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# Various modified solutions of the Randall–Sundrum model with the Gauss–Bonnet interaction

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## Abstract

The Gauss–Bonnet interaction is the only consistent quadratic interaction below the Planck scale in the Randall–Sundrum compactification. We study various static and inflationary solutions including this Gauss–Bonnet interaction. © 2000 Elsevier Science B.V. All rights reserved.

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## 1. Introduction

Recently, Randall and Sundrum (RS) proposed a compactification scheme with nonvanishing cosmological constant in the bulk [1] which has immediately attracted a great deal of attention [2–5]. The most simple compactification studied in superstring models before the RS proposal has been the orbifold compactification [6] in which the compactified space is flat. On the other hand, the Randall–Sundrum compactification allows a nonflat compactified space, but the analysis is relatively simple. Because of the nonflat nature of the bulk between two branes, there exists an exponential warp factor for metrics going from one brane to the other [1]. This exponential warp factor has been suggested for a large hierarchy between the Planck scale  $M_P = 2.44 \times 10^{18}$  GeV and the electroweak scale  $v \simeq 250$  GeV.

Among two branes, let Brane 1 (B1) be the hidden-sector brane and Brane 2 (B2) the visible sector brane. An exponential warp factor suppresses the soft mass in the visible brane B2, and it is possible to obtain this small ratio because the Higgs mass term at B2 is a dimension two operator. Thus, in the RS world, one changes the traditional gauge hierarchy problem to a problem in geometry. Using the same argument, the nonrenormalizable

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operators are suppressed not by  $M_P$  but by  $v$ . Thus one has to make sure that the theory has a high degree of symmetry to suppress sufficiently the unwanted operators.

Another problem is the problem of inflation. Generally, inflation occurs unless one fine-tunes the bulk cosmological constant and the brane tensions [2–4]. For the fine-tuned relations [3–5], there exist static solutions. So far it has not been shown that any of the static solutions is the  $t \rightarrow \infty$  limit, not allowing a graceful exit from the inflationary period. In addition, there is a possibility that the separation between the branes is expanding or shrinking exponentially. However, this last problem may be understood by introducing a scalar field in the bulk [7].

The most interesting point of the RS world is the interplay of the bulk and the brane world. In particular, the bulk cosmological constant,  $k$ , and the brane tensions,  $k_i$  ( $i = 1, 2$ ), must be related,  $k_1 = k = -k_2$  and  $k_1 > 0$ . But the expansion rate of the observable universe is measured by the Hubble parameter which is a function of  $k$  and  $k_2$ . These  $k$ 's are the appropriately defined from the original bulk cosmological constant  $\Lambda_b$  and the brane tensions  $\Lambda_1, \Lambda_2$  at B1 and B2.<sup>2</sup> One interesting point of the RS compactification is that there may exist a possibility of understanding the old cosmological constant problem.

Below the Planck scale, the higher order effective interaction in the RS model is known to be the Gauss–Bonnet interaction [5]. In contrast to the models without the Gauss–Bonnet interaction, this model allows solutions with a positive  $\Lambda_2$  at the visible brane, which is suitable for a proper expansion in the standard big bang cosmology after the inflationary period.

In Section 2, the RS compactification with the Gauss–Bonnet term is explored. In Section 3, the static background solutions with the Gauss–Bonnet interaction are presented. In Section 4, simple inflationary solutions are given. In Section 5, we present other possible inflationary solutions. In Section 6, metric perturbation near the static background geometry is discussed.

## 2. Gauss–Bonnet interaction

We will neglect the matter interaction, and consider only the gravitational interaction with cosmological constants in the bulk and at the branes. The space–time dimension is  $D = 5$ . The fifth dimension  $x^5 \equiv y$  is compactified with an  $S_1/Z_2$  orbifold. The five-dimensional index is  $M, N = 0, 1, 2, 3, 5$  and the four-dimensional brane world index is  $\mu, \nu = 0, 1, \dots, 3$ . The fifth dimension variable  $y$  ranges in the region  $[0, 1/2]$ . The  $S_1/Z_2$  orbifold is used to locate the two branes at  $y = 0$  and  $y = 1/2$ . The periodicity of  $y$  is 1.

Below the Planck scale, the higher order gravity effects can be added as effective interaction terms. Since we are neglecting the matter interactions, the possible terms in the Lagrangian is, up to  $O(R^2/M^2)$ ,

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<sup>2</sup> Notations for  $k$ 's are borrowed from Refs. [3,4].

$$\begin{aligned}
 S = & \int d^5x \sqrt{-g} \left( \frac{M^3}{2} R - \Lambda_b + \frac{1}{2} \alpha M R^2 + \frac{1}{2} \beta M R_{MN} R^{MN} + \frac{1}{2} \gamma M R_{MNPQ} R^{MNPQ} \right) \\
 & + \sum_{i=1,2 \text{ branes}} \int d^4x \sqrt{-g^{(i)}} (\mathcal{L}_i - \Lambda_i), \tag{1}
 \end{aligned}$$

where  $g, g^{(i)}$  are the determinants of the metrics in the bulk and the branes,  $M$  is the five-dimensional gravitational constant,  $\Lambda_b$  and  $\Lambda_i$  are the bulk and brane cosmological constants, and  $\alpha, \beta, \gamma$  are the effective couplings. We assume that the three-dimensional space is homogeneous and isotropic, and hence the metric is parametrized by  $n, a,$  and  $b$

$$ds^2 = -n^2(\tau, y) d\tau^2 + a^2(\tau, y) \delta_{ij} dx^i dx^j + b^2(\tau, y) dy^2, \tag{2}$$

where the Roman characters  $i, j$  denote the space indices 1, 2, and 3.

It has been found that there exist solutions consistent with the Randall–Sundrum setup if the additional interaction is of the Gauss–Bonnet type [5], namely,  $\beta = -4\alpha$  and  $\gamma = \alpha$  are satisfied. In this case, the higher  $y$  derivative terms,  $n''''$ ,  $n'''n'$ ,  $n''n''$ ,  $a''''$ ,  $a'''a'$ ,  $a''a''$ ,  $\dots$ ,<sup>3</sup> are absent in the left-hand side of the Einstein equation. These conditions for vanishing higher  $y$  derivative terms are necessary since the right-hand side of the Einstein equation contains only one power of the Dirac delta function and the higher derivative terms diverge more rapidly than the delta function at the branes. We find that this result is highly nontrivial.

However, it can be anticipated from the fact that the Gauss–Bonnet term can be rewritten as a pseudoscalar quantity,  $\epsilon^{MNOP} \epsilon_{QRSTU} R_{MN}{}^{QS} R_{OP}{}^{TU}$ . From the antisymmetric property of the Riemann tensor, we observe that there are no  $(a'')^2$ ,  $(n'')^2$  and  $a''n''$  terms<sup>4</sup> in the action which would have given the unwanted fourth order derivatives in the equations of motion. Thus, the Gauss–Bonnet effective interaction does not contain higher  $y$  derivatives.

The Gauss–Bonnet term  $E = R^2 - 4R_{MN}R^{MN} + R_{MNPQ}R^{MNPQ}$  is a total derivative in  $D = 4$  space–time, in which case it does not change the Einstein gravity. On the other hand, for  $D \neq 4$  it is not a topological quantity any more. Still, it does not contribute to the massive poles of the spin-2 propagator [8,9]. It means that the metric variations near the flat space do not give rise to a ghost graviton even with the Gauss–Bonnet term. In general, a combination of the quadratic curvature terms without the Gauss–Bonnet ratio leads to ghosts. But it may not be meaningful if the location of the ghost pole in the graviton propagator is above the Planck scale where the derivative expansion breaks down. However, the Gauss–Bonnet term possibly excites ghost particles near anti-de Sitter space in the sense that the sign of the propagator can be flipped [9].

