D theory, which is contradictory to the flat space solution. If the cure for this singularity is correctly performed, the effective 4D cosmological constant is zero but one needs one fine-tuning [9]. Therefore, it is fair to say that the idea for the self-tuning has been suggested in the RS-type models but thus far a successful model has not appeared.

In this Letter, we present a working self-tuning model with a $1/H^2$ term in the Lagrangian in the 5D RSII model

$$S = \int d^4x \int dy \sqrt{-g} \left(\frac{1}{2} R + \frac{2 \times 4!}{H_{MNPQ} H^{MNPQ}} - \Lambda_b + \mathcal{L}_m \delta(y) \right), \quad (1)$$

where we put the brane B1 at $y = 0$ and the brane tension at B1 is $\Lambda_1 \equiv -\langle \mathcal{L}_m \rangle$. We set the fundamental mass parameter M as 1 and we recover the mass M wherever it is explicitly needed. We assume a Z_2 symmetry of the solution, $\beta(-y) = \beta(y)$. We introduced the three-index antisymmetric tensor field A_{MNP} whose field strength is denoted as H_{MNPQ} . The action contains the $1/H^2$ term which does not make sense if H^2 does not develop a vacuum expectation value. We anticipate that this term constitutes part of the gravitational interactions, and hence the renormalizability is not considered in this paper. If a good solution results for the cosmological constant problem, it can be more seriously considered as a fundamental interaction.

The ansatz for the metric is taken as [10]

$$ds^2 = \beta^2(y) \eta_{\mu\nu} dx^\mu dx^\nu + dy^2, \quad (2)$$

where $(\eta_{\mu\nu}) = \text{diag}(-1, +1, +1, +1)$. With the brane tension Λ_1 at B1 and the bulk cosmological constant Λ_b , the energy momentum tensors are

$$T_{MN} = -g_{MN} \Lambda_b - g_{\mu\nu} \delta_M^\mu \delta_N^\nu \Lambda_1 \delta(y) + 4 \times 4! \left(\frac{4}{H^4} H_{MPQR} H_N^{PQR} + \frac{1}{2} g_{MN} \frac{1}{H^2} \right). \quad (3)$$

H_{MNPQ} was considered previously in connection with the cosmological constant problem [8] and compactification [11]. The specific form for $H^2 \equiv H_{MNPQ} H^{MNPQ}$ in Eq. (1) makes sense only if H^2 develops a vacuum expectation value at the order of the fundamental mass scale. Because of the gauge invariant four-index H_{MNPQ} , four space-time is singled out from the five dimensions [11]. The ansatz for the four form fields is $H_{\mu\nu\rho\sigma} = \sqrt{-g} \epsilon_{\mu\nu\rho\sigma} / n(y)$, where μ, \dots run over the Minkowski indices 0, 1, 2, and 3. In 5D space, the three-index antisymmetric tensor field is basically a scalar field a defined by $\partial_M a = (1/4!) \sqrt{-g} \epsilon_{MNPQR} H^{NPQR}$.

[5]. The basic mechanism of our solution is the following. The solution with a 4D flat space ansatz is regular at the whole y space and introduces two integration constants a and c . One constant a defines the Planck scale. The other integration constant c participates in the boundary condition at B1 and is related to the brane tension and bulk cosmological constant. (Note that, in the RSII model, there appears one integration constant which is not participating in the boundary condition.)

We proceed to discuss the model with the action

In this Letter, we show that there exists a solution for $\Lambda_b < 0$. The two relevant Einstein equations are the (55) and $(\mu\mu)$ components,

$$6 \left(\frac{\beta'}{\beta} \right)^2 = -\Lambda_b - \frac{\beta^8}{A}, \quad (4)$$

$$3 \left(\frac{\beta'}{\beta} \right)^2 + 3 \left(\frac{\beta''}{\beta} \right) = -\Lambda_b - \Lambda_1 \delta(y) - 3 \frac{\beta^8}{A}, \quad (5)$$

where the prime denotes the derivative with respect to y , and A is a positive constant in view of the ansatz of $H_{\mu\nu\rho\sigma}$. It is easy to check that Eq. (5) in the bulk is obtained from Eq. (4) for any Λ_b , Λ_1 , and A . If we took H^2 (instead of the $1/H^2$ term) in the Lagrangian, this statement will still hold but the resulting solutions do not lead to a self-tuning solution [12]. If we use both $1/H^2$ and H^2 terms, a self-tuning solution does not result. Near B1 (the $y = 0$ brane), the δ function must be generated by the second derivative of β . The Z_2 symmetry, $\beta(-y) = \beta(y)$, implies $(d/dy)\beta(y)|_{0^+} = -(d/dy)\beta(y)|_{0^-}$. Thus, we can write $(d^2/dy^2)\beta(|y|)$ as $(d^2/dy^2)\beta(|y|)|_{y \neq 0} + 2\delta(y)(d/d|y|)\beta(|y|)$. This δ -function condition at B1 leads to a boundary condition

$$\frac{\beta'}{\beta} \Big|_{y=0^+} \equiv -k_1, \quad (6)$$

where we define k 's in terms of the bulk cosmological constant and the brane tension,

$$k \equiv \sqrt{-\frac{\Lambda_b}{6}}, \quad k_1 \equiv \frac{\Lambda_1}{6}. \quad (7)$$

It is sufficient if we find a solution for the bulk equation [Eq. (4)] with the boundary condition [Eq. (6)]. We define a in terms of A ,

$$a = \sqrt{\frac{1}{6A}}. \quad (8)$$

We note that the solution $\beta(y)$ should satisfy the following: (i) the metric is well behaved in the whole region of

the bulk, and (ii) the resulting 4D effective Planck mass is finite.

