ANTI-SU(5)

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We discuss ordinary as well as supersymmetric SU(5) × $\tilde{U}(1)$ models in the hope of accommodating acceptable τ_p and $\sin^2\theta_W$. The ordinary SU(5) × $\tilde{U}(1)$ model does not have the monopole. The supersymmetric SU(5) × $\tilde{U}(1)$ model can be unified in SO(10).

1. Grand unified theories (GUTs) provide a well-defined framework capable of unifying weak, electromagnetic and strong interactions [1]. The "hard" predictions of GUTs include $\sin^2\theta_W \cong 0.215$, $m_b/m_\tau \sim 2.9$ and proton decay mainly to $e^+\pi^0$ with a lifetime $\tau_p \sim 10^{29\pm 1}$ y. On the other hand, GUTs have a rich topological structure such that superheavy monopoles ($M \sim 10^{16}$ GeV) are contained in the particle spectrum of the theory [2]. Present experimental evidence disfavours either proton decay to $e^+\pi^0$ [3] or the existence of monopoles as predicted in the minimal SU(5) [1] [‡1].

It is remarkable that by going supersymmetric [5], both the above problems are naturally eliminated, while $\sin^2\theta_W$ and m_b/m_τ remain unchanged [‡2]. Namely, in SUSY GUTs the $p \to e^+\pi^0$ mode is naturally suppressed (νK or μK modes are the favourable channels) while a delayed phase transition from the GUT to the SU(3) × SU(2) × U(1) phase ($T_c \sim 10^{10}$ GeV) or inflation [7] evades the monopole problem.

Nevertheless, it is of considerable interest and a challenging problem to construct ordinary GUTs which do not suffer from the above diseases. This is the problem that we address in this paper, out of scientific curiosity, since we are fully aware of the "goodies" of SUSY models. It is by considering the introduction of an extra $\tilde{U}(1)$ which contains a part of the electromagnetic gauge group $U(1)_{em}$ that there is no stable monopole in the theory and the monopole problem does not exist. Furthermore, the SU(N) coupling constant and the $\tilde{U}(1)$ coupling constant can be arbitrary and hence the proton lifetime can be made sufficiently longer.

With proper phenomenological inputs, we calculate the SU(N) coupling constant g_N^2 and the $\tilde{U}(1)$ coupling constant $\tilde{g}_1^2$ at the SU(3) × SU(2) unification scale $\tilde{M}$. If $g_N^2 < g_1^2$ the group SU(N) × $\tilde{U}(1)$ is the partial unification group, and we achieve our objectives. If $g_N^2 > \tilde{g}_1^2$, there exists a possibility of further unification of SU(N) × $\tilde{U}(1)$. Then we cannot resist unifying it in a simple group at $M_u > \tilde{M}$, and in this case, the monopole problem is resolved by the inflationary idea [7]. Indeed, we encounter both of these examples in SU(5) × $\tilde{U}(1)$ models with and without supersymmetry.

The paper is organized as follows. In section 2, we set our rules for finding fermion representations in SU(N) × $\tilde{U}(1)$ models and point out that only one class of models is available for our purpose. In sections 3 and 4, we present SU(5) × $\tilde{U}(1)$ models with and without SUSY, respectively. In section 5, we show that an SU(7) × $\tilde{U}(1)$ model with integer charged leptons is not a viable choice.

2. With educated reasons [8–10], we set the following rules for SU(N) × $\tilde{U}(1)$ theories:

(i) There should not exist triangle anomalies.

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[‡2] For recent reviews see ref. [6].

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(ii) The fermion representation must be chiral under $SU(N) \times \tilde{U}(1)$.

(iii) The fermion representation must be real under the subgroup $SU(3)_c \times U(1)_{em}$.