The general curvature squared terms in any space–time dimension  $D$  can be rewritten as [10],

<sup>3</sup> Here, ' denotes the derivative with respect to  $y$ .

<sup>4</sup> Under the metric assumption Eq. (2), the terms including  $b''$  are absent in the action. We can see it from the antisymmetric property of the Riemann tensor,  $R^M{}_{NST} = \partial_S \Gamma^M{}_{TN} - \dots$  and unique non-vanishing Christoffel symbol containing the first derivative of  $b$  with respect to  $y$  is  $\Gamma^5{}_{55} = b'/b$ .

$$\alpha R^2 + \beta R_{MN}R^{MN} + \gamma R_{MNPQ}R^{MNPQ} \tag{3}$$

$$= -\left[\frac{(D-2)\beta + 4\gamma}{4(D-3)}\right]E + \left(\frac{D-2}{D-3}\right)\left(\frac{\beta}{4} + \gamma\right)C^2 + \left[\frac{4(D-1)\alpha + D\beta + 4\gamma}{4(D-1)}\right]R^2, \tag{4}$$

where  $E$  is the Gauss–Bonnet term,  $C^2$  is the square of the Weyl tensor as follows,

$$E = R^2 - 4R_{MN}R^{MN} + R_{MNPQ}R^{MNPQ}, \tag{5}$$

$$C^2 = R_{MNPQ}^2 - \frac{4}{D-2}R_{MN}^2 + \frac{2}{(D-1)(D-2)}R^2. \tag{6}$$

Note that if the metric is conformally flat and  $16\alpha + 5\beta + 4\gamma = 0$  in  $D = 5$ , the curvature squared terms appear necessarily in the Gauss–Bonnet combination because the Weyl tensor vanishes for a conformally flat metric,  $n(\tau, y) = a(\tau, y) = b(\tau, y)$ . Then the coefficient of the resultant Gauss–Bonnet term becomes  $(8\alpha + \beta)/4$ .

For  $n(\tau, y) = a(\tau, y)$  and  $16\alpha + 5\beta + 4\gamma = 0$ , there still exist higher time derivatives  $\sim (4\alpha + \beta)(\ddot{a}/a - \ddot{b}/b)^2$  in the action from the curvature squared terms. Thus, to eliminate higher time derivatives too, in addition to the condition  $16\alpha + 5\beta + 4\gamma = 0$ , we should choose the Gauss–Bonnet form in curvature squared terms or conformally flat metric,  $n(\tau, y) = a(\tau, y) = b(\tau, y)$ .

Variations of the above action with the Gauss–Bonnet term gives, apart from those for the brane Lagrangian,

$$\begin{aligned} &\sqrt{-g}\left[R_{MN} - \frac{1}{2}g_{MN}R - \frac{\alpha}{2M^2}g_{MN}(R^2 - 4R_{PQ}R^{PQ} + R_{STPQ}R^{STPQ})\right. \\ &\quad + \frac{2\alpha}{M^2}(RR_{MN} - 4R_{MP}R_N^P + R_{MQSP}R_N^{QSP}) + \frac{2\alpha}{M^2}(g_{MN}R_{;P}{}^{;P} - R_{;M;N}) \\ &\quad - \frac{4\alpha}{M^2}(g_{MN}R^{PQ}{}_{;P;Q} + R_{MN;P}{}^{;P} - R_M^P{}_{;N;P} - R_N^P{}_{;M;P}) \\ &\quad \left. + \frac{2\alpha}{M^2}(R_M^P{}_{;N}{}^Q{}_{;P;Q} + R_M^P{}_{;N}{}^Q{}_{;Q;P})\right] \\ &= -M^{-3}\left[\Lambda_b\sqrt{-g}g_{MN} + \Lambda_1\sqrt{-g^{(1)}}g_{\mu\nu}^{(1)}\delta_M^\mu\delta_N^\nu\delta(y)\right. \\ &\quad \left. + \Lambda_2\sqrt{-g^{(2)}}g_{\mu\nu}^{(2)}\delta_M^\mu\delta_N^\nu\delta\left(y - \frac{1}{2}\right)\right], \tag{7} \end{aligned}$$

where 1 refers to the brane of the hidden world B1 and 2 refers to the visible brane B2. The left-hand side of the above equation contains the extra term due to the Gauss–Bonnet term,  $\sqrt{-g}X_{MN}$ , in addition to the familiar Einstein tensor  $\sqrt{-g}G_{MN}$ . With the metric given in Eq. (2), the  $G_{MN}$  and  $X_{MN}$  are

$$G_{00} = 3\frac{\dot{a}}{a}\left(\frac{\dot{a}}{a} + \frac{\dot{b}}{b}\right) - \frac{3n^2}{b^2}\left[\frac{a''}{a} + \frac{a'}{a}\left(\frac{a'}{a} - \frac{b'}{b}\right)\right], \tag{8}$$

$$G_{ii} = -\frac{a^2}{n^2} \left[ 2\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} - \frac{\dot{a}}{a} \left( 2\frac{\dot{n}}{n} - \frac{\dot{a}}{a} \right) - \frac{\dot{b}}{b} \left( \frac{\dot{n}}{n} - 2\frac{\dot{a}}{a} \right) \right] \\ + \frac{a^2}{b^2} \left[ \frac{n''}{n} + 2\frac{a''}{a} + \frac{a'}{a} \left( 2\frac{n'}{n} + \frac{a'}{a} \right) - \frac{b'}{b} \left( \frac{n'}{n} + 2\frac{a'}{a} \right) \right], \quad (9)$$

$$G_{55} = -\frac{3b^2}{n^2} \left[ \frac{\ddot{a}}{a} - \frac{\dot{a}}{a} \left( \frac{\dot{n}}{n} - \frac{\dot{a}}{a} \right) \right] + 3\frac{a'}{a} \left( \frac{n'}{n} + \frac{a'}{a} \right), \quad (10)$$

$$G_{05} = 3 \left( \frac{\dot{a} n'}{a n} + \frac{\dot{b} a'}{b a} - \frac{\dot{a}'}{a} \right), \quad (11)$$

$$X_{00} = \frac{12\alpha}{M^2} \left( \frac{\dot{a}^3 \dot{b}}{a^3 n^2 b} - \frac{\dot{a}^2 a''}{a^3 b^2} + \frac{\dot{a}^2 a' b'}{a^3 b^3} - \frac{\dot{a} \dot{b} a'^2}{a^3 b^3} + \frac{a'^2 a'' n^2}{a^3 b^4} - \frac{a'^3 b' n^2}{a^3 b^5} \right), \quad (12)$$