The solution of Eq. (4), consistent with the Z_2 symmetry, is

$$\beta(|y|) = \left(\frac{k}{a}\right)^{1/4} [\cosh(4k|y| + c)]^{-1/4}, \quad (9)$$

where c is an integration constant to be determined by the boundary condition [Eq. (6)]. This solution, consistent with (i), is possible for a finite range of the brane tension Λ_1 . Note that c can take any sign. This solution gives a localized gravity consistent with the above condition (ii). The boundary condition (6) determines c in terms of Λ_b and Λ_1 ,

$$c = \tanh^{-1}\left(\frac{k_1}{k}\right) = \tanh^{-1}\left(\frac{\Lambda_1}{\sqrt{-6\Lambda_b}}\right). \quad (10)$$

We note that the solution exists for a finite range of parameters, $-\sqrt{-6\Lambda_b} < \Lambda_1 < \sqrt{-6\Lambda_b}$.

The effective 4D Planck mass is finite:

$$M_{P,\text{eff}}^2 = 2M^3 \left(\frac{k}{a}\right)^{1/2} \int_0^\infty dy \frac{1}{\sqrt{\cosh(4ky + c)}}. \quad (11)$$

Note that the Planck mass is given in terms of the integration constant a , or the integration constant is expressed in terms of the fundamental mass M and the 4D Planck mass $M_{P,\text{eff}}$,

$$a = \left(\frac{M^3}{M_{P,\text{eff}}^2 \sqrt{2k}} F\right)^2, \quad (12)$$

where F is an elliptic integral of the first kind and is known to be finite [12].

To obtain the field equation for A_{MNPQ} , we note that the variation of the Lagrangian (1) with respect to A_{NPQ} gives

$$\begin{aligned} \delta \mathcal{L} \supset & -4 \times 4! \partial_M \left(\sqrt{-g} \frac{H^{\text{MNPQ}} \delta A_{\text{NPQ}}}{H^4} \right) \\ & + 4 \times 4! \left[\partial_M \left(\sqrt{-g} \frac{H^{\text{MNPQ}}}{H^4} \right) \right] \delta A_{\text{NPQ}}. \end{aligned} \quad (13)$$

To cancel the first term of the above equation, we add a surface term in the action

$$\begin{aligned} S_{\text{surface}} &= \int d^4x dy 4 \times 4! \partial_M \left(\sqrt{-g} \frac{H^{\text{MNPQ}} A_{\text{NPQ}}}{H^4} \right) \\ &= \int d^4x dy 4 \times 4! \left[\frac{\sqrt{-g}}{H^2} + A_{\text{NPQ}} \partial_M \left(\sqrt{-g} \frac{H^{\text{MNPQ}}}{H^4} \right) \right], \end{aligned} \quad (14)$$

where the variation of the derivative of A_{NPQ} vanishes at the boundary [13]. Then the field equation for A_{NPQ} is

$$\partial_M \frac{\sqrt{-g} H^{\text{MNPQ}}}{H^4} = 0, \quad (15)$$

which can be integrated to give

$$\frac{\sqrt{-g} H^{\text{MNPQ}}}{H^4} = \text{function of } y \text{ only}. \quad (16)$$

Because of our ansatz for the 4D homogeneous space, H_{MNPQ} can have nonvanishing values only for $H_{\mu\nu\rho\sigma}$, as discussed earlier. Thus, A in Eqs. (4) and (5), and hence a in Eq. (8), is an integration constant. Field equations do not determine a , namely, a is not dynamically determined. However, a can take any value. Then, for a given a , the Planck mass is given in terms of a , as shown in Eq. (11). It is clear that this integration constant a itself does not

participate in the self-tuning. On the other hand, the integration constant c does participate in the self-tuning.

Suppose that we are given a Lagrangian with Λ_1 and Λ_b . We can then always find a solution for $\Lambda_b < 0$ and $|\Lambda_1| < \sqrt{-6\Lambda_b}$. Namely, there exists a 4D flat space solution (2) with c given by Eq. (10). If we add some constant vacuum energy at B1, then Λ_1 is shifted to, say, Λ'_1 . For this new set of Λ'_1 and Λ_b , we can again find a solution, but with a different integration constant c' given with Λ'_1 through Eq. (10). In other words, the dynamics of gravity and the antisymmetric tensor field adjust the solutions a little, i.e., self-tune the above integration constant from c to c' , to satisfy the field equations.

