COSMOLOGICAL GRAVITINO REGENERATION AND DECAY

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We present a detailed study of gravitino production and decay subsequent to cosmological inflation. We calculate the cross sections for gravitino production in collisions of particles in the supersymmetric standard model, and use them to calculate the regenerated abundance of gravitinos as a function of the maximum reheating temperature T_{max}. An upper limit on the gravitino mass density during cosmological nucleosynthesis requires $T_{max} < 0.90 \times 10^{16}$ GeV and considerations of the entropy released when gravitinos decay require $T_{max} < 2.2 \times 10^{13}$ GeV, while more careful analyses of their decay products' disruptive effects on light nuclei and on the microwave background radiation suggest $T_{max} < 10^{9}-10^{10}$ GeV.

Many phenomenological supersymmetry models [‡1] employ the super-Higgs mechanism to break supersymmetry spontaneously, and envisage the existence of a gravitino with mass $m_{3/2} = O(M_W) = O(100)$ GeV. Such a gravitino could be a severe cosmological embarrassment [2], since its lifetime is longer than the age of the universe during cosmological nucleosynthesis. One must ensure that it does not alter the rate of expansion of the universe during that epoch, which would alter the primordial abundances of light nuclei. Furthermore, subsequent gravitino decays should not disrupt conventional cosmology, for example by their decay products producing excess entropy, dissociating the light nuclei previously produced, or distorting the microwave background. It has been proposed that this cosmological gravitino problem may be solved by an inflationary epoch which suppresses the primordial gravitino abundance. Initial calculations [3] underestimated [4] the abundance of gravitinos produced after inflation but before nucleosynthesis, but it still seems that the abundance of gravitinos during and after nucleosynthesis may be acceptably low if the maximum temperature T_{max} to which the universe reheated at the end of the inflationary epoch was sufficiently low [5].

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[‡1] For a recent review see ref. [1].

The purpose of this paper is to investigate this possibility in more detail than has been done previously [2–5], making an analysis which is as quantitative as possible. This we do in three steps. First we set up the Feynman rules for gravitino interactions, and use them to calculate gravitino production cross sections at high energies, and the gravitino decay rate. Next we use these results to estimate the gravitino number density produced subsequent to inflation, which is essentially linear in the maximum reheating temperature T_{max}. Finally, we set upper limits on the gravitino number density, and hence on T_{max}, by requiring (a) that their mass density at the moment of neutrino decoupling during nucleosynthesis be less than the energy density in a single neutrino species, (b) that the entropy released in gravitino decay be less than the total entropy present in other particles during the decay epoch, (c) that the gravitino's decay products do not break up light nuclei, and (d) that they do not distort the microwave background. All these constraints are quite plausible, though the bounds (c) and (d) may be subject to more uncertainties than are (a) and (b). Finally, we discuss the implications of our results, commenting in particular on the implications of our results for inflationary models [‡2].

[‡2] For a recent review see ref. [6].

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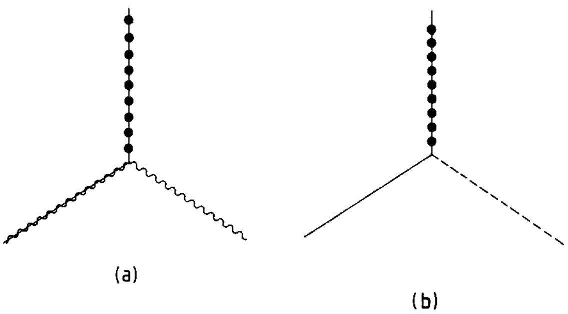

Fig. 1. Feynman diagrams for gravitino interaction.