$$X_{ii} = \frac{4\alpha}{M^2} \left( -2\frac{\ddot{a}\dot{a}\dot{b}}{n^4 b} + 3\frac{\dot{a}^2 \dot{n}\dot{b}}{n^5 b} + 2\frac{\ddot{a}a''}{n^2 b^2} - 2\frac{\ddot{a}a'b'}{n^2 b^3} + \frac{\ddot{b}a'^2}{n^2 b^3} + \frac{\dot{a}^2 n''}{n^3 b^2} \right. \\ - \frac{\dot{a}^2 b''}{n^4 b} - 2\frac{\dot{a}^2 n'^2}{n^4 b^2} - \frac{\dot{a}^2 n' b'}{n^3 b^3} - 2\frac{\dot{a}\dot{n}a''}{n^3 b^2} + 2\frac{\dot{a}\dot{n}a'b'}{n^3 b^3} - 2\frac{\dot{b}^2 a'^2}{n^2 b^4} \\ - 2\frac{\dot{a}\dot{b}a'n'}{n^3 b^3} - \frac{\dot{b}\dot{n}a'^2}{n^3 b^3} - 2\frac{(\dot{a}')^2}{n^2 b^2} + 4\frac{\dot{a}'\dot{a}n'}{n^3 b^2} + 4\frac{\dot{a}'\dot{b}a'}{n^2 b^3} - 2\frac{a''a'n'}{nb^4} \\ \left. - \frac{n''a'^2}{nb^4} + 3\frac{a'^2 n' b'}{nb^5} \right), \quad (13)$$

$$X_{55} = \frac{12\alpha}{M^2} \left( \frac{\dot{a}^3 \dot{n} b^2}{a^3 n^5} - \frac{\ddot{a} \dot{a}^2 b^2}{a^3 n^4} + \frac{\ddot{a} a'^2}{a^3 n^2} + \frac{\dot{a}^2 a' n'}{a^3 n^3} - \frac{\dot{a}\dot{n}a'^2}{a^3 n^3} - \frac{a'^3 n'}{a^3 n b^2} \right), \quad (14)$$

$$X_{05} = \frac{12\alpha}{M^2} \left( \frac{\dot{a}^3 n'}{a^3 n^3} + \frac{\dot{a}^2 \dot{b} a'}{a^3 n^2 b} - \frac{\dot{a} a'^2 n'}{a^3 n b^2} - \frac{\dot{b} a'^3}{a^3 b^3} - \frac{\dot{a}^2 \dot{a}'}{a^3 n^2} + \frac{\dot{a}' a'^2}{a^3 b^2} \right), \quad (15)$$

where ' denotes the derivative with respect to  $y$  and  $\dot{\phantom{x}}$  denote the derivatives with respect to  $\tau$ .

The equation of motions Eq. (7) are

$$G_{MN} + X_{MN} = T_{MN}, \quad (16)$$

where

$$T_{00} = \frac{n^2}{M^3} \left[ \Lambda_b + \frac{\delta(y)}{b} \Lambda_1 + \frac{\delta(y - \frac{1}{2})}{b} \Lambda_2 \right], \\ T_{ii} = -\frac{a^2}{M^3} \left[ \Lambda_b + \frac{\delta(y)}{b} \Lambda_1 + \frac{\delta(y - \frac{1}{2})}{b} \Lambda_2 \right], \\ T_{55} = -\frac{b^2}{M^3} \Lambda_b, \quad T_{05} = 0. \quad (17)$$

### 3. Static solutions

To find static solutions, let us assume that the metric takes the following form,

$$ds^2 = e^{-2\sigma(y)} \eta_{\mu\nu} dx^\mu dx^\nu + b_0^2 dy^2, \tag{18}$$

where the length parameter  $b_0$  is a constant. Note that the modified Einstein equation for the (00) component is identical to the (ii) component in Eq. (16). Thus, the (00), (ii), and (55) components of Eq. (16) lead to two equations,

$$\frac{3\sigma''}{b_0^2} \left( 1 - \frac{4\alpha}{M^2 b_0^2} (\sigma')^2 \right) = \frac{\Lambda_1}{M^3 b_0} \delta(y) + \frac{\Lambda_2}{M^3 b_0} \delta\left(y - \frac{1}{2}\right), \tag{19}$$

$$\frac{6(\sigma')^2}{b_0^2} \left( 1 - \frac{2\alpha}{M^2 b_0^2} (\sigma')^2 \right) = \frac{-\Lambda_b}{M^3}. \tag{20}$$

There exist two solutions of Eq. (20), consistent with the orbifold symmetry  $y \rightarrow -y$ ,

$$\sigma^\pm = b_0 |y| \left[ \frac{M^2}{4\alpha} \left( 1 \pm \left( 1 + \frac{4\alpha \Lambda_b}{3M^5} \right)^{1/2} \right) \right]^{1/2} \equiv k_\pm b_0 |y|. \tag{21}$$

Let us call  $\sigma^+$  (or  $k_+$ ) and  $\sigma^-$  (or  $k_-$ ) solutions ‘+’ and ‘-’ solutions, respectively. Note that both positive and negative bulk cosmological constants are possible for the ‘+’ solution  $\sigma^+$ . If  $\Lambda_b = 0$ , we have an AdS space for the ‘+’ solution irrespective of  $\alpha$ , and a Minkowski space for the ‘-’ solution [9]. The RS solution is obtained by taking  $\alpha \rightarrow 0$  in the ‘-’ solution.

These solutions exist for:

- (i)  $\alpha < 0$  and  $\Lambda_b < 0$  allows only  $\sigma^-$ , and
- (ii)  $\alpha > 0$  allows both  $\sigma^\pm$  solutions. The  $\sigma^+$  solution is possible for both  $\Lambda_b > 0$  and  $\Lambda_b < 0$ . The  $\sigma^-$  solution is possible only for  $\Lambda_b < 0$ . In any case, there exists the lower limit of  $\alpha \Lambda_b$ ,  $\alpha \Lambda_b \geq -3M^5/4$ .

Comparing our results with that of Randall and Sundrum  $k = (-\Lambda_b/6M^3)^{1/2}$ , the “effective” bulk cosmological constant by the Gauss–Bonnet interaction can be defined as

$$\Lambda_{\text{eff}}^{(b)} \equiv -\frac{3M^5}{2\alpha} \left( 1 \pm \sqrt{1 + \frac{4\alpha \Lambda_b}{3M^5}} \right) = -6M^3 k_\pm^2. \tag{22}$$

This is because our geometry of the AdS space guarantees a negative bulk cosmological constant effectively. For  $\sigma^\pm$  to be a real number it should have the negative sign, which is the same as in the RS case.

Considering the discontinuities at the branes,

$$|y'| = 2 \left( \theta(y) - \theta\left(y - \frac{1}{2}\right) \right) - 1, \tag{23}$$

which comes from the periodicity in  $y$  direction and the orbifold symmetry, we obtain two solutions if the following relations among the brane cosmological constants are satisfied

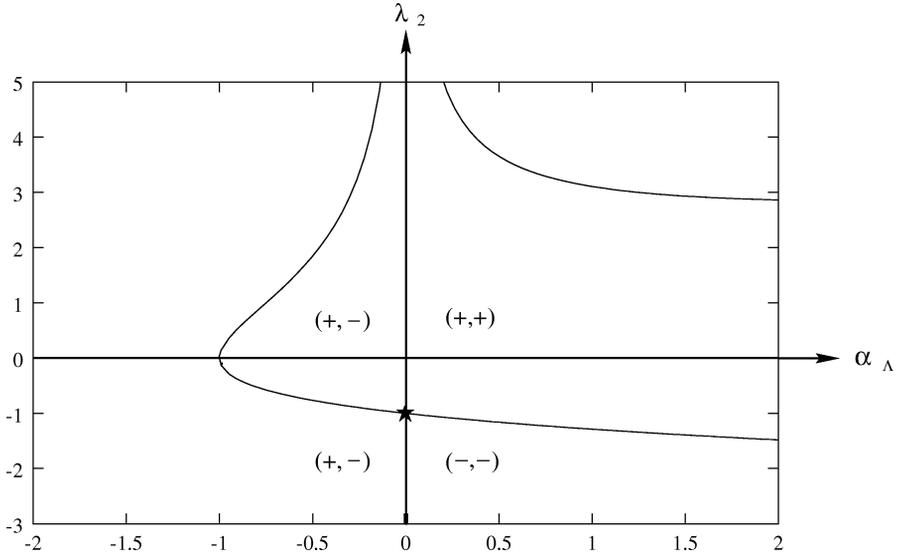


Fig. 1. Possible solutions for  $\lambda_2 \equiv \Lambda_2/\sqrt{6M^3|\Lambda_b|}$  as a function of  $\alpha_\Lambda \equiv 4\alpha\Lambda_b/(3M^5)$ . The star point is the RS solution. The four quadrants have different sets of signs of  $\alpha$  and  $\Lambda_b$ , denoted as (sign of  $\alpha$ , sign of  $\Lambda_b$ ).