Even though we obtained a flat space solution for the 4D Minkowski space, it is worthwhile to check explicitly that the effective cosmological constant vanishes. From action (1), the 4D gravity with vacuum energy is effectively described by

$$\begin{aligned} S &= \int d^4x dy \sqrt{-\eta} \beta^4 \left[\frac{1}{2} \beta^{-2} \tilde{R}_4 - 4 \left(\frac{\beta''}{\beta} \right) - 6 \left(\frac{\beta'}{\beta} \right)^2 \right. \\ &\quad \left. - \Lambda_b + \frac{2 \times 4!}{H^2} - \Lambda_1 \delta(y) \right] + S_{\text{surface}}, \end{aligned} \quad (17)$$

where the 4D metric is $\tilde{g}_{\mu\nu} = \beta^2 \eta_{\mu\nu}$, η is the determinant of $\eta_{\mu\nu}$, and \tilde{R}_4 is the 4D Ricci scalar. Then $-\Lambda_{\text{eff}}$ is given by the y integral of Eq. (17), except the \tilde{R}_4 term,

$$\Lambda_{\text{eff}} = \int_{-\infty}^{\infty} dy \beta^4 \left[4 \left(\frac{\beta''}{\beta} \right) + 6 \left(\frac{\beta'}{\beta} \right)^2 + \Lambda_b + \frac{\beta^8}{A} + \Lambda_1 \delta(y) + \frac{2\beta^8}{A} \right]. \quad (18)$$

Using Eqs. (4) and (5), we can rewrite Λ_{eff} as

$$\Lambda_{\text{eff}} = \Lambda_{\text{eff}}^{(1)} + \Lambda_{\text{eff}}^{(2)}, \quad (19)$$

where

$$\Lambda_{\text{eff}}^{(1)} = - \int_{-\infty}^{\infty} dy \left(\frac{2}{3} \Lambda_b + \frac{1}{3} \Lambda_1 \delta(y) \right) \beta^4, \quad (20)$$

$$\Lambda_{\text{eff}}^{(2)} = - \frac{8}{3A} \int_0^{\infty} dy \beta^{12}.$$

Using solution (9), and conditions (7), (8), and (10), we can show that

$$\Lambda_{\text{eff}}^{(1)} = -2 \frac{kk_1}{a} \frac{1}{\cosh(c)} + \left[2 \frac{k^2}{a} \tan^{-1} \sinh(4ky + c) \right]_0^{\infty},$$

$$\Lambda_{\text{eff}}^{(2)} = 2 \frac{k^2}{a} \frac{\sinh(c)}{\cosh^2(c)} - \left[2 \frac{k^2}{a} \tan^{-1} \sinh(4ky + c) \right]_0^{\infty}, \quad (21)$$

which leads to $\Lambda_{\text{eff}}^{(1)} + \Lambda_{\text{eff}}^{(2)} = 0$, in agreement with the flat 4D metric [Eq. (2)].

So far we have presented a model for a simple Lagrangian with a $1/H^2$ term only. However, we can show that more general Lagrangians containing only negative powers of H^2 can have the self-tuning solutions. If the Lagrangian contains $\sum_n [a_n / (\langle H^2 \rangle)^n]$ with $a_1 > 0$ and larger for the $1/\langle H^2 \rangle$ dominance, the last terms of the (55) and $(\mu\mu)$ Einstein equations, (4) and (5), are changed to

$$- \sum_n \frac{C_n(\beta)}{A_n}, \quad - \sum_n \frac{(2n+1)C_n(\beta)}{A_n}, \quad (22)$$

respectively, with $A_1 > 0$. Then, by checking the two equations, we obtain $C_n(\beta) = \beta^{8n}$ for consistency. The (55) equation then gives

$$|\beta'| = \sqrt{-\frac{\Lambda_b}{6} \beta^2 - \sum_n \frac{\beta^{8n+2}}{6A_n}}. \quad (23)$$

Equation (23) gives $\beta' \rightarrow 0$ as $\beta \rightarrow 0$ if $n \geq 0$, which guarantees the existence of the solution. But, for $n < 0$, there exists a naked singularity and there is no solution. Suppose that $1/\langle H^2 \rangle$ is given and other corrections are powers of H^2 . Then, these small corrections in $(H^2)^m$ ($m > 0$) can be brought approximately into the form $\sum_{n \geq 0} a_n / (\langle H^2 \rangle + H^2)^n$, where the $n = 1$ term is dominant. Thus, if the corrections contain only the $(H^2)^n$ -type terms, the existence of the self-tuning solution is intact.

Before concluding, we must point out that the self-tuning solution [Eq. (9)] is stable in the sense that the

metric perturbation around the solution does not lead to tachyons [12].

In conclusion, we obtained a solution for self-tuning of the cosmological constant in 5D theory with the Z_2 symmetry. For the self-tuning solution to exist, the bulk cosmological constant must be negative, $\Lambda_b < 0$, and we need a specific form for the gravitational interaction of the three-index antisymmetric tensor field A_{MNP} .

This work is supported in part by the BK21 program of the Ministry of Education, Korea Research Foundation Grant No. KRF-2000-015-DP0072, CTP Research Fund of Seoul National University, and Center for High Energy Physics(CHEP), Kyungpook National University.

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