Gravitino production cross sections and decay rates. The couplings of gravitinos to ordinary and supersymmetric partners can be read from the lagrangian of ref. [7]. The dominant contributions to high energy cross sections come from the helicity ±3/2 components of the gravitino ψ_μ, not the helicity $\pm\frac{1}{2}$ goldstino components. The dimension-five operator coupling the helicity ±3/2 components of the gravitino to a vector boson A_μ and its gaugino partner λ is

$$(1/4M)\, \bar{\lambda}^a \gamma^\mu \sigma^{\rho\sigma} \psi_\mu F^a_{\rho\sigma} + (\text{h.c.})\,, \qquad (1)$$

where $M = M_{\text{Pl}}/\sqrt{8\pi}$, which gives the Feynman rule shown in fig. 1a. The dimension-five operator coupling the helicity ±3/2 components of the gravitino to a chiral scalar z_i and its fermionic partner f_i is

$$(1/2M)\, \bar{\psi}_\mu \not{\partial} z^{*i} \gamma^\mu f_{iL} + (\text{h.c.})\,, \qquad (2)$$

which gives the Feynman rule shown in fig. 1b. However, we do not use (2) in our calculation because it involves the Z field and is not so important as (1). We have used these together with conventional gauge theory Feynman rules to calculate all the high energy $2 \to 2$ cross sections for pairs of supersymmetric standard model particles colliding to produce gravitinos, with the results shown in table 1.

We see that, as expected from the forms of the couplings (1) and (2), all the high energy cross sections are $O(1/M^2)$. We have exhibited the dependence on the centre-of-mass scattering angle θ, and note that the processes $\lambda + \lambda$, $\lambda + f$, and $\lambda + \tilde{f}$ exhibit forward peaks, and the processes $A + \lambda$ and $\lambda + \lambda$ exhibit both forward and backward peaks at $t \sim 1 \mp \cos\theta \to 0$. In order to integrate over this singularity to get the total gravitino production, we have incorporated in the massless t-channel propagator an effective mass m_{eff} due to plasma effects:

$$(1 \mp \cos\theta)_{\min} = m_{\text{eff}}^2/2T^2\,. \qquad (3)$$

We do not expect this ratio to be many orders of magnitude less than unity. Accordingly, the resulting logarithmic term in the total cross section in the second row of the third column of table 1 is not expected to be large. Numerically, we take $\ln(m_{\text{eff}}^2/T^2) = 0$.

We evaluate the cross sections of table 1 using the particles of the minimal supersymmetric standard model (MSSM): SU(3) × SU(2) × U(1) gauge bosons and gauginos, three generations of quarks and squarks, leptons and sleptons, and two light doublets of Higgses and shiggses. The total number N of degrees of freedom in the MSSM is:

$$N = \sum_{\text{B}} + \frac{7}{8}\sum_{\text{F}} = 122(1 + \tfrac{7}{8}) = \tfrac{915}{4}\,. \qquad (4)$$

The total cross sections in the MSSM are exhibited in the fourth column of table 1 with a weighting factor 3/4 for every initial state fermion. Adding together all these entries we find

$$\Sigma_{\text{tot}} = \sum_{\substack{\text{initial}\\ \text{states}}} \sigma_t$$

$$= M^{-2}(15.59 g_3^2 + 5.25 g_2^2 + 1.65 g'^2)\,. \qquad (5)$$

We expect the lightest SUSY particle to be the photino, in which case the gravitino can decay: $\psi_\mu \to \gamma + \tilde{\gamma}$. If the gravitino is heavy enough, other decay channels may be open to it, but the $\psi_\mu \to \gamma + \tilde{\gamma}$ mode calculated using fig. 1a gives a lower bound on the gravitino decay rate:

$$\Gamma(\psi_\mu \to \gamma + \tilde{\gamma}) = (m_{3/2}^3/4M_{\text{Pl}}^2)(1 - m_{\tilde{\gamma}}^2/m_{3/2}^2)^3\,. \qquad (6)$$

From this we deduce the partial lifetime for $m_{\tilde{\gamma}} \ll M_{3/2}$:

$$\tau(\psi_\mu \to \gamma + \tilde{\gamma}) = 4 \times 10^8\ \text{s}(100\ \text{GeV}/m_{3/2})^3\,. \qquad (7)$$

If the gluinos are also lighter than the gravitino, we obtain the following gravitino lifetime:

$$\tau(m_{\tilde{\gamma}}, m_{\tilde{g}} \ll m_{3/2}) = 4.4 \times 10^7\ \text{s}(100\ \text{GeV}/m_{3/2})^3. \qquad (8)$$

It is clear that cosmologically produced gravitinos with masses in the range of interest to phenomenological supergravity models, namely 20 GeV $< m_{3/2} <$ O(1) TeV, would have decayed sometime between cosmological nucleosynthesis and recombination.