$$\begin{aligned} \Lambda_1^\mp &= -\Lambda_2^\pm = \mp 6k_\pm M^3 \sqrt{1 + \frac{4\alpha\Lambda_b}{3M^5}} \\ &= \mp 6M^3 \left[ \frac{M^2}{4\alpha} \left( 1 \pm \left( 1 + \frac{4\alpha\Lambda_b}{3M^5} \right)^{1/2} \right) \left( 1 + \frac{4\alpha\Lambda_b}{3M^5} \right) \right]^{1/2}, \end{aligned} \tag{24}$$

where  $k_\pm > 0$ . The RS solution is obtained by taking  $\alpha \rightarrow 0$  in the ‘-’ solution. *Note that the visible brane can take a positive cosmological constant.* It will be important in the later stage of the evolution of the universe, which will be considered in the following section.

Possible solutions are depicted in Fig. 1 as a function of the Gauss–Bonnet coupling  $\alpha$ . The vertical axis ( $\equiv \lambda_2$ ) is the solution for  $\Lambda_2$  in the visible brane in units of  $\sqrt{6M^3|\Lambda_b|}$ , and the horizontal axis ( $\equiv \alpha_\Lambda$ ) is defined as  $4\alpha\Lambda_b/(3M^5)$ .

Similarly, we can define the “effective” brane cosmological constants as

$$\Lambda_{\text{eff}(i)}^\pm \equiv \frac{\Lambda_i^\pm}{\sqrt{1 + 4\alpha\Lambda_b/3M^5}} \equiv 6M^3 k_{i,\pm}. \tag{25}$$

Thus we see the RS-like fine tuning conditions again,

$$\frac{\Lambda_{\text{eff}(1)}^\mp}{6M^3} = -\frac{\Lambda_{\text{eff}(2)}^\pm}{6M^3} = \mp \sqrt{\frac{-\Lambda_{\text{eff}}^{(b)}}{6M^3}}. \tag{26}$$

The Planck constant at B2 (visible sector) is given by [1]

$$M_P^2 = M^3 b_0 \int_{-1/2}^{1/2} dy e^{-2k_\pm b_0 |y|} = \frac{M^3}{k_\pm} [1 - e^{-k_\pm b_0}]$$

$$= M^2 \left[ \frac{1}{4\alpha} \left( 1 \pm \left( 1 + \frac{4\alpha\Lambda_b}{3M^5} \right)^{1/2} \right) \right]^{-1/2} [1 - e^{-k_{\pm}b_0}]. \tag{27}$$

The Higgs boson mass parameter at the visible sector is obtained by redefining the Higgs field such that the kinetic energy term of the Higgs boson takes a standard form [1]. Thus the Higgs mass parameter is given by

$$m \equiv e^{-\frac{b_0}{2}k_{\pm}}m_0 = m_0 \exp\left(-\frac{b_0}{2}\left[\frac{M^2}{4\alpha}\left(1 \pm \left(1 + \frac{4\alpha\Lambda_b}{3M^5}\right)^{1/2}\right)\right]^{1/2}\right) \tag{28}$$

where  $m_0$  is the mass given in the fundamental Lagrangian, before redefining the Higgs field.

For  $k_+b_0 \simeq 74$ , the ‘+’ solution gives a needed large mass hierarchy through the warp factor  $e^{-\frac{b_0}{2}k_+}$  from the input mass parameter ( $M$ ) of order  $10^{19}$  GeV, leading to a TeV scale observable mass. To achieve a sufficient hierarchy, Randall and Sundrum set  $k^2 = -\Lambda_b/(6M^3) \approx M^2 \approx M_P^2$  and  $b_0/2 \approx 37/M$ . These results can be reproduced for our ‘+’ solution with  $\alpha = O(1)$  and  $\Lambda_b = O(1) \times M^5$ . For example,  $k_+ = M \approx M_P$  for  $\Lambda_b/M^5 = 12\alpha - 6$  and  $\alpha > 1/4$ . In this case, the brane cosmological constants are given by  $\Lambda_1^- = -\Lambda_2^+ = -6M^4|4\alpha - 1|$ . Thus, we have to set  $b_0/2 = 37/M$  to explain the hierarchy between the Planck and TeV scale. Note that in this case  $k_+$  is not zero even for  $\Lambda_b = 0$  or  $\alpha = 1/2$ .

The value of  $k_+$  can be smaller or larger using the parameter  $\alpha$  and the bulk cosmological constant  $\Lambda_b$ . Smaller  $k_+$  require longer interval length to explain the hierarchy between those scales, which results in lighter Kaluza–Klein (KK) modes of the graviton. As the KK modes must interact with the standard model particles through the gravitational interaction, the lighter KK mode has the longer lifetime. And sufficiently longer life time of the KK modes could have an effect on nucleosynthesis. According to Ref. [11], the masses of the KK modes should be larger than about a few GeV, which corresponds to  $k_+b_0/2 \lesssim 40$  to be consistent with the nucleosynthesis scenario. Therefore, smaller  $k_+$  than  $M$  cannot be consistent with the current cosmology.

On the other hand, a larger  $k_+$  corresponds to a larger curvature and then it would locate our theory out of perturbative regime. Thus, it isn’t desirable. We will show, however, in Section 6 that if quadratic curvature terms have the Gauss–Bonnet ratio, at least the quadratic corrections do not affect linearized 4-dimensional Einstein gravity or non-relativistic Newtonian gravity regardless of the curvature’s magnitude.

For  $k_-b_0 \simeq 74$ , the ‘-’ solution has the same behavior. The case of  $\Lambda_b/M^5 = 12\alpha - 6$  and  $\alpha < 1/4$  corresponds to  $k_- = M \approx M_P$  and  $\Lambda_1^+ = -\Lambda_2^- = +6M^4|1 - 4\alpha|$ . Then we have only to set  $b_0/2 = 37/M$  to explain the hierarchy between two scales too. Of course, we also have the freedom to make the  $k_-$  smaller or larger than  $M$  depending on the parameters  $\alpha$  and  $\Lambda_b$ .

The small warp factor [1] makes it possible to generate a TeV scale mass from the fundamental parameter of  $O(M)$ . But these TeV scale masses also appear in the other mass parameters of the effective operators. In particular, the operators leading to proton decay are also parametrized by a TeV scale mass. Therefore, one has to suppress

sufficiently the low dimensional proton decay operators such that it is sufficiently long-lived ( $\tau_p > 10^{32}$  years), allowing operators with  $D > 14$  only.

