ORBIFOLD COMPACTIFICATIONS WITH THREE FAMILIES OF SU(3)×SU(2)×U(1)^n

L.E. IBÁÑEZ, Jihn E. KIM [1], H.P. NILLES and F. QUEVEDO
CERN, CH-1211 Geneva 23, Switzerland

Received 10 March 1987

We construct several $N=1$ supersymmetric three-generation models with SU(3)×SU(2)×U(1)^n gauge symmetry, obtained from orbifold compactification of the heterotic string in the presence of constant gauge-background fields. This Wilson-line mechanism also allows us to eliminate extra colour triplets which could mediate fast proton decay.

1. Introduction. Recent studies of four-dimensional string theories [1–4] have led to a generalized impression that, starting from heterotic strings, it is possible to obtain almost any four-dimensional model we want. However, phenomenologically interesting models have not yet emerged from the different analyses of consistent string theories in four dimensions. Even though chiral supersymmetric models are easily obtained, the gauge group and/or the number of generations generally appears to be unrealistic.

Recently, a mechanism was found [3] with which it is possible to reduce the number of generations and at the same time break the gauge group in orbifold compactification of the heterotic string. This was done through the consideration of Wilson lines, i.e. constant gauge-background fields corresponding to the non-contractible loops of the torus underlying the orbifold. In the present note, we use the above mechanism to construct three-generation models with the gauge symmetry SU(3)×SU(2)×U(1)^n. Our aim is – more than stressing the phenomenological virtues of the models – to illustrate how the existence of Wilson lines allows us to have good control over the types of models we have, in such a way that we can select the "interesting ones" by appropriate choices of the Wison lines and the different embeddings of the point group of the orbifold in the gauge group. Using this mechanism, we can eliminate extra colour triplets and thus avoid the risk of them inducing fast proton decay.

2. Orbifolds and Wilson lines. For definiteness, we will restrict our discussion to the Z orbifold, which is the six-torus defined by the $[SU(3)]^3$ lattice, modded out by the discrete group $P=Z_3$. This group acts as $2\pi/3$ rotations of the lattice vectors. The Z orbifold has 27 (fixed) points which are invariant, up to a lattice vector, under the action of P. The translations on the lattice e_i, together with the discrete rotations θ on P, form the space group S. The action of S can be extended to the gauge degrees of freedom, where both the discrete rotations and translations in the space–time lattice are represented by shifts in the $E_8\times E_8$ lattice. To a discrete rotation θ we assign a shift v^I ($I=1, ..., 16$), and to a translation by e_i^μ ($i=1, ..., 6$) we assign the shifts a_i^I corresponding to the Wilson lines $\int_i A_\mu^I \, dx^\mu = 2\pi A_\mu^I e_i^\mu = 2\pi a_i^I$. These shifts cannot be chosen arbitrarily. The fact that P is of order 3 implies that $3v^I$ is a lattice vector; this, together with the group law on S, implies that $3a_i^I$ is also a lattice vector. Also, since a rotation by θ relates two SU(3) lattice vectors, we have

$$a_i^I = a_{i+1}^I\ , \quad i=1, 3, 5\ . \qquad (1)$$

Therefore, there are only three independent Wilson lines. Modular invariance also imposes restrictions on the choices of v^I and a_i^I, as we will see later.

[1] Permanent address: Department of Physics, Seoul National University, Seoul 151, Korea.

0370-2693/87/$ 03.50 © Elsevier Science Publishers B.V. (North-Holland Physics Publishing Division)

The Hilbert space of the string in an orbifold splits into two types of sectors: twisted and untwisted. The untwisted sector corresponds to the string closed in the original torus. The mass formula in this sector is that of the original $E_8 \times E_8$ string,

$$\tfrac{1}{4}m_R^2 = \tfrac{1}{4}m_L^2 = \tfrac{1}{2}(p^I)^2 + N_L - 1 , \tag{2}$$

where p^I ($I=1, ..., 16$) are the momenta in the $E_8 \times E_8$ lattice, and N_L is the left-movers number operator. The massless states are given by the $E_8 \times E_8$ roots $[(p^I)^2=2]$ projected onto those which are Wilson-line singlets, i.e. [5]

