# COMPACTIFICATION AND AXIONS IN $E_8 \times E_8'$ SUPERSTRING MODELS

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We investigate the axions of $E_8 \times E_8'$ superstring models under the simple dimensional reduction scheme. We find that $E_8'$ gaugino condensation must occur above $10^{11}$ GeV from consideration of the axion energy density problem. In addition, we find that the gauge coupling constant of an abelian gauge interaction descending from $E_6$ is smaller than that of non-abelian interactions at the compactification scale.

Recently discovered anomaly-free superstring theories with gauge groups O(32) or $E_8 \times E_8'$ have attracted a great deal of attention [1–12]. It renders the hope that they may unify the known interactions with gravity.

The $E_8 \times E_8'$ models are favored over the O(32), because the latter models predict vectorlike fermions or massless $Q = \frac{2}{3}$ quarks [4,5]. Requiring $d = 4, N = 1$ supersymmetry to solve the gauge hierarchy problem, we therefore study $E_8 \times E_8'$ models on Calabi–Yau manifolds.

It has been known that this type of models has two axions. Strong *CP* invariance is guaranteed with these two axions [3]. In this paper we investigate the axion physics in the simple dimensional reduction scheme which was proposed by Witten [7]. The resulting $d = 4$ theory can be considered to be qualitatively equivalent to the effective $d = 4$ theory with Calabi–Yau internal space. We also investigate the effect in the low energy effective theory due to the Wilson loop vacuum configuration of the Yang–Mills potential [4,8]. In these investigations, we find *a unification scale dependent correction to the abelian gauge couplings descending from $E_6$*.

The basic method of dimensional reduction is to include only $SU(3) \oplus \widetilde{SU(3)}$ singlet fluctuations around the presumed vacuum configuration where SU(3) denotes the holonomy and $\widetilde{SU(3)}$ is the holonomy embedding in $E_8$ [7]. The relevant $SU(3) \oplus \widetilde{SU(3)}$ singlet bosonic degrees can be written as

$$g_{\mu\nu}^{(10)} = \exp(-3\sigma/M_c) g_{\mu\nu}^{(4)}, \quad g_{mn}^{(10)} = \exp(\sigma/M_c)\delta_{mn},$$

$$H_{\mu\nu\rho} = M_1 \epsilon_{\mu\nu\rho\sigma}\partial^\sigma a_1, \quad H_{\mu mn} = M_2 \epsilon_{mn} \partial_\mu a_2,$$

$$A_\mu^{(10)} = A_\mu, \quad A_m^{(10)} = \langle A_m \rangle + (C_m^a h_a + \text{h.c.}), \tag{1}$$

where $g_{MN}^{(10)}$ is the $d = 10$ metric, $H_{MNP}$ is the three-form gauge invariant field strength and $A_M^{(10)}$ denotes the $d = 10$ Yang–Mills potential. We use $d = 10$ indices $M, N, P$; $d = 4$ indices $\mu, \nu, \rho$; $d = 6$ indices $m, n, p$; SU(3) indices $a, b, c$. $\epsilon_{mn} = \mathrm{diag}(i\sigma_2, i\sigma_2, i\sigma_2)$ corresponds to the complex structure of Calabi–Yau space and the $C_m^a$ satisfy $C_m^a C_m^b = C_{ma}^* C_{mb}^* = 0$ and $C_m^a C_{mb}^* = \delta_b^a$. The $SU(3) \oplus \widetilde{SU(3)}$ singlet fluctuation $h_a$ transforms as 27 of $E_6$. $\langle A_m \rangle$ is the VEV of the Yang–Mills connection which is the same as the spin connection. We define $M_c$, $M_1$ and $M_2$ as mass parameters which make the kinetic terms of $\sigma$, $a_1$ and $a_2$ as canonical ones. Note that $\langle\sigma\rangle$ is not determined at the classical level.