In non-Gauss–Bonnet cases satisfying the condition  $16\alpha + 5\beta + 4\gamma = 0$ , the RS type solution is still valid except for the substitution  $4\alpha \rightarrow 8\alpha + \beta$  in Eq. (21) because the RS metric can be redefined to be conformally flat.

#### 4. Inflationary solutions

For inflationary solutions we impose an ansatz,

$$n = f(y), \quad a = g(\tau)f(y), \quad b = b_0, \tag{29}$$

where  $b_0$  is a constant. Now adding the (00) and (ii) equations in Eq. (16), we obtain

$$-2\left(\frac{\dot{g}}{g}\right) \left[ 1 - \frac{4\alpha}{M^2 b_0^2} \frac{f''}{f} \right] = 0. \tag{30}$$

Since  $f''$  necessarily gives rise to a delta function we should take  $(\dot{g}/g)' = 0$ . So we define  $(\dot{g}/g) \equiv H_0 = \text{constant}$ . Then the (55) equation gives

$$\left[ \left(\frac{H_0}{f}\right)^2 - \left(\frac{f'}{f}\right)^2 \frac{1}{b_0^2} \right] + \frac{2\alpha}{M^2} \left[ \left(\frac{H_0}{f}\right)^2 - \left(\frac{f'}{f}\right)^2 \frac{1}{b_0^2} \right]^2 = \frac{\Lambda_b}{6M^3}. \tag{31}$$

After little algebra, we obtain

$$\left(\frac{f'}{b_0}\right)^2 = H_0^2 + k_{\pm}^2 f^2, \tag{32}$$

where the  $k_{\pm}^2$  is defined in Eq. (22). For the ‘-’ case with  $\alpha = 0$ , we arrive at the solution given in Ref. [4],  $k_{-}^2 = -\Lambda_b/6M^3$ . Note that  $k_{+}^2$  contains the cases of both positive and negative cosmological constants in the bulk and the  $k_{\pm}^2$  can take both positive and negative signs. Inflationary solutions were obtained for a flat bulk geometry and for an AdS bulk geometry [2,3,17], which can solve the hierarchy problem in the static limit. In our case, as one can see below, inflationary solutions exist also for a positive bulk cosmological constant.

For  $k_{\pm}^2 > 0$ , the solution consistent with the orbifold symmetry is

$$f = \frac{H_0}{k_{\pm}} \sinh(-k_{\pm} b_0 |y| + c_0). \tag{33}$$

The (00) or (ii) equations of Eq. (16) just give a boundary condition for the solution. Using the relation given in Eq. (23), one can find that the imposed conditions determine the extra dimension scale  $b_0$  and the integration constant  $c_0$  as follows,

$$k_{1,\mp} = \mp k_{\pm} \coth(c_0), \quad k_{2,\pm} = \pm k_{\pm} \coth\left(-\frac{1}{2} k b_0 + c_0\right) \tag{34}$$

where the  $k_i$  are defined in Eq. (25). Here, the solutions are valid only for  $k_{\pm} < |k_{1,\pm}| < |k_{2,\pm}|$  in case  $c_0 > \frac{1}{4} k b_0$  and  $k_{\pm} < |k_{2,\pm}| < |k_{1,\pm}|$  in case  $c_0 < \frac{1}{4} k b_0$ . One can also check

easily that the  $k_i$  tends to those of Ref. [4] in the limit of  $\alpha \rightarrow 0$  in the lower case (i.e., the ‘-’ solution). In general, inflation occurs if parameters  $\alpha$ ,  $\Lambda_b$ ,  $\Lambda_1$ , and  $\Lambda_2$  do not satisfy the two relations implied by Eq. (24). From the above relations the  $k_{1,\mp}(k_{2,\pm})$  diverges as  $k_{2,\pm}(k_{1,\mp}) \rightarrow \mp k_{\pm} \coth(\frac{1}{2}kb_0)$ .

Then the metric is

$$ds^2 = \left(\frac{H_0}{k_{\pm}}\right)^2 \sinh^2(-k_{\pm}b_0|y| + c_0)[-d\tau^2 + e^{2H_0\tau} \delta_{ij} dx^i dx^j] + b_0^2 dy^2. \tag{35}$$

To obtain the RS static solution with the warp factor in the visible brane, we should take  $H_0 \rightarrow 0$  and  $c_0 \rightarrow +\infty$  while keeping the ratio  $(H_0 e^{c_0})/(2k_{\pm}) \rightarrow 1$  fixed. Then we obtain the fine tuning condition  $k_{1,\mp} = -k_{2,\pm} = \mp k_{\pm}$  from Eq. (25), which is the same result as Eq. (24). Here one can see the possibility of the warp factored brane with the positive cosmological constant again.

After 4-dimensional coordinate transformation at a given  $y$  to make the 4-dimensional metric be in the form  $ds_4^2 = -dt^2 + e^{2H(y)t} \delta_{ij} dx^i dx^j$  [4], we get the Hubble parameter expressed in terms of the cosmological constant and the energy density,  $H_{\text{vis},\pm} = \sqrt{(k_{\text{vis},\pm})^2 - k_{\pm}^2}$ . Here  $k_{\text{vis},\pm}^2 = k_{\pm}^2$  for the static solutions and the two parameters corresponding to the ‘+’ and ‘-’ solutions at the visible brane,  $k_{\text{vis},\pm}$ , are given by

$$k_{\text{vis},\pm} = \frac{(\Lambda_2^{\pm} + \rho_{\text{vis}})}{6M^3 \sqrt{1 + (4\alpha \Lambda_b / 3M^5)}}, \tag{36}$$

where  $\Lambda_2^{\pm} \gg \rho_{\text{vis}}$ . Thus the Hubble parameter at B2 is given by [5]

$$H_{\text{vis},\pm}^2 = \frac{\rho_{\text{vis}}(\rho_{\text{vis}} + 2\Lambda_2^{\pm})}{36M^6(1 + 4\alpha \Lambda_b / 3M^5)} = \frac{\pm \rho_{\text{vis}}}{3M_{Pl}^2 \sqrt{1 + 4\alpha \Lambda_b / 3M^5}} \left[1 + \frac{\rho_{\text{vis}}}{2\Lambda_2^{\pm}}\right]. \tag{37}$$

The second equation above is derived with the use of Eqs. (24) and (27). With  $\rho_{\text{vis}} = 0$ , we obtain the previous static solution. But with  $\Lambda_2^- = \Lambda_b < 0$  where the original RS solution sits, there exists a possibility that  $\rho_{\text{vis}}(2\Lambda_2^- + \rho_{\text{vis}}) < 0$  at a sufficiently low temperature, and hence it is difficult to obtain a real Hubble parameter [12–16]. But with a positive  $\Lambda_2^+$ , there does not exist such a problem. This is possible for our ‘+’ solution for  $\alpha > 0$ . *Therefore, with the ‘+’ solution we can obtain a plausible Friedmann–Robertson–Walker universe after inflation ends.*

In Eq. (37), the  $\rho_{\text{vis}}^2$  term gives a correction to the conventional Friedmann equation but in the limit  $\Lambda_2^+ \rightarrow \infty$  and  $\alpha \Lambda_b / M^5 \rightarrow 0$  we recover the standard 4-D general relativistic result. The modified Friedmann equation leads to the modified inflation condition,

$$\frac{\ddot{a}}{a} = \frac{-1}{3M_{Pl}^2 \sqrt{1 + 4\alpha \Lambda_b / 3M^5}} \left( \frac{\rho_{\text{vis}}}{2} \left(1 + \frac{2\rho_{\text{vis}}}{\Lambda_2^+}\right) + \frac{3p_{\text{vis}}}{2} \left(1 + \frac{\rho_{\text{vis}}}{\Lambda_2^+}\right) \right) > 0 \tag{38}$$

or

$$p_{\text{vis}} < -\frac{\rho_{\text{vis}}}{3} \left[ \frac{\Lambda_2^+ + 2\rho_{\text{vis}}}{\Lambda_2^+ + \rho_{\text{vis}}} \right], \tag{39}$$

where the  $\rho_{\text{vis}}$  and  $p_{\text{vis}}$  satisfy the fluid equation,  $\dot{\rho}_{\text{vis}} + 3H(\rho_{\text{vis}} + p_{\text{vis}}) = 0$ .