$$p^I a_i^I = n_i , \quad n_i \in \mathbb{Z} . \tag{3}$$

These roots are split into three groups, according to how they transform under $e^{2\pi i p \cdot v}$. The invariant states

$$p^I v^I = m , \quad m \in \mathbb{Z} , \tag{4}$$

combine with the right-movers $|i\rangle_R$ and $|a\rangle_R$ (i, $a=1, ..., 8$), which are invariant under P, to make the gauge bosons' multiplet. The roots with

$$p^I v^I = \tfrac{2}{3} \bmod 1 \tag{5}$$

make invariant states after combining with the three right-moving states $|i\rangle_R$, $|a\rangle_R$ which transform as $e^{-2\pi i/3}$ under P. They make three copies of matter fields transforming under some representation of the gauge group. Their antiparticles are obtained from the roots satisfying $p^I v^I = -\tfrac{2}{3} \bmod 1$.

The twisted sectors correspond to strings closed up to an action of P. They are required for modular invariance because they can be obtained from a $\tau \to -1/\tau$ transformation on the untwisted sector partition function. This transformation shifts the $E_8 \times E_8$ lattice by

$$p^I \to p^I + v^I + n_i a_i^I , \quad i=1, 3, 5 , \tag{6}$$

where $n_i = 0, \pm 1$. Different values of n_i lead to different twisted sectors. Since in the Z orbifold there are 27 twisted sectors, corresponding to the 27 fixed points, this means that turning on the three independent Wilson lines will make all the twisted sectors inequivalent. This is a consequence of the fact that different fixed points are invariant up to different lattice vectors. Invariance under $\tau \to \tau + 3$ on these sectors imposes the constraints [6]

$$3(v^I + n_i a_i^I)^2 = 2m , \quad m \in \mathbb{Z} . \tag{7}$$

There is then a modular-invariance constraint for every twisted sector. The mass formulae for the twisted sectors are modified by the shifts in the $E_8 \times E_8$ lattice as well as by the modded oscillators' contribution to the zero point energy. In our case we have

$$\tfrac{1}{4}m_R^2 = \tfrac{1}{4}m_L^2 = \tfrac{1}{2}(p^I + v^I + n_i a_i^I)^2 + N_L - \tfrac{2}{3} , \tag{8}$$

where now $N_L = 0, \tfrac{1}{3}, ...$. Similar to the case of the untwisted sector, a projection which selects only the invariant states should be performed in the twisted sectors. A careful treatment of the partition functions shows that all the states satisfying the massless conditions coming from eq. (8) survive the projection. The antiparticles come from the shifts $p^I - v^I - n_i a_i^I$.

3. $SU(3) \times SU(2) \times U(1)^n$ models. For a given orbifold, many different models that are consistent with the constraints of the types (1) and (7) can be constructed, depending on the different choices of embeddings v^I and Wilson lines a_i^I (and other background fields in more general models). The number of consistent models is then very large, and it would be interesting to have a general classification of them. On the other hand, the mechanism for constructing the models is simple enough, so it is relatively easy to make a good selection. Here we present different values of v^I and a_i^I which lead to three-family models and to the $SU(3) \times SU(2) \times U(1)^n$ gauge group in the observable sector, starting with the Z orbifold. We will describe in some detail the model in which it is also possible to eliminate the unwanted extra colour triplets; two extra models are given in the appendix.

We take the embedding

$$(v^I) = (\tfrac{1}{3}\tfrac{1}{3}\tfrac{1}{3}\tfrac{1}{3}\tfrac{2}{3}0\tfrac{1}{3}\tfrac{1}{3})(\tfrac{1}{3}\tfrac{1}{3}000\tfrac{1}{3}\tfrac{1}{3}\tfrac{2}{3}) ,$$

and add the Wilson lines

$$(a_1^I) = (\tfrac{1}{3}\tfrac{1}{3}\tfrac{1}{3}\tfrac{2}{3}\tfrac{1}{3}000)(00000\tfrac{2}{3}00) ,$$

$$(a_3^I) = (0000000\tfrac{2}{3})(\tfrac{1}{3}\tfrac{1}{3}\tfrac{1}{3}\tfrac{2}{3}000\tfrac{1}{3}) .$$