Table 1
Differential and total cross sections for gravitino production. The angle between momenta of the first initial particle and the gravitino (ψ_μ) is denoted as θ. In the last column, we multiplied 3/4 for each initial fermion.

	Process	$d\sigma/d(\cos\theta) \sim$	$\sigma_{tot} = (1/128\pi M^2) \times$	$\sigma_t^{MSSM} = (1/128\pi M^2) \times$
(A)	$A^a + A^b \to \lambda^c + \psi_\mu$	$5/2 + \cos^2\theta$	$g^2 \sum_c f_{abc} f_{abc} (17/3)$	$(34) \times (4g_3^2 + g_2^2)$
(B)	$A^a + \lambda^c \to A^b + \psi_\mu$	$10/(1+\cos\theta)$ $+(-15+9\cos\theta)/4$	$\left(g^2 \sum_b f_{abc} f_{abc}\right)$ $\times [10 \ln(T^2/m_{eff}^2) + 20 \ln 2 - 15/2]$	$(3/4) \times (24 g_3^2 + 6 g_2^2)$ $\times [10 \ln(T^2/m_{eff}^2) + 20 \ln 2 - 15/2]$
(C)	$A^a + \tilde{f}_i \to f_j + \psi_\mu$	flat	$2g^2 C_2(f)$	$96 g_3^2 + 42 g_2^2 + 22 g'^2$
(D)	$A^a + f_i \to \tilde{f}_j + \psi_\mu$	$1 + \cos\theta$	$g^2 C_2(f)$	$(3/4) \times (48 g_3^2 + 21 g_2^2 + 11 g'^2)$
(E)	$f_i + \tilde{f}_j \to A^b + \psi_\mu$	$1 - \cos\theta$	$g^2 C_2(f)$	$(3/4) \times (48 g_3^2 + 21 g_2^2 + 11 g'^2)$
(F)	$\lambda^a + \lambda^b \to \lambda^c + \psi_\mu$	$46/(1-\cos^2\theta)$ $-(3+4\cos\theta+\cos^2\theta)$	$\left(g^2 \sum_c f_{abc} f_{abc}\right)$ $\times [(23/4) \ln(T^2/m_{eff}^2) + (23/2) \ln 2 - 20/3]$	$(9/16) \times (24 g_3^2 + 6 g_2^2)$ $\times [(23/4) \ln(T^2/m_{eff}^2) + (23/2) \ln 2 - 20/3]$
(G)	$\lambda^a + f_i \to f_j + \psi_\mu$	$2/(1-\cos\theta) - 1$	$g^2 C_2(f)$ $\times [2 \ln(T^2/m_{eff}^2) + 4 \ln 2 - 2]$	$(9/16) \times (48 g_3^2 + 21 g_2^2 + 11 g'^2)$ $\times [2 \ln(T^2/m_{eff}^2) + 4 \ln 2 - 2]$
(H)	$\lambda^a + \tilde{f}_i \to \tilde{f}_j + \psi_\mu$	$1/(1-\cos\theta)$	$g^2 C_2(f)$ $\times [(1/2) \ln(T^2/m_{eff}^2) + \ln 2]$	$(3/4) \times (48 g_3^2 + 21 g_2^2 + 11 g'^2)$ $\times [(1/2) \ln(T^2/m_{eff}^2) + \ln 2]$
(I)	$f_i + \bar{f}_j \to \lambda^a + \psi_\mu$	$1 - \cos^2\theta$	$g^2 C_2(f) \times (1/3)$	$(9/16) \times (16 g_3^2 + 7 g_2^2 + 11 g'^2/3)$
(J)	$\tilde{f}_i + \tilde{f}_j \to \lambda^a + \psi_\mu$	$\cos^2\theta$	$g^2 C_2(f) \times (1/6)$	$8 g_3^2 + 7 g_2^2/2 + 11 g'^2/6$