Let us concentrate on completely antisymmetric fermion representations of the $SU(N)$ groups. This is reasonable because the quarks are believed to be 3 and 3* of $SU(3)_c$. An irreducible fermion representation with m antisymmetrized indices is denoted as R^N_m. There exist three types of triangle anomalies

$$A_1[AAA]^N_m, \quad A_2[AA\tilde{Y}]^N_m, \quad A_3[\tilde{Y}\tilde{Y}\tilde{Y}]^N_m \tag{1}$$

for a fermion loop R^N_m where A and $\tilde{Y}$ inside the bracket represent the external $SU(N)$ gauge bosons or $\tilde{U}(1)$ gauge bosons. Therefore, we satisfy three anomaly free conditions. From conditions of vanishing A_1, A_2 and A_3 anomalies, we obtain

$$\sum_{m=1}^{N-1} n_m \frac{(N-2m)(N-3)!}{(N-m-1)!(m-1)!} = 0, \tag{2}$$

$$\sum_{m=1}^{N-1} n_m \binom{N-2}{m-1} \tilde{Y}_m = 0, \tag{3}$$

$$\sum_{m=1}^{N-1} n_m d_m \tilde{Y}^3_m + n_0 \tilde{Y}^3_0 = 0, \tag{4}$$

where n_m is the number of irreducible representations R^N_m, and $\tilde{Y}_m$ is the $\tilde{U}(1)$ charge of the representation R^N_m. We also introduce an $SU(N)$ singlet R^N_0 whose $\tilde{Y}$ values is $\tilde{Y}_0$. Note that it is generally difficult to satisfy eqs. (2)–(4) without a singlet. With singlet(s), eq. (4) is merely a defining equation for $\tilde{Y}_0$. This definition is possible because a cubic equation has always a real root.

Let us first find out the simplest solution to eqs. (2)–(4). Eq. (4) is satisfied by the introduction of $SU(N)$ singlet(s). The simplest solution is obtained by precise matching of each term in the sum of (2) and (3), which results in the condition

$$\tilde{Y}_m = N - 2m. \tag{5}$$

This solution is equivalent to the hypercharges of irreducible representations of $SU(N)$ when a spinor representation of $SO(2N)$ breaks into $SU(N) \times \tilde{U}(1)$ [11]. The simplest choice is therefore $n_m = 0$ for m = odd and $n_m = 1$ for m = even, which is obtainable from one spinor representation of $SU(2N)$. This representation then satisfies the properties (ii) and (iii) too.

Let us next consider possibilities of more general solutions. For this purpose, the constraints (ii) and (iii) play important roles. Indeed, there exists a study of this problem in the literature [10]. The conclusion is that (reducible) fermion representations with properties (i)–(iii) are possible only for the spinor representations of $SU(2n+1)$, i.e., the representations obtained from the spinor representation of $SO(4n+2)$. In ref. [10], the conclusion was drawn without $\tilde{U}(1)$. Nevertheless, we will get the same result with the inclusion of $\tilde{U}(1)$ also since the result of ref. [10] led to the spinor of $SU(2n+1)$. Thus, possible fermion spectra satisfying properties (i)–(iii) are expected to be:

$$\psi_\alpha + \psi^{\alpha\beta}: SU(5) \times \tilde{U}(1),$$

$$\psi_\alpha + \psi^{\alpha\beta} + \psi_{\alpha\beta\gamma}: SU(7) \times \tilde{U}(1),$$

$$\psi_\alpha + \psi^{\alpha\beta} + \psi_{\alpha\beta\gamma} + \psi^{\alpha\beta\gamma\delta}: SU(9) \times \tilde{U}(1). \tag{6}$$