Now let us consider the case that the only matter in the 4-D universe is a self interacting scalar field, inflaton  $\phi$ . Then the  $\rho_{\text{vis}}$  and  $p_{\text{vis}}$  are given by  $\rho_{\text{vis}} = \frac{1}{2}\dot{\phi}^2 + V(\phi)$  and  $p_{\text{vis}} = \frac{1}{2}\dot{\phi}^2 - V(\phi)$ , respectively, and the Eq. (39) becomes [19]

$$\dot{\phi}^2 - V(\phi) + \left[ \frac{\dot{\phi}^2 + 2V(\phi)}{8\Lambda_2^+} (5\dot{\phi}^2 - 2V(\phi)) \right] < 0, \tag{40}$$

which reduces to  $\dot{\phi}^2 < V$  when  $\dot{\phi}^2 + 2V \ll \Lambda_2^+$ . Assuming that the inflaton field rolls down to a true vacuum very slowly, the energy density is dominated by the potential  $V$  and the inflaton field evolution is strongly damped, which implies

$$H^2 \simeq \frac{V}{3M_{Pl}^2 \sqrt{1 + 4\alpha\Lambda_b/3M^5}} \left[ 1 + \frac{V}{2\Lambda_2^+} \right], \tag{41}$$

$$\dot{\phi} \simeq -\frac{V'}{3H}, \tag{42}$$

where we use ‘ $\simeq$ ’ to denote equality within the slow-roll approximation [18]. Our brane physics modifies also the e-folding number as follows;

$$N = \int_{t_i}^{t_f} dt H \simeq \frac{-1}{M_{Pl}^2 \sqrt{1 + 4\alpha\Lambda_b/3M^5}} \int_{\phi_i}^{\phi_f} d\phi \frac{V}{V'} \left[ 1 + \frac{V}{2\Lambda_2^+} \right]. \tag{43}$$

For a large  $\Lambda_2^+$  and small  $\alpha\Lambda_b/M^5$  we obtain the standard e-folding formula again.

Next, let us consider the corrections to the scalar and tensor density perturbations. The scalar density perturbation can be related to the curvature perturbation  $\zeta$  on uniform density hypersurfaces when modes re-enter the Hubble radius during the matter dominated era [18,19],

$$A_S^2 = \frac{4}{25} \langle \zeta^2 \rangle, \quad \zeta = \frac{H\delta\phi}{\dot{\phi}}, \tag{44}$$

where the scalar field fluctuation at Hubble crossing ( $k = aH$ ) are given by  $\langle \delta\phi^2 \rangle \simeq (H/2\pi)^2$ . Thus, using the slow-roll conditions Eqs. (41) and (42), the amplitude of scalar perturbations becomes

$$A_S^2 \simeq \frac{1}{75\pi^2} \left( \frac{1}{M_{Pl}^2 \sqrt{1 + 4\alpha\Lambda_b/3M^5}} \right)^3 \frac{V^3}{V'^2} \left[ 1 + \frac{V}{2\Lambda_2^+} \right]^3 \Big|_{k=aH}. \tag{45}$$

So the amplitude of scalar perturbations is increased relative to the standard result. Of course, we recover the standard one for a large  $\Lambda_2^+$  and small  $\alpha\Lambda_b/M^5$ .

The amplitude of tensor (gravitational wave) perturbation at Hubble crossing is given by [18,19]

$$A_T^2 = \frac{1}{50\pi^2} \left( \frac{H}{M_{Pl}} \right)^2 \Big|_{k=aH}. \tag{46}$$

In the slow-roll approximation, this yields

$$A_T^2 \simeq \frac{1}{150\pi^2} \left( \frac{1}{M_{Pl}^4 \sqrt{1 + 4\alpha \Lambda_b / 3M^5}} \right) V \left[ 1 + \frac{V}{2\Lambda_2^+} \right] \Big|_{k=aH} \tag{47}$$

which is increased by brane effects, but with a smaller factor than in the case the scalar perturbation. This tends to the standard form as  $\Lambda_2^+ \rightarrow \infty$  and  $\alpha \Lambda_b / M^5 \rightarrow 0$ .

In the previous section, we discussed the phenomenologically favored values of  $\Lambda_b / M^5$ ,  $\Lambda_2^+$  and  $\alpha$ . The result was  $k_+ = M \approx M_P$ ,  $\Lambda_b / M^5 = 12\alpha - 6$ ,  $\Lambda_2^+ = 6M^4 |4\alpha - 1|$  and  $\alpha > 1/4$ . If we take  $\Lambda_b = 0$  or  $\alpha = 1/2$ , we could recover the existing results in cosmology.

### 5. Other solutions

If we take a non-separable ansatz for the metric tensor,

$$n(\tau, y) = a(\tau, y) = \frac{1}{\tau f(y) + g_0}, \quad b(\tau, y) = k_{\pm} b_0 \tau a(\tau, y), \tag{48}$$

the solution is

$$ds^2 = \frac{-d\tau^2 + \delta_{ij} dx^i dx^j + (k_{\pm} b_0 \tau)^2 dy^2}{[k_{\pm} \tau \sinh(k_{\pm} b_0 |y| + c_0) + g_0]^2}, \tag{49}$$

where  $b_0$  and  $c_0$  are constants and determined by the boundary wall’s conditions.

$$c_0 = \cosh^{-1} \left( \frac{\mp k_{1,\mp}}{k_{\pm}} \right),$$

$$k_{\pm} b_0 = 2 \left[ \cosh^{-1} \left( \frac{\pm k_{2,\pm}}{k_{\pm}} \right) - \cosh^{-1} \left( \frac{\mp k_{1,\mp}}{k_{\pm}} \right) \right]. \tag{50}$$

Actually this has the same form as in Ref. [4] except that the cosmological constants are, as before, given by Eq. (22) and Eq. (25). The constant  $g_0$  remains as a free parameter and its physical role is discussed in [4]. Setting  $g_0 = 0$ , we get the solution given in Ref. [3] and  $b_0$  becomes independent of  $\tau$ .

If  $16\alpha + 5\beta + 4\gamma = 0$  is satisfied but the Gauss–Bonnet conditions are not, the inflationary solutions in given Eq. (35) (with a separable metric) and Eq. (49) (with a nonseparable metric) are still valid except the substitution  $4\alpha \rightarrow 8\alpha + \beta$  in our solutions Eq. (22), etc., even though there exist higher time derivatives in the equations of motion.

Taking a different ansatz,  $n(\tau, y) = a(\tau, y) = b(\tau, y)$ , which is conformally flat, the solution is given by

$$ds^2 = \frac{-d\tau^2 + \delta_{ij} dx^i dx^j + dy^2}{[-(k_{1,\mp}^2 - k_{\pm}^2)^{1/2} \tau + k_{1,\mp} |y| + c_0]^2}, \tag{51}$$

where  $c_0$  is a constant. This metric describes inflation in both the spatial dimensions and the extra dimension. The  $k$ ’s are given by Eqs. (22) and (25) as before. For non-Gauss–Bonnet case with  $16\alpha + 5\beta + 4\gamma = 0$ , the solution is still valid also except for the substitution  $4\alpha \rightarrow 8\alpha + \beta$  because it is conformally flat.