From the action

$$\int d^{10}x(\sqrt{-g^{(10)}})\{-(1/2k^2)R^{(10)}$$
$$-(1/120g^2\phi)\mathrm{Tr}\,F^{MN}F_{MN}$$
$$-[3k^2/2(g^2\phi)^2]H^{MNP}H_{MNP}\}$$
$$=\int d^4x(\sqrt{-g^{(4)}})[-(M_P^2/16\pi)R^{(4)}$$
$$-(1/120\tilde{g}^2)\mathrm{Tr}\,F^{\mu\nu}F_{\mu\nu}-\tfrac{1}{2}(\partial_\mu a_1)^2$$
$$-\tfrac{1}{2}(\partial_\mu a_2)^2-\tfrac{1}{2}(\partial_\mu \sigma)^2+...], \qquad (2)$$

we find

$$M_c^2=(3/4\pi)M_P^2, \quad M_1^2=(\tilde{g}^4/144\pi)M_P^2,$$
$$M_2'^2=(x^8/3)M_1^2,$$
$$g^2\phi/k^2=(x^3\tilde{g}^2/8\pi)M_P^2, \qquad (3)$$

where $x=\exp(\langle\sigma\rangle/M_c)$ and $\tilde{g}$ is the $d=4$ Yang–Mills coupling constant at the compactification scale. It is known that $a_1$ and $a_2$ correspond to axions. We will see later that $M_1$ and $M_2'$ are related to the axion decay constants. Since the radius of the internal space can be regarded as $\sqrt{x}$ times $M_c^{-1}$, the grand unification scale can be identified as [9]

$$\langle A_m\rangle \sim M_{GUT} \sim x^{-1/2}M_c. \qquad (4)$$

With this relation, let us consider the axions $a_1$ and $a_2$. Because we expect two non-abelian gauge interactions [‡1] in the $d=4$ effective theory of the $E_8 \times E_8'$ superstring, the existence of two axions is desirable for strong *CP* invariance [13]. Hereafter, $SU(3)_c \times E_8'$ is assumed to be an exact $d=4$ gauge symmetry. $E_8'$ takes the role of the hidden sector for dynamical supersymmetry breaking [6,7]. Then we must diagonalize two axions by their $SU(3)_c \times E_8'$ instanton couplings. The instanton coupling of the model independent axion $a_1$ comes from the loop effect through the Wess–Zumino terms in the $d=10$ lagrangian. This is manifest from the Bianchi identity for the three-form field strength $H_{MNP}$. For the model dependent axion $a_2$, its instanton coupling resides in the Wess–Zumino term itself.

The Wess–Zumino action is

$$S_c=c\int(-3BX_8+2X_3^0X_7^0), \qquad (5)$$

where the notation of ref. [1] is used. From this, the $d=10$ equation of motion for the two-form potential $B_{MN}$ can be written as

$$\partial_M\{-[3e/(g^2\phi)^2]H^{MM_1M_2}+\Sigma^{MM_1M_2}\}$$
$$=(3c/32k^2)\epsilon^{M_1M_2M_3...M_{10}}$$
$$\times(\tfrac{1}{24}\mathrm{Tr}\,F_{M_3M_4}F_{M_5M_6}F_{M_7M_8}F_{M_9M_{10}}$$
$$-\tfrac{1}{7200}\mathrm{Tr}\,F_{M_3M_4}F_{M_5M_6}\mathrm{Tr}\,F_{M_7M_8}F_{M_9M_{10}}$$
$$-\tfrac{1}{240}\mathrm{Tr}\,F_{M_3M_4}F_{M_5M_6}\mathrm{tr}\,R_{M_7M_8}R_{M_9M_{10}}$$
$$+\tfrac{1}{8}\mathrm{tr}\,R_{M_3M_4}R_{M_5M_6}R_{M_7M_8}R_{M_9M_{10}}$$
$$+\tfrac{1}{32}\mathrm{tr}\,R_{M_3M_4}R_{M_5M_6}\mathrm{tr}\,R_{M_7M_8}R_{M_9M_{10}}), \qquad (6)$$

where $\Sigma_{MNP}$ is a fermion bilinear three-form. The Bianchi identity is

$$dH=-\tfrac{1}{30}\mathrm{Tr}\,F^2+\mathrm{tr}\,R^2. \qquad (7)$$

From eqs. (6) and (7), we obtain $d=4$ axion equations. If we consider only the Yang–Mills fluctuation around the vacuum, we find