We show this property explicitly for $SU(5) \times \tilde{U}(1)$ and $SU(7) \times \tilde{U}(1)$. The complexity property is apparent from the representations (6). The reality property is equivalent to the existence of all possible Yukawa couplings which can give masses to all $SU(3)_c \times U(1)_{em}$ non-trivial fermions. Therefore, we study all possible Yukawa couplings instead of checking the $SU(3)_c \times U(1)_{em}$ property. We know that $SU(2n+1)$ fermions are real if all possible Yukawa couplings are allowed. Thus, we prove the reality property by the following dictum. First, write down all possible Yukawa couplings allowed by the $SU(2n+1)$ gauge symmetry only. Then, assing $\tilde{U}(1)$ hypercharge by the formula (3). If some couplings are forbidden by the $\tilde{U}(1)$ hypercharge, there is a chance that some fermions do not get masses. If $\tilde{U}(1)$ hypercharges of Higgs fields are not completely determined, there will remain a global symmetry which forbids mixings between the fermion generations or even some fermions would not get masses. However, if the $\tilde{U}(1)$ hypercharges of the Higgs fields are uniquely determined, then we obtain the desired reality property.

For $SU(5) \times \tilde{U}(1)$, the hypercharge assignment by eq. (3) is idential to the one by eq. (5), i.e.,

$$\tilde{Y}(\psi_\alpha) = -3, \quad \tilde{Y}(\psi^{\alpha\beta}) = 1. \tag{7}$$

The Yukawa couplings are,

$$\psi_\alpha \psi^{\alpha\beta} H_\beta, \quad \psi^{\alpha\beta}\psi^{\gamma\delta} H^\epsilon \epsilon_{\alpha\beta\gamma\delta\epsilon}, \tag{8a,b}$$

which uniquely determine

$$\widetilde{Y}(H^\alpha) = -2. \tag{9}$$

Therefore, we satisfy the reality condition. For SU(7) $\times$ $\widetilde{\mathrm{U}}(1)$, we can satisfy eq. (3) by the following assignment

$$\widetilde{Y}(\psi_{\alpha\beta\gamma}) = -1, \quad \widetilde{Y}(\psi^{\alpha\beta}) = y, \quad \widetilde{Y}(\psi_\alpha) = 10 - 5y. \tag{10}$$

The relevant Yukawa couplings are

$$\psi_\alpha \psi^{\alpha\beta} H_\beta, \quad \psi^{\alpha\beta}\psi_{\alpha\beta\gamma} H^\gamma,$$

$$\psi_\alpha \psi_{\beta\gamma\delta} H_{\mu\nu\rho} \epsilon^{\alpha\beta\gamma\delta\mu\nu\rho}, \quad \psi^{\alpha\beta}\psi^{\gamma\delta} H^{\mu\nu\rho}\epsilon_{\alpha\beta\gamma\delta\mu\nu\rho}. \tag{11}$$

Eqs. (10) and (11) are satisfied with $y = 3$, i.e.,

$$\widetilde{Y}(\psi_\alpha) = -5, \quad \widetilde{Y}(\psi^{\alpha\beta}) = 3, \quad \widetilde{Y}(\psi_{\alpha\beta\gamma}) = -1,$$

$$\widetilde{Y}(H^\alpha) = -2, \quad \widetilde{Y}(H^{\alpha\beta\gamma}) = -6, \tag{12}$$

which agrees with the assignment (5). The spinor representation is real.

We have also checked this reality property for SU(9) $\times$ $\widetilde{\mathrm{U}}(1)$. It is believed that the spinor representation of SU($2n + 1$) $\times$ $\widetilde{\mathrm{U}}(1)$ with the hypercharge given by eq. (5) satisfies properties (i)–(iii) provided a SU($2n + 1$) singlet has $\widetilde{Y}(\psi_0) = 2n + 1$.

3. Let us consider an SU(5) $\times$ $\widetilde{\mathrm{U}}(1)$ model [‡3] without SUSY. This model has the same particle assignment as Barr's [13], but we differ in philosophy from his by not unifying $\widetilde{\mathrm{U}}(1)$ within SO(10). The electromagnetic charge Q_{em} is given by

$$Q_{\mathrm{em}} = I_3 - \tfrac{1}{5}Y' + \tfrac{1}{5}\widetilde{Y}, \tag{13}$$

where

$$1: \quad Y' = 0, \quad \widetilde{Y} = 5, \tag{14}$$

$$\bar{5}: \quad Y' = (\tfrac{1}{3}, \tfrac{1}{3}, \tfrac{1}{3}; -\tfrac{1}{2}, -\tfrac{1}{2}),$$