Cosmology with gravitinos. If the primordial abundance of gravitinos has been suppressed by inflation, the total rate of gravitino production subsequently is given by

$$dn_{3/2}/dt = \Sigma_{tot}[n(T)]^2 . \tag{9}$$

Using

$$t = (90/32\pi^3 N)^{1/2} M_{Pl}/T^2 , \tag{10}$$

where N is given by eq. (4), we have

$$dn_{3/2}/dT = -\Sigma_{tot}[2\zeta(3)/\pi^2]^2 (90/\pi^3 N)^{1/2} M T^3. \tag{11}$$

Hence the gravitino density is mostly produced at temperatures close to the maximum reheating temperature T_{max}, and the density at low temperatures T_f is by integrating (11)

$$n_{3/2}(T_f) = \Sigma_{tot}[2\zeta(3)/\pi^2]^2 (90/\pi^3 N)^{1/2} \times M T_{max} T_f^3 . \tag{12}$$

Evaluating the expression (5) for Σ_{tot} using the standard renormalization group evolutions for α_3, α_2 and α':

$$\alpha_3(T) \simeq 0.0635(1 - 0.024 \ln T_9) ,$$
$$\alpha_2(T) \simeq 0.0400(1 + 0.00675 \ln T_9) ,$$
$$\alpha'(T) \simeq 0.0115(1 + 0.0237 \ln T_9) , \tag{13}$$

where $T_9 \equiv T/10^9$ GeV, we evaluate

$$n_{3/2}(T_f) = 3.35 \times 10^{-12} T_9^{max} T_f^3 \times (1 - 0.018 \ln T_9^{max}) , \tag{14}$$

which is essentially linear in T^{max} in the range of interest.

Our lifetime estimate (8) told us that gravitinos did not decay before cosmological nucleosynthesis. In order to maintain the validity of conventional models of primordial nucleosynthesis, we must therefore ensure that their energy density just after neutrino decoupling at $T \simeq 0.8$ MeV can be no larger than that of an additional neutrino species

$$m_{3/2} n_{3/2} < \rho_\nu = \tfrac{7}{8} \cdot 2(\pi^2/30)\, T_\nu^4 = 0.061 \text{ MeV}^4, \tag{15}$$

where we have taken $(915/43)^{1/3} T_f = T_\nu = (0.8$ MeV/1.4). Using eqs. (14) and (15) we deduce the constraint

$$m_{100} T_9^{max}(1 - 0.018 \ln T_9^{max}) < 3.65 \times 10^7 , \tag{16}$$

where $m_{100} = (m_{3/2}/100$ GeV). Thus the helium abundance condition at 0.8 MeV would tolerate T_{max} up to 5.4×10^{16} GeV if $m_{3/2} = 100$ GeV.

However, the energy bound on non-relativistic particles at the time of nucleosynthesis is more involved because of the faster expansion rate after the neutrino decoupling. This case is different from the massless neutrino case. Another effect is that the ^{4}He formation rate is not as effective as the standard case after passing through the deuterium bottleneck because the universe might be expanding faster. Kolb and Scherrer [8] have analyzed this problem for the case of massive neutrinos and found that the heavy neutrino should be heavier than 25 MeV. This translates in our case to the constraint that the gravitinos can only dominate the matter density after the temperature falls below 3.3×10^{-2} MeV. For the gravitino number density given by eq. (14), we obtain:

$$m_{100} T_9^{max}(1 - 0.018 \ln T_9^{max}) < 6.38 \times 10^6 , \tag{17}$$

where we used $T_\gamma = [(11/4) \cdot (915/43)]^{1/3} T_f$. Thus we can tolerate T_{max} up to 9.0×10^{15} GeV if $m_{3/2} = 100$ GeV.