$$\partial^2 a_1=-(1/2M_1)(F^i_{\mu\nu}\tilde{F}^i_{\mu\nu}+F'^i_{\mu\nu}\tilde{F}'^i_{\mu\nu}), \qquad (8)$$
$$\partial^2 a_2=-(1/2M_2)(F^i_{\mu\nu}\tilde{F}^i_{\mu\nu}-F'^i_{\mu\nu}\tilde{F}'^i_{\mu\nu}), \qquad (9)$$

where

$$M_2'=(cx^2g^4\phi^2/3840k^2)\epsilon_{mn}\epsilon^{mnpqrs}$$
$$\times\mathrm{Tr}\langle F_{pq}\rangle\langle F_{rs}\rangle M_2, \qquad (10)$$

and $F^i_{\mu\nu}$ ($F'^i_{\mu\nu}$) denotes the $E_6$ ($E_8'$) field strength and $\tilde{F}^i_{\mu\nu}$ ($\tilde{F}'^i_{\mu\nu}$) is its dual. Note the sign difference of $E_6$ and $E_8'$ Pontryagin densities in eqs. (8) and (9). Because of these independent $a_1$ and $a_2$ couplings to field strengths, there are two independent axions and strong *CP* invariance is guaranteed, even though there is another Yang–Mills interaction $E_8'$. From eqs. (8) and (9), we obtain the axion lagrangian

‡1 One can imagine that $E_8'$ is broken down to $U(1)^8$ by a Wilson line element at compactification, but dynamical SUSY breaking prefers an unbroken non-abelian subgroup of $E_8'$.

$$\mathcal{L}_{\text{axions}} = \tfrac{1}{2}(\partial_\mu a_1)^2 + \tfrac{1}{2}(\partial_\mu a_2)^2 - \tfrac{1}{2}(a_1/M_1 + a_2/M_2)F^i_{\mu\nu}\widetilde{F}^i_{\mu\nu} - \tfrac{1}{2}(a_1/M_1 - a_2/M_2)F'^i_{\mu\nu}\widetilde{F}'^i_{\mu\nu}$$
$$= \tfrac{1}{2}(\partial_\mu a)^2 + \tfrac{1}{2}(\partial_\mu a')^2 - (a/2M)F\widetilde{F} - (a'/2M')[F'\widetilde{F}' + (M_2^2 - M_1^2)/(M_1^2 + M_2^2)F\widetilde{F}], \quad (11)$$

where

$$a = (M_1 a_1 + M_2 a_2)/(M_1^2 + M_2^2)^{1/2}, \quad (12a)$$

$$a' = (M_2 a_1 - M_1 a_2)/(M_1^2 + M_2^2)^{1/2}, \quad (12b)$$

and

$$M = \tfrac{1}{2}(M_1^2 + M_2^2)^{1/2}, \quad (13a)$$

$$M' = M_1 M_2/(M_1^2 + M_2^2)^{1/2}. \quad (13b)$$

Now we can see that a$'$ corresponds to the $E_8'$ axion [5], and a corresponds to the QCD invisible axion [14]; both $SU(3)_c$ and $E_8'$ vacuum angles are set to a *CP* conserving value by these axions.

The $E_8'$ is assumed to be stronger than QCD. Then the potential for a$'$ is dominated by the $E_8'$. Using the result on the gluino mass of ref. [6], we get the axion potential $V$

$$V = (\pi/4M_P^2)\Lambda_8^6 |1 - \exp(ia'/F)|^2, \quad (14)$$

where $F = 15M'/16\pi^2$ and $\Lambda_8$ is the scale of $E_8'$ gluino condensation [9]. Therefore, the a$'$ mass is