$$\widetilde{Y} = (-3, -3, -3; -3, -3), \tag{15}$$

$$10: \quad Y' = (-\tfrac{2}{3}, -\tfrac{2}{3}, -\tfrac{2}{3}; \tfrac{1}{6}, \tfrac{1}{6}, \tfrac{1}{6}, \tfrac{1}{6}, \tfrac{1}{6}, \tfrac{1}{6}; 1),$$

$$\widetilde{Y} = (1, 1, 1; 1, 1, 1, 1, 1, 1; 1). \tag{16}$$

Defining the coupling constants associated with T_3, Y' and $\widetilde{Y}$ by g_2, g' and $\widetilde{g}$, respectively, we have a relation [14]

$$1/e^2 = 1/g_2^2 + 1/25g'^2 + 1/25\widetilde{g}^2. \tag{17}$$

To study and compare the evolution of coupling constants, it is useful to define properly normalized generators $Y_1' = C'Y'$ and $\widetilde{Y}_1 = \widetilde{C}\widetilde{Y}$ such that (on the sixteen states)

$$\mathrm{Tr}(Y_1'^2) = \mathrm{Tr}(\widetilde{Y}_1^2) = \mathrm{Tr}(I_3^2) = 2 \tag{18}$$

(i.e., $C'^2 = 3/5$ and $\widetilde{C}^2 = 1/40$) and the associated coupling constants verify

$$1/e^2 = 1/g_2^2 + 1/15g_1'^2 + 8/5\widetilde{g}_1^2. \tag{19}$$

In particular, we obtain for $\sin^2\theta_{\mathrm{W}}$ at the SU(5) unification scale $\widetilde{M}$ where $g_2(\widetilde{M}) = g_1'(\widetilde{M}) \equiv g_5$

$$\sin^2\theta_{\mathrm{W}}^0 = \tfrac{3}{8}\{1 + \tfrac{3}{5}[(g_5^2/\widetilde{g}_1^2)_{\widetilde{M}} - 1]\}^{-1}, \tag{20}$$

where g_5 is the unification coupling constant at $\widetilde{M}$. If $\widetilde{g}_1 = g_5$, $\sin^2\theta_{\mathrm{W}}^0 = 3/8$ as expected. To have a larger proton decay rate than the one of the SU(5) model, we must start with a relation $|g_5| < |\widetilde{g}_1|$ at $\widetilde{M}$ so that the prediction of $\sin^2\theta_{\mathrm{W}}(M_{\mathrm{W}})$ is untouched with a larger GUT gap, $M_{\mathrm{W}} - \widetilde{M}$. Because of the condition $|g_5| < |\widetilde{g}_1|$, we cannot further unify SU(5) $\times$ $\widetilde{\mathrm{U}}(1)$.

The evolution of coupling constants is

$$1/g_3^2(M_{\mathrm{W}}) = 1/g_5^2 + (1/8\pi^2)(-11 + \tfrac{4}{3}N_{\mathrm{g}})\ln(\widetilde{M}/M_{\mathrm{W}}), \tag{21}$$

$$1/g_2^2(M_{\mathrm{W}}) = 1/g_5^2 + (1/8\pi^2)(-\tfrac{22}{3} + \tfrac{4}{3}N_{\mathrm{g}} + \tfrac{1}{6}N_{\mathrm{H}})\ln(\widetilde{M}/M_{\mathrm{W}}), \tag{22}$$

$$1/\widetilde{g}_1^2(M_{\mathrm{W}}) = 1/\widetilde{g}_1^2(\widetilde{M}) + (1/8\pi^2)(\tfrac{4}{3}N_{\mathrm{g}} + \tfrac{1}{10}N_{\mathrm{H}})\ln(\widetilde{M}/M_{\mathrm{W}}), \tag{23}$$

where N_{g} and N_{H} are numbers of families and Higgs

‡3 In another context, SU(5) $\times$ U(1) was considered in ref. [12].