It can easily be checked that the modular-invariance conditions (7) are satisfied for this model, and following the discussion in the previous section, we can find its massless spectrum. In order to obtain the gauge group, we have to look for the $E_8 \times E_8$ roots which satisfy relations (3) and (4). They are $\pm(\underline{1, -1, 0}, 0, 0, 0, 0, 0)$, $\pm(0, 0, 0, 1, 1, 0, 0, 0,)$ from the first E_8, and $\pm(1, -1, 0, 0, 0, 0, 0, 0)$, $\pm(0, 0, 1, 1, 0, 0, 0, 0)$ from the second E_8, where the underlining of the numbers means that all permutations are included. These roots correspond to the gauge group

$$[\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)^5] \\ \times[\mathrm{SU}(2)\times\mathrm{SU}(2)\times\mathrm{U}(1)^6]' .$$

The matter fields of the untwisted sector are obtained from the $E_8\times E_8$ roots ($\boldsymbol{p}^2=2$) satisfying eqs. (3) and (5). They are $(\underline{1, 0, 0}, 1, 0, 0, 0, 0)$, $(\underline{1, 0, 0}, 0, -1, 0, 0, 0)$ and $(0, 0, 0, 0, 0, \pm1, -1, 0)$ for the first E_8, and $(\underline{-1, 0}, 1, 0, 0, 0, 0, 0)$, $(\underline{-1, 0}, 0, -1, 0, 0, 0, 0)$, $(\underline{1, 0}, 0, 0, 0, 0, 0, -1)$, $(0, 0, 0, \pm1, 0, 0, 0, 1)$, $(0, 0, 0, 0, -1, 0, -1, 0)$ and $(0, 0, 0, 0, 1, 0, -1, 0)$ for the second E_8. Therefore, the matter fields coming from the untwisted sector are

$$3[(3, 2)(1, 1)'+4(1, 1)(1, 1)'+(1, 1)(2, 2)' \\ +(1, 1)(2, 1)'+(1, 1)(1, 2)'] ,$$

where the numbers in the parentheses are the $[\mathrm{SU}(3)\times\mathrm{SU}(2)]\times[\mathrm{SU}(2)\times\mathrm{SU}(2)]'$ quantum numbers; as mentioned in the previous section, the factor of 3 comes from the right-moving modes. The three left-handed quark doublets are assigned to the above representation, as may also be the case for the three right-handed charged-lepton singlets. Certainly, these fermions alone have SU(3) non-abelian and SU(2) global anomalies which must be cancelled by contributions of the twisted-sector fermions. This is a manifestation of the modular-invariance requirement for the twisted sectors.

Let us consider the twisted sectors. Since we have only two non-vanishing Wilson lines, there are nine sets of twisted sectors (three equivalent sectors on each set). There are massless fields coming from every sector, but we will describe only the sets of sectors that generate the right-handed quark singlets: the sectors with $n_1=1$, $n_3=0$ [see eq. (6)]. For this sector the total shift is

$$\boldsymbol{v}+\boldsymbol{a}_1=(\tfrac{2}{3}\tfrac{2}{3}\tfrac{2}{3}110\tfrac{1}{3}\tfrac{1}{3})(\tfrac{1}{3}\tfrac{1}{3}0001\tfrac{1}{3}\tfrac{2}{3}) .$$

The zero modes satisfy $\frac{1}{2}(\boldsymbol{p}+\boldsymbol{v}+\boldsymbol{a}_1)^2+N_{\mathrm{L}}-\frac{2}{3}=0$. For $N_{\mathrm{L}}=\frac{1}{3}$, this equation does not have a solution, and for $N_{\mathrm{L}}=0$ we obtain the following values for p^I:

$$(\underline{-1-1\quad 0}-1-1\quad 0\quad 0\quad 0) \\ \otimes(00000-10-1) ,$$

$$(-\tfrac{1}{2}-\tfrac{1}{2}-\tfrac{1}{2}-\tfrac{1}{2}-\tfrac{1}{2}-\tfrac{1}{2}-\tfrac{1}{2}-\tfrac{1}{2}) \\ \otimes(00000-10-1) ,$$

$$(-\tfrac{1}{2}-\tfrac{1}{2}-\tfrac{1}{2}-\tfrac{3}{2}-\tfrac{3}{2}-\tfrac{1}{2}-\tfrac{1}{2}-\tfrac{1}{2}) \\ \otimes(00000-10-1) ,$$