### 6. Metric perturbation near the RS background geometry

It is also of interest to study the gravitational interaction with the RS background. In fact, Randall and Sundrum demonstrated that the Newton’s force law does not imply only four non-compact dimensions in the presence of a non-factorizable background geometry [20–26]. The example they studied is the case of a *single* 3-brane embedded in non-compact five dimension. In this section, let us reconsider the case with the Gauss–Bonnet interaction.

The graviton is a linearized tensor fluctuation near the background geometry,

$$g_{\mu\nu} = e^{-2k_{\pm}|y|}\eta_{\mu\nu} + h_{\mu\nu}(x, y), \tag{52}$$

where the  $x$  indicates the coordinate for the 4-dimensional space embedded in the 5-dimensional bulk. Since we are interested in the 4-dimensional graviton only, which is the longitudinal component of the metric fluctuation, we set  $h_{\mu 5} = h_{5\mu} = h_{55} = 0$ . Inserting Eq. (52) into Eq. (7) and taking only the linear terms in  $h_{\mu\nu}$ , we obtain

$$\begin{aligned} \underline{G}_{\mu\nu} + \underline{X}_{\mu\nu} = & \left[ -\frac{1}{2}\left(1 - \frac{4\alpha k_{\pm}^2}{M^2}\right)\partial_y^2 + \frac{8\alpha k_{\pm}^2}{M^2}\delta(y)\operatorname{sgn}(y)\partial_y \right. \\ & - \frac{\square_4}{2}e^{2k_{\pm}|y|}\left(1 - \frac{4\alpha k_{\pm}^2}{M^2} + \frac{8\alpha k_{\pm}}{M^2}\delta(y)\right) \\ & \left. - 8k_{\pm}\delta(y)\left(1 - \frac{6\alpha k_{\pm}^2}{M^2}\right) + 4k_{\pm}^2\left(2 - \frac{5\alpha k_{\pm}^2}{M^2}\right) \right] h_{\mu\nu}(x, y) \end{aligned} \tag{53}$$

and

$$\begin{aligned} \underline{T}_{\mu\nu} = & -\frac{1}{M^3}\left[\Lambda_b + \Lambda_1^{\mp}\delta(y)\right]h_{\mu\nu}(x, y) \\ = & \left[6k_{\pm}^2\left(1 - \frac{2\alpha k_{\pm}^2}{M^2}\right) - 6k_{\pm}\delta(y)\left(1 - \frac{4\alpha k_{\pm}^2}{M^2}\right) \right] h_{\mu\nu}(x, y), \end{aligned} \tag{54}$$

where the underlined quantities denote the linear part in  $h_{\mu\nu}$  in the full expressions and  $\square_4$  is  $\eta^{\mu\nu}\partial_{\mu}\partial_{\nu}$ . Here we set  $b_0 = 1$  for simplicity. Eq. (54) is obtained by the use of Eq. (22) and Eq. (24). Here we choose the traceless transverse gauge conditions,  $\partial^{\mu}h_{\mu\nu} = h_{\mu}^{\mu} = 0$  [20–26]. Under this gauge condition all components of  $h_{\mu\nu}$  satisfy the same equation of motion, and hence we will omit the  $\mu\nu$  indices below. Here we note again that in the Gauss–Bonnet case the unwanted higher derivative terms disappear in the linear approximation as in the background case.

To perform a Kaluza–Klein reduction down to 4-dimension and get an understanding of all modes that appear in the assumed 4D effective theory, we separate the variables;  $h(x, y) = \psi(y)e^{ip\cdot x}$ , where the  $p^{\mu}$  is a 4-dimensional momentum. Since the 4-dimensional mass  $m^2$  of the KK excitation is  $p^2 = -m^2$ , Eq. (53) = Eq. (54) leads to

$$\begin{aligned} & \left[ -\frac{1}{2}\left(1 - \frac{4\alpha k_{\pm}^2}{M^2}\right)\partial_y^2 + 2k_{\pm}^2\left(1 - \frac{4\alpha k_{\pm}^2}{M^2}\right) \right. \\ & \left. - 2k_{\pm}\delta(y)\left(-\frac{4\alpha k_{\pm}}{M^2}\operatorname{sgn}(y)\partial_y + 1 - \frac{12\alpha k_{\pm}^2}{M^2}\right) \right] \psi(y) \end{aligned}$$

$$= \frac{m^2}{2} e^{2k_{\pm}|y|} \left[ 1 - \frac{4\alpha k_{\pm}^2}{M^2} + \frac{8\alpha k_{\pm}}{M^2} \delta(y) \right] \psi(y). \tag{55}$$

Note that the above equation remains the same regardless of the sign of the brane’s cosmological constant.

In the bulk, we can easily check that the Gauss–Bonnet interaction does not modify the equation of motion because all terms have the exactly same common factor  $(1 - 4\alpha k_{\pm}^2/M^2)$  neglecting the Dirac delta functions. Therefore, we obtain *the same eigenfunctions and eigenvalues as in the RS’s solutions* [20–23] except for the definition of  $k_{\pm}$ . On the other hand, the Dirac delta functions gives the boundary condition, so the Gauss–Bonnet interaction modifies only the boundary condition in the order of magnitude of  $\alpha k_{\pm}^2/M^2$ . Thus, one can imagine that there exists a possibility that the massless mode has not only an exponentially decaying component but also an exponentially growing one at order  $\alpha k_{\pm}^2/M^2$ . Note that the coefficient of the growing mode is exactly zero in the absence of the higher curvature terms in the action [20–23]. We have found, however, that the exponentially growing mode does not appear even in the presence of the Gauss–Bonnet interaction.

To follow the RS process, let us make change of variables;  $z \equiv \text{sgn}(y)(e^{k_{\pm}|y|} - 1)/k_{\pm}$ ,  $\hat{\psi}(z) \equiv \psi(y)e^{k_{\pm}|y|/2}$  and  $\hat{h}(x, z) \equiv h(x, y)e^{k_{\pm}|y|/2}$ . Then, Eq. (55) reads

$$\begin{aligned} & \left[ -\frac{1}{2}\partial_z^2 + \frac{15k_{\pm}^2}{8(k_{\pm}|z| + 1)^2} + \frac{k_{\pm}}{2}\delta(z) \left( \frac{2B}{Ak_{\pm}} \text{sgn}(z)\partial_z - \frac{3C}{A} \right) \right] \hat{\psi}(z) \\ &= \frac{m^2}{2} \left[ 1 + \frac{B}{Ak_{\pm}}\delta(z) \right] \hat{\psi}(z), \end{aligned} \tag{56}$$

where  $A = 1 - 4\alpha k_{\pm}^2/M^2$ ,  $B = 8\alpha k_{\pm}^2/M^2$  and  $C = 1 - 12\alpha k_{\pm}^2/M^2$ . Note that  $A = B + C$ .