Table 1
$\widetilde{M}$ and $(g_5^2/\widetilde{g}_1^2)_{\widetilde{M}}$ in ordinary SU(5) × $\widetilde{\mathrm{U}}(1)$.

N_H	$\sin^2\theta_W(M_W)$	$\alpha_c(M_W)$	$\widetilde{M}$	$\sin^2\theta_W(\widetilde{M})$	$(g_5^2/\widetilde{g}_1^2)_{\widetilde{M}}$
1	0.215	0.10	2.18×10^{14}	0.359	1.074
1	0.215	0.13	9.58×10^{15}	0.382	0.970
1	0.215	0.16	1.02×10^{17}	0.397	0.910
1	0.225	0.10	1.73×10^{15}	0.382	0.969
1	0.225	0.13	7.59×10^{16}	0.405	0.875
1	0.225	0.16	8.07×10^{17}	0.420	0.820
2	0.215	0.10	6.63×10^{13}	0.351	1.116
2	0.215	0.13	2.49×10^{15}	0.372	1.012
2	0.215	0.16	2.40×10^{16}	0.386	0.952
2	0.225	0.10	4.81×10^{14}	0.373	1.009
2	0.225	0.13	1.81×10^{16}	0.395	0.914
2	0.225	0.16	1.74×10^{17}	0.410	0.859

doublets. The value $g_1^2(\widetilde{M})$ is related to g_5^2 and $\widetilde{g}_1^2(\widetilde{M})$ by

$$1/g_1^2(\widetilde{M}) = 1/25g_5^2(\widetilde{M}) + 24/25\widetilde{g}_1^2(\widetilde{M}). \tag{24}$$

From (21) and (22), we obtain a useful relation

$$\ln(\widetilde{M}/M_W) = 2\pi(\sin^2\theta_W/\alpha_{em} - 1/\alpha_c)/(\tfrac{11}{3} + \tfrac{1}{6}N_H)|_{M_W}. \tag{25}$$

For various input parameters of N_H, $\sin^2\theta_W(M_W)$ and $\alpha_c(M_W)$, we present in table 1 the values $\widetilde{M}$, $\sin^2\theta_W(\widetilde{M})$, and $(g_5^2/\widetilde{g}_1^2)_{\widetilde{M}}$. For example, for $N_g = 3$, $N_H = 1$, $\sin^2\theta_W(M_W) = 0.215$, $\alpha_c(M_W) = 0.13$ and $\alpha_{em}(M_W)^{-1} = 128$, we obtain $\widetilde{M} \cong 9 \times 10^{15}$ GeV, $\tau_p \approx 10^{35}$ y, and $(g_5^2/\widetilde{g}_1^2)_{\widetilde{M}} = 0.97$. If higher order effect does not change the relation $(g_5^2/\widetilde{g}_1^2)_{\widetilde{M}} < 1$, we cannot unify SU(5) × $\widetilde{\mathrm{U}}(1)$ in a simple group for this attractive set of input parameters [‡4]. In this case, there would not exist a monopole and τ_p is too long to be observed by current proton decay detectors.

4. For the case of SUSY SU(5) × $\widetilde{\mathrm{U}}(1)$, we obtain

$$1/g_3^2(M_W) = 1/g_5^2 + (1/8\pi^2)(-9 + 2N_g)\ln(\widetilde{M}/M_W), \tag{26}$$

$$1/g_2^2(M_W) = 1/g_5^2 + (1/8\pi^2)(-6 + 2N_g + \tfrac{1}{2}N_H)\ln(\widetilde{M}/M_W), \tag{27}$$

$$1/g_1^2(M_W) = 1/g_1^2(\widetilde{M}) + (1/8\pi^2)(2N_g + \tfrac{3}{10}N_H)\ln(\widetilde{M}/M_W), \tag{28}$$

$$\ln(\widetilde{M}/M_W) = 2\pi(\sin^2\theta_W/\alpha_{em} - 1/\alpha_c)/(3 + \tfrac{1}{2}N_H)|_{M_W}. \tag{29}$$