$$(-\tfrac{1}{2}-\tfrac{1}{2}-\tfrac{1}{2}-\tfrac{1}{2}-\tfrac{3}{2}\quad \tfrac{1}{2}-\tfrac{1}{2}-\tfrac{1}{2}) \\ \otimes(00000-10-1) ,$$

$$(-\tfrac{1}{2}-\tfrac{1}{2}-\tfrac{1}{2}-\tfrac{3}{2}-\tfrac{1}{2}\quad \tfrac{1}{2}-\tfrac{1}{2}-\tfrac{1}{2}) \\ \otimes(00000-10-1) ,$$

$$(-1-1-1-1-1\quad 0-1\quad 0) \\ \otimes(00000-10-1) .$$

These $E_8\times E_8$ weights transform as $3[(3, 1)+(1, 2)+4(1, 1)]$ under $\mathrm{SU}(3)\times\mathrm{SU}(2)$ and have chiralities opposite to those in the untwisted sector [3]. Thus we obtain three right-handed quark singlets, and lepton or Higgs doublets. The other three right-handed quark singlets are obtained from the three sectors for which $n_1=1$, $n_3=-1$.

Putting the contributions from all the sectors together, we finally obtain

$$3\{(3, 2)+(1, 2)+2(3^*, 1)+(1, 1)\}+12(1, 2) \\ +3(2, 2)'+18(2, 1)'+18(1, 2)'+\text{singlets} ,$$

which is the standard model content plus extra singlets and doublets.

Notice that unlike other known phenomenologically interesting string compactifications, there are no colour triplet fields which could give rise to fast proton decay. The particular form of the Wilson lines chosen is such that those fields disappear from the massless spectrum. This mechanism for getting rid of the dangerous colour triplets is different from the one described in ref. [7], in which only some of the dangerous colour triplets are projected out.

It could also be possible to consider a non-vanishing a_5^l and kill some or all of the extra doublets. Note that the minimum number of doublets we could obtain is one third of the number we have (15), which is also the minimum allowed by phenomenology.

These chiral fermions have neither non-abelian nor SU(2) anomalies. However, as frequently occurs when we adopt non-standard embeddings [5], there are U(1) anomalies. A Green–Schwarz-type mechanism is believed to give a mass to the anomalous U(1) boson [8] and probably induce supersymmetry-breaking by a D-term [9,5]. This may or may not affect the standard model particles directly, depending on the anomalous U(1) assignments of quarks and leptons.

4. Conclusions. We have succeeded in constructing phenomenologically interesting models based on the Z orbifold with Wilson lines. Many similar models can probably be constructed in this manner, using different orbifolds. Our formalism, in which the embeddings are made in an abelian way, has the limitation of leaving the rank of the gauge group unchanged, implying that we have to deal with many extra U(1)'s. It would be interesting to find a mechanism that lowers the rank of the gauge group, and to see if models closer to the standard one can be obtained in that way [10].

The large amount of, up to now, consistent string theories in four dimensions is in some sense discouraging because of the lack of predictive power of the theory. Hopefully, more restrictive constraints will be found which would kill most or all of what are at present thought to be consistent models – if we are lucky enough, only one model will survive containing the world we live in. We may also have a vacuum that is not unique. In any case, proof of the existence of phenomenologically interesting consistent models, and further investigations in this direction, are certainly needed in order to understand the physical implications (if any) of string theories.

One of us (J.E.K.) thanks the CERN Theory Division for the kind hospitality extended to him. J.E.K. is supported in part by the Korean Science and Engineering Foundation and the Ministry of Education.

Appendix.

Model 2.

We take the embedding and the Wilson lines as

$$\boldsymbol{v}=(\tfrac{1}{3}\tfrac{1}{3}\tfrac{1}{3}\tfrac{1}{3}\tfrac{2}{3}000)(\tfrac{2}{3}0000000)\,,$$

$$\boldsymbol{a}_1=(0000000\tfrac{2}{3})(0\tfrac{1}{3}00\tfrac{1}{3}\tfrac{1}{3}\tfrac{1}{3}\tfrac{2}{3})\,,$$

$$\boldsymbol{a}_3=(\tfrac{1}{3}\tfrac{1}{3}\tfrac{1}{3}\tfrac{2}{3}\tfrac{1}{3}0\tfrac{1}{3}\tfrac{1}{3})(\tfrac{1}{3}\tfrac{1}{3}000000).$$