For  $m^2 = 0$ , the eigenfunctions in the bulk satisfying the orbifold symmetry are  $(k_{\pm}|z| + 1)^{-3/2}/k_{\pm}$  ( $= \exp(-\frac{3}{2}k_{\pm}|y|)/k_{\pm}$ ) and  $(k_{\pm}|z| + 1)^{5/2}/k_{\pm}$  ( $= \exp(\frac{5}{2}k_{\pm}|y|)/k_{\pm}$ ). Therefore, the solution is a linear combination of them

$$a \frac{(k_{\pm}|z| + 1)^{-3/2}}{k_{\pm}} + b \frac{(k_{\pm}|z| + 1)^{5/2}}{k_{\pm}}. \tag{57}$$

To satisfy the boundary condition at  $z = 0$ , let us insert Eq. (57) into Eq. (56) and assemble the coefficients of the Dirac delta functions. Then, we obtain

$$a(-A + B + C) + b(A - B + C) = 0, \tag{58}$$

where  $A = B + C$ . Thus,  $b = 0$ , i.e., the massless graviton is confined on the brane and the Newton’s force law on the brane holds good *even under the non-compact extra dimension*. Particularly, we note that the result is not changed even though the brane’s cosmological constant is negative.

As

$$h_{\mu\nu}(x, y) \propto e^{-\frac{k_{\pm}}{2}|y|} \hat{\psi}(z) e^{ipx} \propto e^{-2k_{\pm}|y|} e^{ipx},$$

the fluctuation near the background metric can be written down as

$$g_{\mu\nu}^0 = e^{-2k_{\pm}|y|}(\eta_{\mu\nu} + \epsilon_{\mu\nu}e^{ipx}), \tag{59}$$

where the superscript 0 denotes massless fluctuation and the  $\epsilon_{\mu\nu}$  is a polarization tensor of the graviton wave function and from which we can see that the massless mode fluctuates only in the longitudinal direction to the brane. Besides, from Eq. (55) we can get the 4-dimensional linearized Einstein equation in the Minkowski space,

$$-\frac{\square_4}{2}\epsilon_{\mu\nu}e^{ipx} = 0, \tag{60}$$

which is of course true also after 4-dimensional coordinate transformation at a given  $y$  to make the 4-dimensional background metric be in the form  $ds_4^2 = \eta_{\mu\nu} dx^\mu dx^\nu$ . Note that *the above result is not affected by the Gauss–Bonnet correction.*

We usually worry about the instability of anti-de Sitter space due to excitations of ghost particles [9]. In our case, we still have such a problem since the sign of the kinetic term for the  $k_+$  background is opposite to that of the case without the Gauss–Bonnet term, viz.

$$-\frac{1}{2}\left(1 - \frac{4\alpha k_{\pm}^2}{M^2}\right)(e^{2k_{\pm}|y|\square_4 + \partial_y^2})h_{\mu\nu} = \pm\frac{1}{2}\sqrt{1 + \frac{4\alpha\Lambda_b}{3M^5}}(e^{2k_{\pm}|y|\square_4 + \partial_y^2})h_{\mu\nu}. \tag{61}$$

However, the equation of motion itself describes the same behavior of gravity localization on the hidden sector brane (B1) because the brane cosmological constant contributing to energy momentum tensor changes its sign as well. Thus, we have no ghost problem as far as the brane cosmological constant is concerned as energy density. But we cannot regard the brane with negative cosmological constant at  $z = 0$  as our universe due to the later cosmological problem that was discussed in Section 4.

For  $m^2 > 0$ , the solutions for the above equation of motion in the bulk are

$$a(|z| + 1/k_{\pm})^{1/2}J_2(m(|z| + 1/k_{\pm})) + b(|z| + 1/k_{\pm})^{1/2}Y_2(m(|z| + 1/k_{\pm})), \tag{62}$$

which is the same solution as in the RS case except for the definition of  $k_{\pm}$ . The imposed boundary condition at  $z = 0$  fixes the ratio of  $a$  and  $b$ ,

$$\frac{a}{b} = \frac{4k_{\pm}^2}{\pi m^2} \frac{1 - 4\alpha k_{\pm}^2/M^2 + 2\alpha m^2/M^2}{1 - 12\alpha k_{\pm}^2/M^2 + \alpha m^2/M^2}. \tag{63}$$

In this case, the Gauss–Bonnet interaction modifies Newton’s non-relativistic gravitational potential *through the KK states* as follows,

$$\begin{aligned} V &\sim G_N \frac{m_1 m_2}{r} + \pi \int_0^\infty \frac{dm}{k_{\pm}} \frac{m}{k_{\pm}} G_N \frac{m_1 m_2 e^{-mr}}{r} \left[ \frac{1 - 12\alpha k_{\pm}^2/M^2 + \alpha m^2/M^2}{1 - 4\alpha k_{\pm}^2/M^2 + 2\alpha m^2/M^2} \right]^2 \\ &\sim G_N \frac{m_1 m_2}{r} \left[ 1 + \frac{\pi}{(k_{\pm} r)^2} \left( \frac{1 - 12\alpha k_{\pm}^2/M^2}{1 - 4\alpha k_{\pm}^2/M^2} \right)^2 \right]. \end{aligned} \tag{64}$$

The above result is obtained through the RS technique given in Ref. [20–23]. Of course, the potential given above is not ruled out yet [16].

For the case of two branes and bulk with  $S^1/Z_2$  symmetry, the Eq. (55) is modified into

$$\begin{aligned}
 & \left[ -\frac{1}{2} \left( 1 - \frac{4\alpha k_{\pm}^2}{M^2} \right) \partial_y^2 + 2k_{\pm}^2 \left( 1 - \frac{4\alpha k_{\pm}^2}{M^2} \right) \right. \\
 & \quad - 2k_{\pm} \delta(y) \left( -\frac{4\alpha k_{\pm}}{M^2} \operatorname{sgn}(y) \partial_y + 1 - \frac{12\alpha k_{\pm}^2}{M^2} \right) \\
 & \quad \left. + 2k_{\pm} \delta \left( y - \frac{1}{2} \right) \left( -\frac{4\alpha k_{\pm}}{M^2} \operatorname{sgn}(y) \partial_y + 1 - \frac{12\alpha k_{\pm}^2}{M^2} \right) \right] \psi(y) \\
 & = \frac{m^2}{2} e^{2k_{\pm}|y|} \left[ 1 - \frac{4\alpha k_{\pm}^2}{M^2} + \frac{8\alpha k_{\pm}}{M^2} \delta(y) - \frac{8\alpha k_{\pm}}{M^2} \delta \left( y - \frac{1}{2} \right) \right] \psi(y), \tag{65}
 \end{aligned}$$

where the  $\operatorname{sgn}(y) \equiv |y|' = 2(\theta(y) - \theta(y - \frac{1}{2})) - 1$  and we can check the solution Eq. (57) with  $b = 0$  satisfies the above equation (65) regardless of the length scale by use of the relation  $h_{\mu\nu} = e^{-\frac{k_{\pm}}{2}|y|} \hat{\psi}(z)$  and Eq. (23). Therefore, from Eq. (60) the non-relativistic Newtonian gravity could be restored at the visible sector brane for sufficiently small interval length  $b_0 \ll r$ .

### 7. Conclusion

We studied various static and inflationary solutions in the Randall–Sundrum framework with the Gauss–Bonnet term added to the standard Hilbert action. It has been argued that in this RS framework the Gauss–Bonnet term is the only acceptable curvature square term. Then there exist additional coupling  $\alpha$ , the coefficient of the Gauss–Bonnet term. Depending on various values of  $\alpha$ , there exist static solutions and also the inflationary solutions. In particular, there exist solutions for a positive visible sector tension  $\Lambda_2$  for  $\alpha > 0$ , which makes it possible to transit to a standard Big Bang cosmology after inflation.

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