$$m_{a'}^2 = 64\pi^5\Lambda^6/225M_P^2 M'^2. \quad (15)$$

As is well known, the axion potentials can give rise to cosmological energy density problems [15]. The nondissipated axion energy density at present is proportional to $(\Lambda)^\delta$ where $\Lambda$ is the scale of the phase transition and $\delta$ is a model dependent constant which is larger than $\frac{1}{2}$ in general. For an axion decay constant $F_a$ larger than $10^{12}$ GeV and $\Lambda = \Lambda_{QCD}$, the consideration of cosmological axion energy density requires a fine-tuning of the axion VEV before the phase transition. For the case of a$'$ which corresponds to $F_{a'} \sim 10^{16}$ GeV and $\Lambda_8 \gg \Lambda_{QCD}$, the axion energy density problem is much worse than that of a if the lifetime of a$'$ is larger than the age of the universe [‡2]. The main decay mode of a$'$ is to two massless $E_6$ gauge bosons through the $a'F\widetilde{F}$ term of eq. (11). For the estimate of the a$'$ lifetime, we take

$$M' = M_1 = (\tilde{g}^2 M_P/12\sqrt{\pi}) \simeq 1.1 \times 10^{17}\ \text{GeV},$$

$$(M_2^2 - M_1^2)/(M_1^2 + M_2^2) \simeq 1. \quad (16)$$

The lifetime of a$'$ is

$$\tau_{a'} = (32\pi/N)M'^2/m_{a'}^3 \simeq \{8 \times 10^{115}/[\Lambda\,(\text{GeV})]^9\}\,\text{s}, \quad (17)$$

where $N$ ($\approx 100$) is the number of effective decay channels. Requiring $\tau_{a'} \leqslant 1.5 \times 10^{10}$ yr (1 s), we obtain

$$\Lambda \geqslant 9.7 \times 10^{10}\ \text{GeV}\ (7.5 \times 10^{12}\ \text{GeV}). \quad (18)$$

Note that our bound (18) is consistent with the value $\Lambda \simeq 5 \times 10^{13}$ GeV of ref. [6] which uses the relation $m_{\text{gaugino}} \simeq M_W$. The bound (18) or the bound for $\Lambda$ of ref. [5] is quite general because of the model independent axion. It does not depend on the details of compactification and low energy physics.

Finally let us consider the coupling constants of $d = 4$ gauge groups at the compactification scale. $E_6$ can be broken down to its subgroup by Wilson line elements which preserve $N = 1$ supersymmetry [4,8, 16]. In the simple dimensional reduction scheme, this can be implemented most simply by $\langle A_m \rangle$ along an abelian direction transforming as 78 of $E_6$. Let us decompose $\langle A_m \rangle$ as

$$\langle A_m \rangle = \langle A_m[1,8] \rangle + \langle A_m[78,1] \rangle = \langle A_m[1,8] \rangle + M_{\text{GUT}}\Sigma, \quad (19)$$

where $A_m[1,8]$ ($A_m[78,1]$) transforms as [1, 8] ([78, 1]) under $E_6 \times SU(3)$ and $\Sigma$ is the generator of the $U(1)_\Sigma$ subgroup descending from $E_6$. Because $\Sigma$ is embedded in the adjoint representation of $E_8$, we normalize $\text{Tr}\,\Sigma^2 = 30$. We will consider $G \times U(1)_\Sigma \times E_8'$ where G is a subgroup of $E_6$, which commutes with $U(1)_\Sigma$. Note that the U(1)-breaking mechanism of ref. [3] does not occur because the field strength for $\langle A_m[78,1] \rangle$ vanishes.

‡2 Such a possibility has been considered in ref. [5] under an assumption which is different from that of the present analysis. The basic assumption of ref. [5] was that supersymmetry is badly broken by some unknown reason and gauginos are extremely heavy.

There is a term in the $d = 10$ lagrangian which contributes only to the kinetic energy of the $U(1)_\Sigma$ gauge field. Note that

$$H_{\mu\nu m} = -\tfrac{1}{30}\,\mathrm{Tr}\langle A_m\rangle F_{\mu\nu} + \ldots = -M_{\mathrm{GUT}} G_{\mu\nu} + \ldots, \tag{20}$$

where $G_{\mu\nu}$ is the field strength of the $U(1)_\Sigma$ gauge field. From $3k^2/2g^4\phi^2$, $H_{MNP}H^{MNP}$ and eq. (20), we can calculate the kinetic energy term of the U(1) gauge boson. Using eqs. (3) and (4), we obtain [17]

$$\tilde{g}^2/\tilde{g}_\Sigma^2 = 1 + 576\pi x^{-4} M_{\mathrm{GUT}}^2/M_{\mathrm{P}}^2 \simeq 1 + 432 x^{-5}, \tag{21}$$

where $\tilde{g}_\Sigma^2$ is the $U(1)_\Sigma$ coupling and $\tilde{g}^2$ is the coupling of the other nonabelian gauge group descending from $E_6$. Therefore,

$$\tilde{g}^2|_{\text{compactification scale}} > \tilde{g}_\Sigma^2|_{\text{compactification scale}}. \tag{22}$$