A more severe restriction on T_{max} follows from the requirement that the entropy produced when the gravitinos eventually decay at $t = O(10^8)$ s [see the estimates (7) and (8)] does not exceed that already present in other particles:

$$B_\gamma m_{3/2} n_{3/2}(T_D) < (\pi^2/30) N_D T_D^4 . \tag{18}$$

In this equation, T_D is the temperature of the universe when gravitinos decay, which we estimate using (7), (8) and (10) to be

$$T_D = (90/32\pi^3 N_D)^{1/4} (m_{3/2}^3/4M_{Pl})^{1/2} \times 1 , \quad \times 3 , \tag{19}$$

where the factors 1 and 3 correspond to eqs. (7) and (8), respectively, and N_D is the number of degrees of freedom at T_D: $e^\pm$, γ, and a reduced contribution from neutrinos gives $N_D = 3.363$. The factor B_γ on the left-hand side of eq. (18) has been included because the photinos $\tilde{\gamma}$ produced by gravitino decay do not interact with the rest of the universe, whereas the photons γ do. Corresponding to eqs. (7) and (8), we take $B_\gamma = 0.5$ and 0.8, respectively. From eqs. (14), (18) and (19), we deduce

$$T_9^{max}(1 - 0.018 \ln T_9^{max}) < m_{100}^{1/2} \times 0.94 \times 10^4 , \quad \times 1.77 \times 10^4 , \tag{20}$$

i.e., the second condition of eq. (20) gives

$$T_{max} < 2.2 \times 10^{13}\ \mathrm{GeV}\,, \tag{21}$$

for $m_{3/2} = 100$ GeV, from the requirement that the entropy in gravitino decay products does not exceed that in all other particles.

This is not, however, the most stringent constraint we can apply. When the gravitinos decay, the universe contains light nuclei such as D, ^{3}He, ^{4}He and ^{7}Li. The observed abundances of all these species are in good agreement with conventional calculations of their abundances due to primordial nucleosynthesis. Therefore, we should ensure that the photons produced in gravitino decay do not dissociate the delicate light nuclei [9]. The best constraint seems to be provided by considering D dissociation.

The most stringent bound coming from D dissociation is obtained in the following way. A gravitino with mass O(100) GeV will dump ultimately 80% of its energy in the form of photons after all hadrons and charged leptons annihilate. The remaining 20 GeV will end up in photinos. The photons lose their energy rapidly until their energy is degraded to ~3 GeV through e^+e^- pair production by scattering with background photons. Then they lose energy by Compton scattering on electrons and nuclei. A naive estimate is the following. One gravitino produces about thirty thousand 2.3 MeV photons (note that the D dissociation cross section is maximum just above the photon energy threshold). One such photon has the following probability to destroy a deuterium nucleus [9]

$$p(E) = n_D \sigma_D / n_e \sigma_T\,, \tag{22}$$

where E is the photon energy, n_D, n_e, σ_D and σ_T are the D number density, electron number density, D dissociation cross section and total cross section (e.g. γe, γp, etc.), respectively. In fact, this naive estimate gives a bound on T_{max} a factor (2.53) times smaller than the one we cite in the following paragraph.