The unbroken gauge group is

$$[\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)^5]\times[\mathrm{SU}(4)\times\mathrm{SU}(2)\times\mathrm{SU}(2)\times\mathrm{U}(1)^3]'\,,$$

and the chiral fermions are

$$3\{(3,2)+(1,2)+2(3^*,1)+(1,1)\}+6\{(3,1)+(3^*,1)\}+18(1,2)$$
$$+3(1,2)(1,1,2)'+3(4,1,2)'+6(4^*,1,1)'+18(1,2,1)'+18(1,1,2)'+\text{singlets}\,.$$

Besides having the unwanted extra quark singlets, this model mixes the non-abelian subgroups of E_8 and E_8', thus ruining the $\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)^n$ structure. However, this can easily be overcome by using the extra Wilson line that we have not yet considered, e.g. $a_5=(0,\ldots,0)(0,0,0,\tfrac{2}{3},0,0,\tfrac{1}{3},\tfrac{1}{3})$, which eliminates the $\mathrm{SU}(2)\times\mathrm{SU}(2)$ symmetry in the hidden sector without changing the main structure of the model – except for the fact that the standard model quark singlets come from independent twisted sectors.

Model 3.

The shift vector and the Wilson lines are taken as

$$\boldsymbol{v}=(\tfrac{1}{3}\tfrac{1}{3}\tfrac{1}{3}\tfrac{1}{3}\tfrac{2}{3}000)(\tfrac{2}{3}0000000)\,,$$

$$\boldsymbol{a}_1=(0000000\tfrac{2}{3})(0\tfrac{1}{3}\tfrac{1}{3}00000)\,,$$

$$\boldsymbol{a}_3=(\tfrac{1}{3}\tfrac{1}{3}\tfrac{1}{3}\tfrac{2}{3}\tfrac{1}{3}0\tfrac{1}{3}\tfrac{1}{3})(\tfrac{1}{3}\tfrac{1}{3}000000)\,.$$

The unbroken gauge group is

$$[\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)^5]\times[\mathrm{SO}(10)\times\mathrm{U}(1)^3]'\,,$$

and the chiral fermions are

$$3\{(3,2)+2(3^*,1)+(1,2)+(1,1)\}$$
$$+3(16)'+12\{(3,1)+3^*,1)\}$$
$$+36(1,2)+\text{singlets}\,.$$

References

[1] K.S. Narain, Phys. Lett. B 169 (1986) 41;
K.S. Narain, M.H. Sarmadi and E. Witten, Nucl. Phys. B 279 (1987) 369;
P. Ginsparg, Harvard preprint HUTP-86/A053 (1986).

[2] H. Kawai, D.C. Lewellen and S.H.H. Tye, Cornell preprint CLNS 86/751 (1986);
W. Lerche, D. Lüst and A.N. Schellekens, Nucl. Phys. B 287 (1987) 447;
I. Antoniadis, C. Bachas and C. Kounnas, Four-dimensional superstrings, École Polytechnique and Berkeley preprint A761.1286/LBL-22709.

[3] L. Dixon, J. Harvey, C. Vafa and E. Witten, Nucl. Phys. B 261 (1985) 678; B 274 (1986) 285.

[4] K. Narain, M.H. Sarmadi and C. Vafa, Harvard preprint HUTP-86/A089 (1987).

[5] L.E. Ibáñez, H.P. Nilles and F. Quevedo, Phys. Lett. B 187 (1987) 25.

[6] C. Vafa, Nucl. Phys. B 273 (1986) 592.

[7] E. Witten, Nucl. Phys. B 258 (1985) 75.

[8] E. Witten, Phys. Lett. B 149 (1984) 637;
K. Pilch and A.N. Schellekens, Nucl. Phys. B 259 (1985) 637.

[9] M. Dine, N. Seiberg and E. Witten, Fayet–Iliopoulos terms in string theory, Princeton preprint (1987).

[10] L.E. Ibáñez, H.P. Nilles and F. Quevedo, Reducing the rank of the gauge group in orbifold compactifications of the heterotic string, preprint CERN-TH.4673 (1987), Phys. Lett. B 192 (1987), to be published.