In table 2, we present the values of $\widetilde{M}$, $\sin^2\theta_W(\widetilde{M})$ and $(g_5^2/\widetilde{g}_1^2)_{\widetilde{M}}$ for several input parameter sets. We note that for the case of $N_H = 2$ reasonable values of $\widetilde{M}$ are obtained from acceptable values of $\sin^2\theta_W(M_W)$. Furthermore, it generally gives $(g_5^2/\widetilde{g}_1^2)_{\widetilde{M}} > 1$, implying a possibility of unification in SO(10). For example, for $N_g = 3$, $N_H = 2$, $\sin^2\theta_W(M_W) = 0.215$, $\alpha_c(M_W) = 0.13$, and $\alpha_{em}(M_W)^{-1} = 128$, we obtain $\widetilde{M} \cong 2.5 \times 10^{15}$ GeV, $\tau_p \cong 10^{33}$ y, and $(g_5^2/\widetilde{g}_1^2)_{\widetilde{M}} \cong 1.32$. The case $N_H = 4$ is not successful.

5. In this section, we present an analysis for the SU(7) × $\widetilde{\mathrm{U}}(1)$ model based on the fermion spectrum of ref. [15]. For the fermion spectrum obtainable from SO(4n + 2), it is generally true to have only two patterns of SU(2n + 1) × $\widetilde{\mathrm{U}}(1)$: one is the usual SU(2$n$ + 1) and the other is the anti-SU(2n + 1) × $\widetilde{\mathrm{U}}(1)$. The Dynkin weight diagram of the spinor of SO(4n + 2) has a distinctive shape [11,15]. The interconnected central part is connected to two strings with two weights on each string. The weight on the end of a string is either the highest or the lowest weight. Only these two weights can be singlets under SU(2n + 1) × $\widetilde{\mathrm{U}}(1)$, since we can disconnect only one simple root from either of these two to make the

[‡4] However, note that we can have $(g_5^2/\widetilde{g}_1^2)_{\widetilde{M}} > 1$ for $N_H = 2$.

Table 2
$\widetilde{M}$ and $(g_5^2/\widetilde{g}_1^2)_{\widetilde{M}}$ in SUSY SU(5) × $\widetilde{\mathrm{U}}(1)$.

N_{H}	$\sin^2\theta_{\mathrm{W}}(M_{\mathrm{W}})$	$\alpha_{\mathrm{c}}(M_{\mathrm{W}})$	$\widetilde{M}$	$\sin^2\theta_{\mathrm{W}}(\widetilde{M})$	$(g_5^2/\widetilde{g}_1^2)_{\widetilde{M}}$
2	0.215	0.10	6.63×10^{13}	0.294	1.456
2	0.215	0.13	2.49×10^{15}	0.315	1.316
2	0.215	0.16	2.40×10^{16}	0.330	1.225
2	0.225	0.10	4.81×10^{14}	0.322	1.275
2	0.225	0.13	1.81×10^{16}	0.347	1.136
2	0.225	0.16	1.74×10^{17}	0.365	1.046
4	0.215	0.10	2.76×10^{11}	0.251	1.823
4	0.215	0.13	5.02×10^{12}	0.261	1.730
4	0.215	0.16	3.07×10^{13}	0.268	1.667
4	0.225	0.10	1.34×10^{12}	0.272	1.631
4	0.225	0.13	2.44×10^{13}	0.284	1.533
4	0.225	0.16	1.49×10^{14}	0.293	1.467

root a singlet under SU($2n+1$) × $\widetilde{\mathrm{U}}(1)$. The analysis presented in this part can be applied to SU(7) × $\widetilde{\mathrm{U}}(1)$ models with fractionally charged leptons.