To see the validity of eq. (21), let us consider the role of other possible higher dimensional operators to the $D = 4$ gauge coupling constants. Any possible discrimination of $\tilde{g}$ and $\tilde{g}_\Sigma$ at the unification scale must appear through $\langle A_m[1,78]\rangle$. For example, the operators which depend only on the field strength $F_{MN}$ do not affect eq. (21) because the field strength of $\langle A_m[1,78]\rangle$ is vanishing. Then we consider gauge invariant or covariant operators which have explicit $\langle A_m[1,78]\rangle$ dependence. Some possible operators are $H_{MNP}$, $D_M F_{NP}$ and the higher order covariant derivatives of $F_{MN}$. Among the gauge invariant and Lorentz invariant operators constructed by them, only $H_{MNP}H^{MNP}$ can distinguish $\tilde{g}$ and $\tilde{g}_\Sigma$. For example, the operator $\mathrm{Tr}\, D_M F_{NP} D^M F^{NP}$ gives a null contribution to $D = 4$ gauge kinetic terms of unbroken gauge interactions:

$$\begin{aligned}&\mathrm{Tr}\, D_M F_{NP} D^M F^{NP}\\ &\quad \to \mathrm{Tr}[\langle A_m[1,78]\rangle, F_{\mu\nu}]\,[\langle A^m[1,78]\rangle, F^{\mu\nu}] = 0,\end{aligned} \tag{23}$$

because the unbroken $D = 4$ gauge field strength $F_{\mu\nu}$ commutes with $\langle A_m[1,78]\rangle$. Therefore we conclude that eq. (21) is not affected by the presence of other higher dimensional operators.

The correction to $\tilde{g}^2$ can be neglected for $x \to \infty$ which is the limiting case of flat $d = 10$ space–time. For example, $x > 6$ implies $\tilde{g}^2/\tilde{g}_\Sigma^2 < 1.06$. But $x$ cannot be arbitrarily large. We note that a large $x$ gives a large decay constant for the QCD axion a in view of eqs. (3), (10) and (13a). hence, a large $x$ makes the problem of QCD axion energy density worse ‡3. On the other hand $x$ cannot be too small for certain channels of spontaneous symmetry breaking. If the hypercharge $Y$ of the standard model is a linear combination of generators of $U(1)_\Sigma$ and a U(1) from G, i.e. $g'_Y = \tilde{g}_\Sigma \cos\alpha$, a small $x$ implies a very small $g'^2_Y$. Namely, $x = 1$ implies $g'^2 < g_2^2/433$ at the compactification scale, which is not acceptable. Therefore, the low energy phenomenology is very sensitive to the details of compactification. Even though the ratio (21) is highly dependent on the compactification, it gives an interesting low energy implication. In particular, the renormalization group analysis in these theories [16,18] a la Georgi, Quinn and Weinberg [19] must take this effect into account.

In conclusion, we find the following. Strong *CP* invariance is guaranteed because there exist two axions $a_1$ and $a_2$ whose instanton couplings are not proportional. The $E_8'$ axion a′ must decay rapidly enough so that it does not cause a cosmological energy density problem. The axion (a′) energy density problem is quite general, which does not depend on the compactification schemes. The abelian and non-abelian gauge couplings descending from $E_6$ are different at the compactification scale. Even though their ratio is highly dependent upon compactification, this will be useful for a phenomenological application to $\sin^2\theta_{\mathrm{w}}$.

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‡3 For the QCD axion a, the cosmological energy density problem exists irrespective of the $x$ value because of the relation $M = \frac{1}{2}(M_1^2 + M_2^2)^{1/2} > M_1/2$.

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