A better estimate is obtained by integrating the probability distribution given in eq. (22) over the photon energy distribution expected from 3 GeV photons degraded by Compton scattering. The corresponding calculation has been performed by Lindley [10] and the result is

$$(m_{3/2} f)\,\beta n_{3/2}/n_e E_* \lesssim 1\,. \tag{23}$$

In our case, $f = 0.8$ for the case (8), $\beta = 0.23$ corresponds to a value $T \simeq 10^{-4}$ MeV at the time of the gravitino decay, $n_e = \frac{7}{8} n_B$, and $E_* = 100$ MeV [10]. Therefore, from eqs. (14) and (22), we obtain

$$m_{100} T_9^{max}(1 - 0.018 \ln T_9^{max}) \lesssim 2.03 \times 10^{10} \delta_B\,, \tag{24}$$

where $\delta_B = n_B/n_\gamma$ at the time of the gravitino decay. For $\delta_B = 10^{-11}$, 10^{-10}, 10^{-9}, and 10^{-8}, we obtain $T_{max} = 2.0 \times 10^8$ GeV, 2.1×10^9 GeV, 2.2×10^{10} GeV, and 2.3×10^{11} GeV, respectively. Therefore, we can tolerate $T_{max} = 10^9$–10^{10} GeV. Note that if the gluino channel is not open [case (7)], the above numbers should be multiplied by 1.6.

Finally, we discuss the bound coming from the shape of the background radiation spectrum. If 100 GeV gravitino injects high energy photons into the background spectrum, $\Delta\rho_\gamma = 0.8\, m_{3/2} n_{3/2}$ at $t \simeq 10^8$ s, Compton scattering will create a Bose–Einstein photon spectrum with a negative chemical potential μ, $n_\gamma = \{\exp[(h_\nu/kT_1) + \mu] - 1\}^{-1}$. Soft photons generated by bremsstrahlung and/or double Compton scattering will slowly restore a Planck spectrum with a rate proportional to the baryon density or Ω_B. For small perturbations ($\Delta\rho_\gamma \ll aT_0^4$) we can express [11] $\Delta\rho_\gamma/\rho_\gamma \simeq 0.714\mu$ for $|\mu| \ll 1$. Recent observations of the microwave background spectrum may be fitted [12] with $T = 2.92 \pm 0.03$ K and

$$|\mu| = (5 \pm 3) \times 10^{-3} \quad \text{for } \omega = 0.1\,,$$

$$= (1.4 \pm 0.9) \times 10^{-2} \quad \text{for } \omega = 1.0\,, \tag{25}$$

where $\omega = \Omega_B (H/50)^2$. This corresponds to limits on the energy release,

$$\Delta\rho_\gamma/\rho_\gamma < (3.6 \pm 2.1) \times 10^{-3} \quad \text{for } \omega = 0.1\,,$$

$$< (1.0 \pm 0.6) \times 10^{-2} \quad \text{for } \omega = 1.0\,. \tag{26}$$

On the other hand, from eq. (14), we have

$$\Delta\rho_\gamma/\rho_\gamma = 1.01 \times 10^{-4}$$

$$\times 1 \times m_{100}^{-1/2} T_9^{max}(1 - 0.018 \ln T_9^{max})\,,$$

$$\times \tfrac{1}{3} \times m_{100}^{-1/2} T_9^{max}(1 - 0.018 \ln T_9^{max})\,, \tag{27}$$

where the factors 1 and $\frac{1}{3}$ correspond to cases (7) and (8), respectively. From eqs. (26) and (27), we obtain, for $\omega = 0.1$,

$$T_{max} = 3.9 \times 10^{10} \sqrt{m_{100}}\ \mathrm{GeV} \quad \text{for } \psi_\mu \to \gamma\tilde{\gamma}\,, \tag{28}$$

$$T_{max} = 1.2 \times 10^{11} \sqrt{m_{100}}\ \mathrm{GeV} \quad \text{for } \psi_\mu \to \tilde{\gamma}\gamma, \tilde{g}g, \qquad (28\ \text{cont'd})$$

respectively. This bound seems to be less stringent than that from deuterium dissociation.

We have calculated the gravitino number density produced by $a + b \to c + \psi_\mu$ after inflation using the $N = 1$ supergravity interaction. This expression has been combined with various constraints from cosmology to deduce the maximum allowable temperature after inflation. At present, the most stringent bound $T_{max} = 10^9$–10^{10} GeV comes from the upper limit on deuterium dissociation. This is a severe constraint on supersymmetric inflationary models.

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