The anti-SU(7) × $\widetilde{\mathrm{U}}(1)$ model has the following relations,

$$Q_{\mathrm{em}} = I_3 + Y' + \tfrac{1}{7}\widetilde{Y}, \tag{30}$$

$$Y'(7) = \mathrm{diag}\left(\tfrac{13}{21}, \tfrac{13}{21}, \tfrac{13}{21}; -\tfrac{3}{14}; -\tfrac{3}{14}; -\tfrac{5}{7}, -\tfrac{5}{7}\right), \tag{31}$$

$$\widetilde{Y}(1) = 7, \quad \widetilde{Y}(\overline{7}) = -5, \tag{32,33}$$

$$\widetilde{Y}(21) = 3, \quad \widetilde{Y}(\overline{35}) = -1, \tag{34,35}$$

$$1/e^2 = 1/g_2^2 + \left(\frac{1}{8}\sum Y'^2\right)\Big/ g_1'^2 + \left(\frac{1}{49}\frac{1}{8}\sum \widetilde{Y}^2\right)\Big/ \widetilde{g}_1^2, \tag{36}$$

where g_1' and $\widetilde{g}_1$ are the coupling constants for properly normalized generators. Since

$$\mathrm{Tr}\, Y'^2(\psi^\alpha) = \tfrac{95}{42}, \tag{37}$$

$$\mathrm{Tr}\, Y'^2(\psi^{\alpha\beta}) = (N-2)\,\mathrm{Tr}\, Y'^2(\psi^\alpha), \tag{38}$$

$$\mathrm{Tr}\, Y'^2(\psi^{\alpha\beta\gamma}) = \tfrac{1}{2}(N-2)(N-3)\,\mathrm{Tr}\, Y'^2(\psi^\alpha), \tag{39}$$

we have

$$\mathrm{Tr}\, Y'^2(64) = 16\times\tfrac{95}{42}, \tag{40}$$

$$\mathrm{Tr}\, \widetilde{Y}^2(64) = 16\times 28. \tag{41}$$

Therefore,

$$1/e^2 = 1/g_2^2 + 95/21 g_1'^2 + 8/7\widetilde{g}_1^2, \tag{42}$$

$$\sin^2\theta_{\mathrm{W}}^0 = \tfrac{21}{116}/[1 + \tfrac{6}{29}(g_7^2/\widetilde{g}_1^2)_{\widetilde{M}}]. \tag{43}$$

Certainly, we obtain $\sin^2\theta_{\mathrm{W}}^0 = 3/20$ for $g_7 = \widetilde{g}_1$.

The renormalization group analysis of coupling constants does not give acceptable intermediate mass scales, and we do not succeed in the SU(7) × $\widetilde{\mathrm{U}}(1)$ model. However, it will be certainly possible to have acceptable intermediate scales for SU(7) × $\widetilde{\mathrm{U}}(1)$ models with fractionally charged leptons.

6. We have seen that adding supersymmetry to the SU(5) × $\widetilde{\mathrm{U}}(1)$ model leaves open the possibility of subsequent unification. At $\widetilde{M}, g_5/\widetilde{g}_1 > 1$, and then these coupling constants will meet at some new scale $M_{\mathrm{u}} > \widetilde{M}$, where unification into SO(10) for instance can occur. We will now discuss the supersymmetric SO(10) model [‡5] and the possible realization of the symmetry breaking pattern

$$\mathrm{SO}(10) \to \mathrm{SU}(5)\times\widetilde{\mathrm{U}}(1)$$
$$\to \mathrm{SU}(3)_{\mathrm{c}}\times \mathrm{SU}(2)_{\mathrm{L}}\times \mathrm{U}(1)_Y. \tag{44}$$

SO(10) breaking into SU(5) × $\widetilde{\mathrm{U}}(1)$ can, in principle, be induced by a real antisymmetric tensorial representation with an even number of indices, i.e., **45** or **210**, denoted generically by ϕ. **45** does not possess a cubic invariant. Thus, the corresponding superpotential is only a mass term $\tfrac{1}{2}M\,\mathrm{Tr}\,\phi^2$, which

[‡5] Supersymmetric SO(10) models with different patterns of symmetry breaking have been considered in refs. [16, 17].

leads to the vanishing vacuum expectation value (VEV). This negative result can be corrected by coupling **45** to other multiplets, price to be paid being that the scale and the little group of the VEV is fixed by this new sector of the model. This problem does not occur with **210** where the superpotential reads $\frac{1}{2}M\,\mathrm{Tr}\,\phi^2 + (\lambda/3)\,\mathrm{Tr}\,\phi^3$ leading to a VEV of order M/λ which breaks, among other possibilities, SO(10) into SU(5) × $\widetilde{\mathrm{U}}(1)$.

For the second step of symmetry breaking, SU(5) × $\widetilde{\mathrm{U}}(1) \rightarrow$ SU(3) × SU(2) × U(1), the two natural Higgs candidates are **16** and **126**, the desired VEV being in the $\mathbf{10}_1$ part of **16**, and in $\mathbf{50}_2$ of **126**. However, the **126** is more attractive, since its coupling to quark and lepton supermultiplets will give a large Majorana mass to right-handed neutrinos. This is not the case using **16**, and since the non-renormalization properties of SUSY suppress radiative corrections, one would get an unacceptable spectrum for neutrinos. Then, using **210** and **126** + $\overline{\mathbf{126}}$ (denoted respectively by ϕ, ψ, $\overline{\psi}$), the most general cubic (i.e., renormalizable) superpotential is

$$W = \tfrac{1}{2}M\,\mathrm{Tr}\,\phi^2 + \tfrac{1}{3}\lambda\,\mathrm{Tr}\,\phi^3 + \alpha\overline{\psi}\phi\psi + \mu\overline{\psi}\psi, \qquad (45)$$

with indices and gamma matrices omitted for clarity. We need both **126** and $\overline{\mathbf{126}}$ chiral multiplets to have a superpotential for these fields and to cancel the VEV of the gauge part of the potential [17]. This latter requirement enforces the VEVs of **126** and $\overline{\mathbf{126}}$ to have the same scale and the same little group. Solving the minimum equations [18], $(\partial W/\partial\phi) = (\partial W/\partial\psi) = 0$ for this superpotential, leads to two difficulties. First, the natural solution is to obtain unbroken SU(5). The $\overline{\psi}\phi\psi$ coupling has in fact a tendency to align the VEVs of **210** (or **45**) and **126**. The second problem is that this superpotential does not possess two actual scales. The VEV of ϕ is of order M/λ. However, $(\partial W/\partial\psi) = 0$ leads to $\langle\phi\rangle \approx \mu/\alpha$ and a tuning of parameters is necessary. To solve these problems, we need non-renormalizable terms like, for instance,

$$(\beta/M_{\mathrm{p}})\overline{\psi}\psi\overline{\psi}\psi. \qquad (46)$$

To obtain $\langle\phi\rangle \approx M/\lambda = \mathrm{O}(10^{17}\ \mathrm{GeV})$ and $\langle\psi\rangle = \mathrm{O}(10^{16}\ \mathrm{GeV})$, we will have to impose $\alpha/\beta < \mathrm{O}(10^{-4})$ and $\mu/\alpha > \mathrm{O}(10^{17}\ \mathrm{GeV})$. Notice that choosing $\alpha = 0$ leads to pseudo-Goldstone multiplets. Such non-renormalizable terms are naturally obtained in supergravity unified models [6].

7. Conclusions. The anti-SU(5) models, the ordinary and supersymmetric ones, can be realistic unified models with acceptable $\sin^2\theta_W$ and τ_p. The ordinary SU(5) × $\widetilde{\mathrm{U}}(1)$ model is not unified in a simple group and hence there is no stable monopole. The supersymmetric SU(5) × $\widetilde{\mathrm{U}}(1)$ model can be realistically unified in the SO(10) group. Other SU(N) × $\widetilde{\mathrm{U}}(1)$ models without fractionally charged leptons cannot be made realistic.

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